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Research Paper

Investigation into Statically Non-linear Behavior of Tapered Reinforced Concrete Beams Using Experimental and Atena-Gid Methods

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ABSTRACT*: Structural members like beams exhibit different behaviour in response to increasing load range (either in form of static or dynamic load) applied on them. The manner by which they respond to loading can be classified into linear and nonlinear. In this study, the linear and nonlinear behaviour of reinforced concrete tapered beams were investigated using finite element analysis method and experimentation. The investigation carried out include the effect of taper on the strength and deflection of the beams. Three beam models were produced; one rectangular beam shape (control model) , two tapered beams with slope 1:12 and 1:24 respectively. The three beams were reinforced with 8mm high yield steel bar (Y8) .The beams were first analysed using finite element method in Atena-Gid computer software, followed by experimental investigation in the laboratory. The results showed that tapered beam with 1:24 slope had lower strength than the control beam in both experimental and Atena-Gid analysis. The result also showed that the higher the slope (taper) the lower the strength. Furthermore, the tapered reinforced concrete beams suffered more deflection than the control concrete beam at nonlinear phase, whereas beam with slope 1:24 exhibited more deflection than the beam with 1:12 slope. However, on the linear phase or region, the strength and deflection results of the experimental beams, with respect to their shape, were relatively similar to that of Atena-Gid analysis.*

KEYWORDS*: Tapered Beams, Atena-Gid Computer Software, Linear and Nonlinear Finite Element.*

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I. INTRODUCTION.

Structural members exhibit different behaviours in response to increasing load range (either in form of static or dynamic load) applied on them. The manner by which they respond to loading are classified into linear and or nonlinear. In structural analysis, members are said to show linear behaviour if linear relationship holds between the force applied on them and displacement or deformation (response to the applied load). Thus, displacements are small such that members have the ability to return to their former form when the loads acting on them are withdrawn. This can be seen or represented as a straight line in Hook's stress –strain graph in fig.1. In nonlinear behaviour, unlike linear behaviour, nonlinear relation holds between applied load and displacement. Here, the displacements are large and the members cannot return to their former state hence the member is said to be deformed. This behaviour or analysis in such form can be seen as plastic or deformed domain in Fig.1.

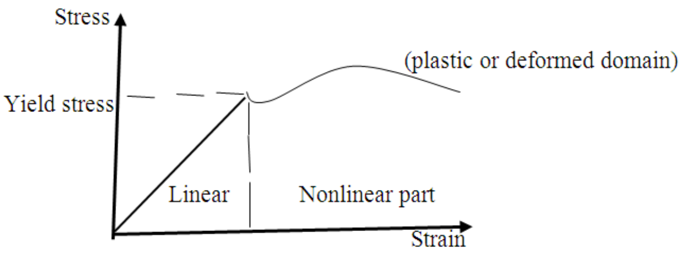


Figure 1: Hook's stress-strain curve

The nonlinear behaviour accounts for nonlinear effects. These nonlinear effects according to [1, 2] are categorized into geometric nonlinearity, material nonlinearity and boundary condition nonlinearity. At the nonlinear stage the member's flexural stiffness changes and the material yields as a result of emergence of cracks in case of concrete which leads to failure or deformation of the member as loading increases. [3] noted that the hypothesis of linear elastic behaviour or analysis of flexural members such as beams has in many cases underestimated the internal forces in the less stressed section of the member or overestimated the internal forces in the more stressed section leading to structures that are less safe or less economical.

The performance of cold formed steel (CFS) of Z section beam under bending and subjected to uniformly distributed load applied at the shear centre of the section was investigated by [4]. He examined the accuracy of the linear and nonlinear solution of the beam by investigating the deformation of the beam with a finite element analysis (FEA) model with ANSYS software program. The study confirms that bending-torsion resulted in the reduction of stiffness contributed additional evidence that suggested that nonlinear analysis played a significant role in deflection and performance of CFS beams.

A numerical simulation analysis of lumped dissipation reinforced concrete continuous beam models was presented by [3]. The study considered the effect of material nonlinearity to access the amount of the reinforcements due to the responses of models. From their analysis, they found out that the nonlinear analysis showed 15% reduction, for the reinforcement needed to resist the ultimate limit state design load at the end support and under the maximum ultimate load of 60kN/m the nonlinear analysis produced the displacement of 5cm compared to 0.5cm produced by linear analysis.

A numerical analysis of two beams with thickness 5mm and 10mm was carried out by [5]. The length and width of the beams were 700mm and 50mm respectively. FEM software SAP 2000 was used to conduct the linear and nonlinear behaviour of the beams. They compared the results of the two beams considering only geometrical nonlinear behaviour of the beam as the loading increased. From their analysis, the beam with lesser thickness tend to deform more under increased load due to its geometric nonlinearity than the thicker beam.

A comparative study of reinforced concrete beam bending failure analysis and experimental test was carried out by [6]. The analysis of the beam was carried out using finite element analysis software ABAQUS. The geometric nonlinearity, material nonlinearity and boundary condition nonlinearity were considered in the analysis. From their study, they observed that before the 8kN of load, the beam was in elastic stage with high stiffness and strength. At the plastic stage, mid-span deflection at deformation load of 24kN was 10.521mm in ABAQUS while it was 12.795mm from the experiment.

An analytical study of a simulated Hot-rolled H-section beam was conducted by [7] using Discrete Element Method (DEM). Their study considered the effects of material and geometric nonlinearity caused by the application of axial load after yield. The results of their analysis was compared with numerical FEM solution in ABAQUS program. They observed that the maximum stress obtained by DEM was 316.94 MPa and that of the FEM analysis was 296.66MPa, while the maximum displacement obtained by PFC was 23.60mm less than that of FEM analysis obtained at 23.91mm.

A nonlinear analysis of composite steel-concrete beam using finite element computer program ANSYS was presented by [8]. They concluded that the FE. Solutions or results were in good agreement with the experimental results throughout the entire range of behaviour and the percentage discrepancy (error ratio) of the F.E. model corresponding to the tested beam was only 2.67%.

A simple laboratory experiment of a cantilever beam in order to introduce the concept of geometric non-linearity in mechanics and strength of materials was investigated by [9] . The experimental analysis of the beam was compared with a numerical analysis of the system using the ANSYS program, a finite element package. From the two analysis, they observed that the deflections calculated under concentrated load coincided linearly with the experimentally measured deflections only when the load is zero. Whereas for all other applied loads the behaviour of the beam is clearly nonlinear.

In this study the behavior (i.e linear and nonlinear analysis) of non-uniform cross-sectional beams (tapered reinforced concrete beam) was investigated and compared with rectangular beam. The slope or the taper of the beams adopted were 1:12 and 1:24. The reinforcement adopted for the beams was high yield steel bar(s) of 8mm (known as Y8).The analysis of the beams was carried out in Atena-Gid finite element software and in the laboratory. The objectives of this study are; to investigate the effects of taper on the strength and deflection of the beam, and to compare the results obtain from FEA with laboratory results.

II. TAPERED BEAM STIFFNESS (EI) EVALUATION

Tapered beams are sometimes referred to as non-prismatic beams or non-uniform beams because of their variable or non-constant cross section. The flexural stiffness EI and the cross sectional area are not constant throughout the length of the beams, unlike prismatic beams. These type of beams are used because of the need for better distribution of strength, reduction in weight or mass and saving in material cost as a result of reduce dead-load of the beam [10]. They are commonly found on bridges, cantilever beams and structural roofs, as in Fig.2.

Figure 2: Tapered R.C beam in Church Structure

III. EVALUATION OF EI FOR TAPERED BEAM

Figures 3 and 4 show beams with constant cross section (prismatic) and non-constant cross sectional area (non-prismatic beam) respectively.

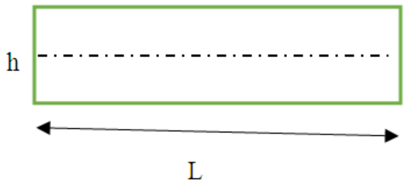


Figure 3: Prismatic beam

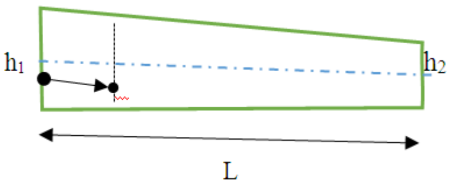


Figure 4: Non-prismatic beam

For a prismatic beam, the EI (flexural rigidity) is constant throughout the beam section.
Where E = Young modulus (which defines the material of the beam) and

I (the second moment area) = $\frac{bh^3}{12}$ (1)

Where, h and b are the height (or depth) and breathe of the beam respectively.
For non-uniform beam in Fig.4,
The following relation is used to determine the depth or height at any point of the beam.

$hx = \left(\frac{h1 - h_2}{L}\right) X + h_2$ (2)

Where, hx is the height varied along the length of the beam
h_1 and h_2 is the height of the beam at the largest cross section and smallest cross section respectively

$X = L - x$ (3)

Where, X is the point located along the length of the beam
And x is the distance measured from the near end to point X
Hence, substituting Eq. (3) into Eq. (2)

$hx = \left(\frac{h1 - h_2}{L}\right) (L-x) + h_2$ (4)

Therefore, the second moment area (I) for the tapered beam(s) is obtained by replacing h with hx in Eq. (5)

Thus, $I = \frac{b(hx)^3}{12}$ (5)

IV. LINEAR FORMATION IN ATENA-GID

According to [11] the formation of nonlinear equations in Atena are implemented by first solving a set(s) of linear algebraic equation using lower and upper triangular matrix solver in the form of Eq.6

$[F] = [k] * [d]$ (6)

In which [F] is the applied load or force vector, [k] is the global stiffness matrix which depends on the geometry, material properties and restraints and [d] stands for vector of unknown variables (this could the displacement, moment or strain).

The stiffness matrix [k] is given by, $[k] = \begin{bmatrix} EA/L & -EA/L \\ -EA/L & EA/L \end{bmatrix}$ (7)

Where, E, A, and L represent elastic modulus, cross sectional area and length of the beam.

V. NONLINEAR FORMATION OR SOLUTION IN ATENA-GID

According to [11] the Atena-Gid software uses full Newton Raphson method to converge the solution of nonlinear finite element analysis. This method uses the concept of increment step by step analysis to obtain the set of nonlinear equations as expressed in Eq.(8). Thus iterative solver method is most efficiently adopted for nonlinear formation.

$$K(P)\,\Delta P = q - f(P) \qquad (8)$$

Where,

q represent vector of total applied joint load,

$f(P)$ represent the vector of internal joint forces

ΔP represent deformation increment due to loading increment and

$K(P)$ represent stiffness matrix, relating to loading increment to deformation.

For repetitive number of iteration for a solution to converge, the set of (i^{th}) iterative equation describing the structural behaviour of a member under load increment is given in Eq. (9)

$$K(P_{i-1})\,\Delta P_i = q - f(P_{i-1}). \qquad (9)$$

VI. RESEARCH METHODOLOGY

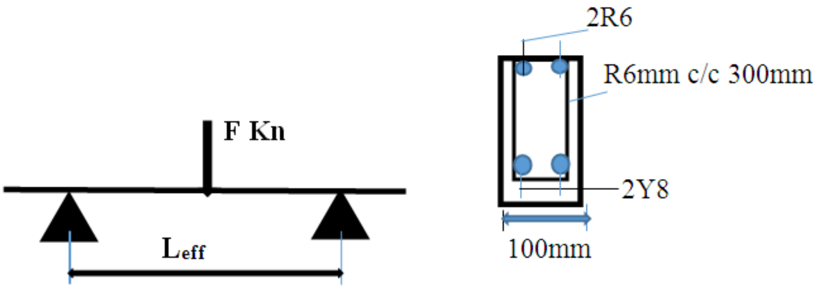
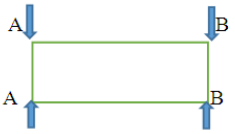
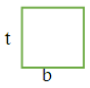
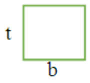
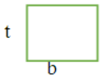
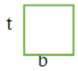


Figure 5: Simply supported beam and cross section of beam reinforcement

The experimental and the computer analysis procedures were adopted in the study of the beams. The experimental procedures consist of casting and testing to failure of the reinforced beams using the universal testing machine UTM while the computer procedures consist of the FE modelling and analyses of the beams.The length of the beams were 600mm, the effective span L_{eff} is 320mm (the allowable span the UTM can support). The beam(s) breath (b) adopted for the study was 100mm. The reinforcements arrangement in Fig.5, were adopted in the study due to the size of the beam and the strength of the available UTM. Cross section of the beams are shown in Table 1.

Table 1: End cross section of the beams

Geometry (L = 600mm)	Cross-section Section A-A	Cross section Section B-B	Slope of beam samples
 Rectangular beam	 150 x 100	 150 x 100	nil
 Tapered beam Tapered 'A' beams (category Tapered 'B' beams (category	 150 x 100 150 x 100	 125 x 100 100 x 100	1:12 1:24

Experiment: The concrete mix design ratio of 1:1:3 with water-cement ratio of 0.6 was adopted. This gave the concrete strength of 31.39N/mm^2 after 28days curing. This concrete strength was adopted in the concrete modelling of the beams in the computer program.

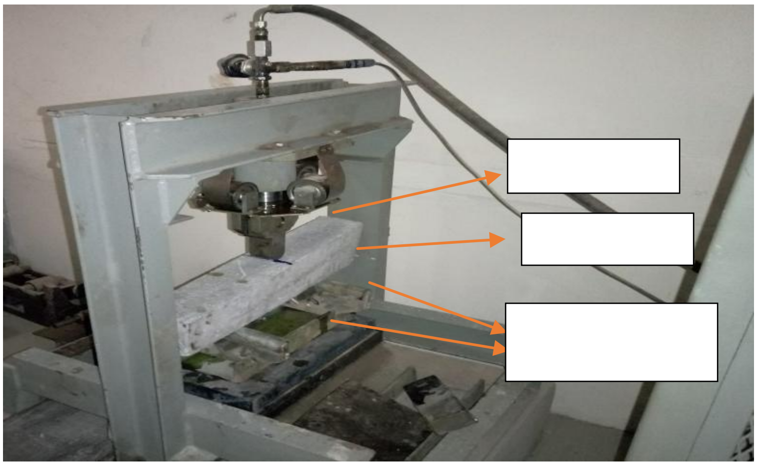


Figure 6: Set-up for the testing of the beam

The deflection for the experimental beams was computed using theoretical formula shown in Eq.10. The formula is based on the assumption that the stiffness rigidity (EI) is constant as loads increases thus the beam behaves in a linear manner. The theoretical formula for the deflection of simply supported beam is given as

$$D_{eff} = \frac{F(L_{eff})^3}{48EI} \qquad (10)$$

Where F, L_{eff}, and EI are load, effective length and beam rigidity stiffness respectively.

Atena- Gid FEA: This FE computer analysis software has two interface Gid interface (where the modelling is carried out) and Atena studio (where the analysis and results are executed). The program consists of three important steps namely; pre-processing (involve modelling of the studied beams, assigning of materials parameters and the support or boundary conditions), analysis and finally the post processing or the results.

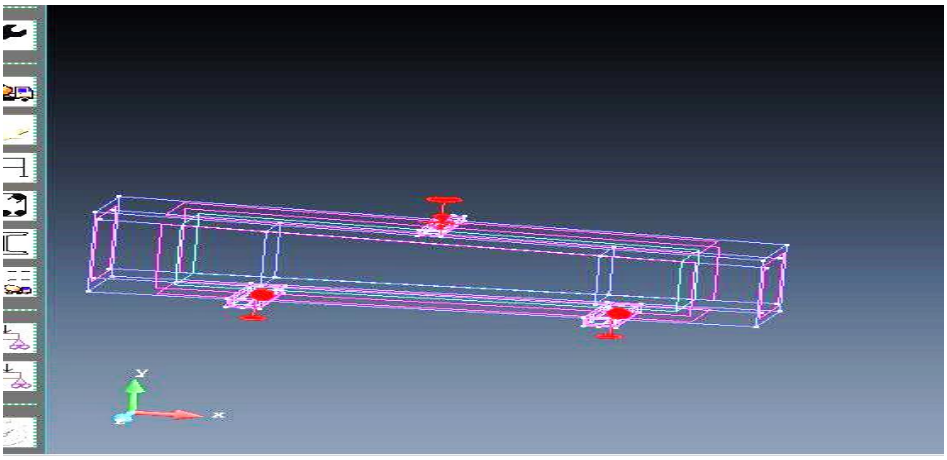


Figure 7: Atena-Gid model diagram

Material parameters of the beam: The modelled structural members (beams) in Atena-Gid consist of three entities or layers namely; loading plate, concrete beam and the reinforcements. Thus, material for these entities were assigned using the following command ‘ **Data|Material.**

Table 2: Material parameters for the reinforced concrete beams model

Concrete properties	**Reinforcement properties**
Strength value - 31.39 Mpa (31.39N/mm^2)	Yield strength - 410 Mpa
Strength type - cylindrical mean	Young modulus (E) - 2.0E +5 Mpa
Young modulus (E) - 31007.02 Mpa	Poisson ratio (v) - 0.3
Poisson ratio (v) - 0.2	Number of multilinear - 2
	Profile (bar size) - , 8mm (high yield strength)

Mesh generation: The mesh for the studied beams is generated through the command ' **mesh|structured|volume'** (selected for the beam). The element type assigned to the beam was ''hexahedra''. The reinforcements and stirrups were meshed as line elements because Atena-Gid denotes lines as reinforcement in solid structures.

Table 3: Element size, nodes and total number of elements assigned to the studied beams

Mesh generation	Rectangular beam	Tapered beam (1:12)	Tapered beam (1:24)
Element size	0.09	0.09	0.09
Nodes	578	558	554
Number of elements	293	293	293

VII. RESULT AND ANALYSIS

Failure Load

Table 4 shows the failure strength relationship between experimental beams and Atena-Gid modelled beams. In the table, the failure load of beam 01 from the experiment was 1041.2kN while it was 816 kN in the Atena-Gid analysis. For beam 02, the failure load obtained from the experiment was 628kN while Atena-Gid gave 728kN. For beam 03, the failure load obtained from the experiment was 435.19kN while it was 641 kN in Atena-Gid.

Moreover, in Table 4, a clear comparison of the experimentally computed failure strength (EFS) and those computed from Atena-Gid analysis (FEA) for tapered and rectangular reinforced concrete beams. It can be seen that (FEA/EFS) ratios vary from 0.78 to 1.47. Thus, it can be said that the result obtained by FE models have close relationship with those of experimental analysis.

Table 4: Failure strength (stress) relationship for experimental beams and Atena-Gid modelled beams.

Beam sample and beam slope	Cross sectional area at mid-span (A) mm	Experiment Failure load KN	Atena-Gid Failure load KN	Failure strength (FS) N/mm^2		Ratio of FEA to EFS
				Experiment (EFS)	Atena-Gid (FEA)	
01	15000	1041.2	816	69.41	54.40	0.78
02 (1:12)	13750	628.46	728	45.71	52.95	1.16
03 (1:24)	12500	435.19	641	34.82	51.28	1.47

The pictorial view of the failure strength comparison between experimental beams and Atena-Gid modelled beams are shown in a Fig. 8.

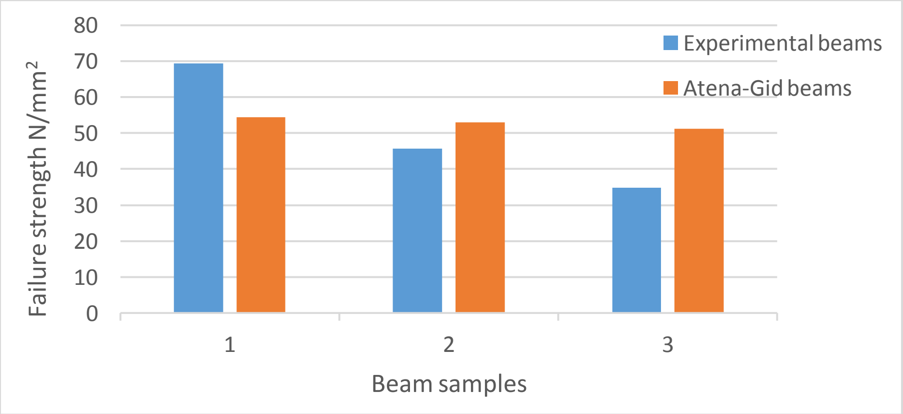
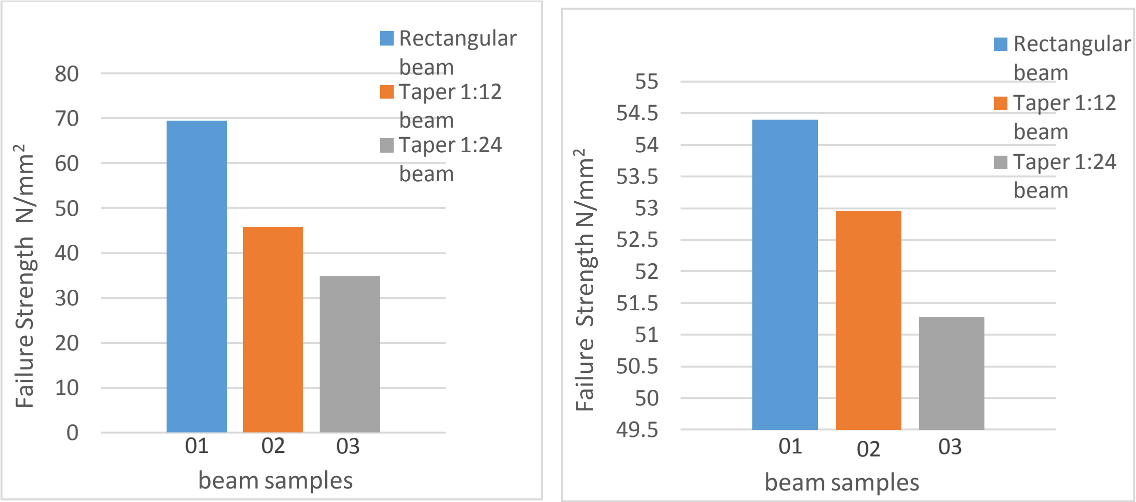


Figure 8: Failure stress (strength) comparison of experimental beams and Atena-Gid beams

Effect of Taper on the Strength of the Beams

Figure 9, shows that rectangular RC beams, have higher strength and can withstand loads more than tapered RC beams whereas, taper 1:12 beams had higher strength and withstood load more than taper 1:24 beams.



(a) **(b)**

Figure 9: Effect of taper on the failure strength for **(a)** Experimental beams, **(b)** Atena – Gid FEA modelled beams

Moreover, it was observed from the experimental approach, in Fig.9(a), the failure strength variation between RC rectangular beam and RC tapered beam of slope 1:12 showed that the failure strength of taper 1:12 beam was 25.2% lower than rectangular beam. Tapered RC beam with slope 1:24 had failure strength variation 44.7% lower than RC rectangular beam while strength variation between slope 1:12 and slope 1:24 beams showed that 1:24 tapered beam was 30% lower.

From the Atena-Gid (FE) analysis approach in Fig.9 (b), the failure strength variation between RC rectangular beam and RC tapered beam showed that taper 1:12 beam was 14.2% lower than the rectangular beam while taper 1:24 beam was 23.4% lower. The strength variation between the tapered beams showed that taper 1:24 beam was 10.8% lower than taper 1:12 beam.

Effect of Taper on the Deflection of the Beams

Figure 10 shows that beams with higher taper are more likely to deflect under less applied load than non-taper or rectangular beam. This is clearly observed at the non-linear region where the studied beam models suffered large deflection. For instance, RC beam 03 (with taper 1:24) gave deflection of 11.1mm at failure load of 641kN, while RC beam 02 (slope 1:12) and rectangular RC beam 01 had deflections of 6.17mm and 4.45mm respectively at same load. It can be said here that beam with taper 1:24 showed deflection 59.9 % higher than rectangular beam while beam with taper 1:12 showed deflection 27.9% higher than rectangular beam.

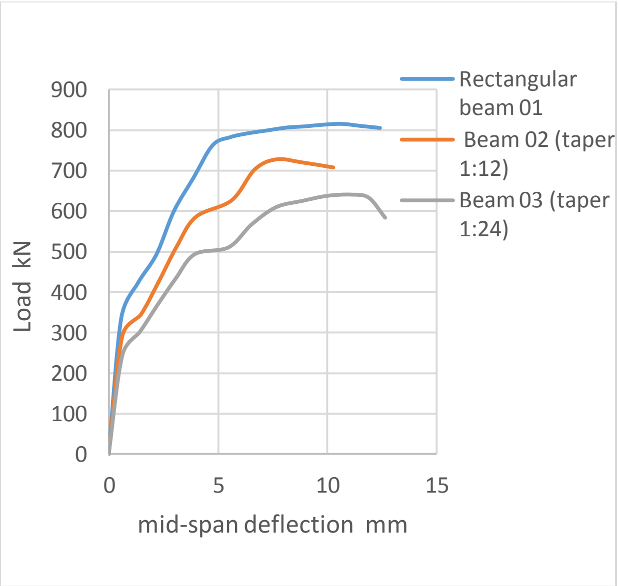
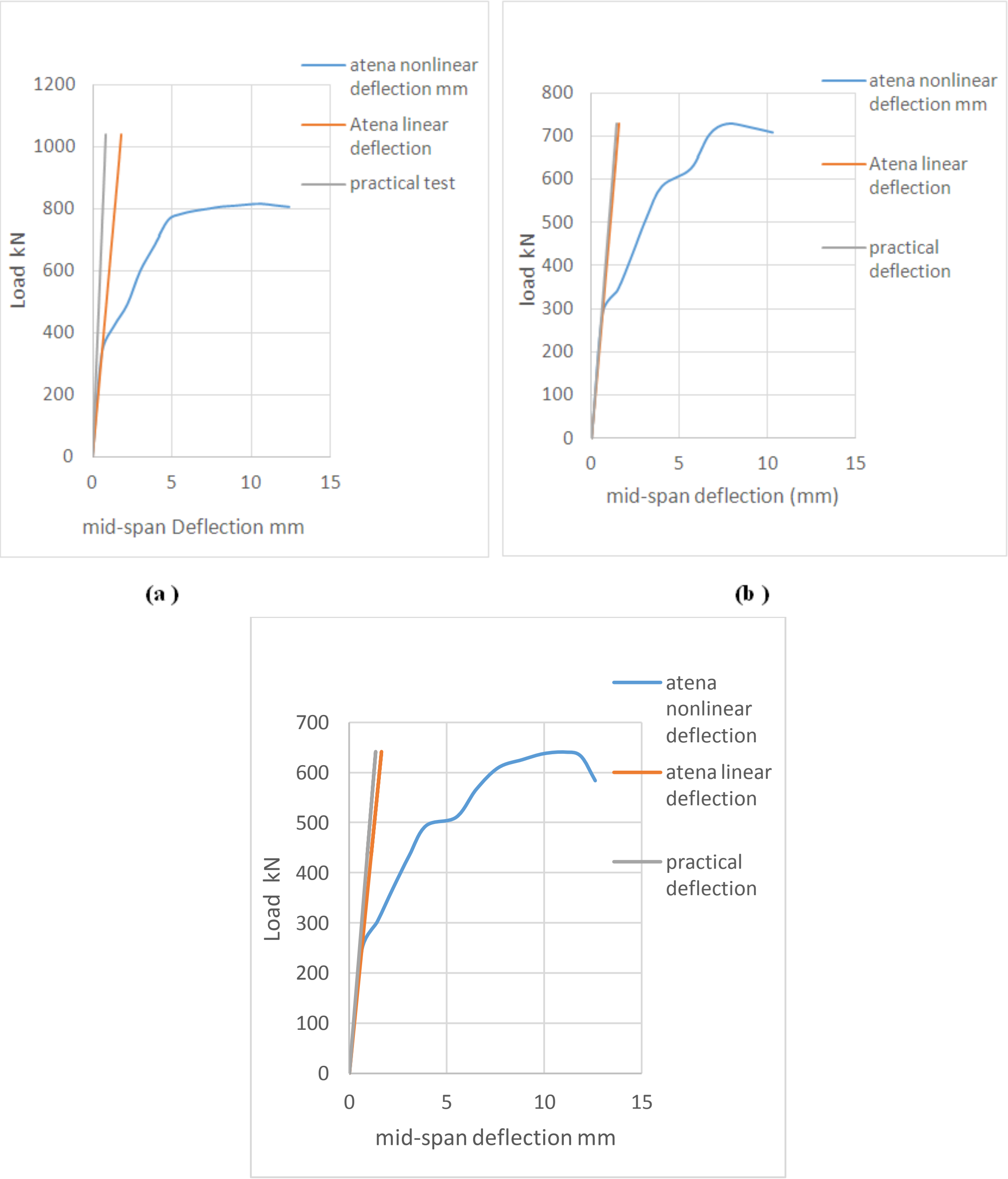


Figure10: deflection curve on the effects of taper in Atena-Gid

Comparing the results of the deflection analysis in both experimental (theoretical linear) and Atena-Gid FE method (linear and non-linear) in Fig.11, it can be seen that nonlinear analysis results in FEM showed significant difference (large deflection) between classical linear FEA and practical (theoretical) linear results (using Eq.7). In Fig. 11a, at the ultimate stress of 54.4 N/mm^2 the nonlinear analysis produced deflection of 10.66mm compared to 1.4mm produced by linear analysis. In Fig. 11b, under ultimate stress of 52.95 N/mm^2 the nonlinear analysis produced deflection of 7.68mm compared to 1.55 produced by linear analysis, whereas in Fig. 11c, under ultimate stress of 51.3 N/mm^2 the nonlinear analysis produced deflection of 11.1mm compared to 1.6mm Produced by linear analysis.

It can be generalised from the study that linear deflection of the studied beams were found within the range of 0 to 1.6mm in the linear region while nonlinear deflection of the beams were found within 0 to 11.1mm in the nonlinear zone. Also, it can be said that linear deflection results for experimental beams and the results obtained from the Atena-Gid modelled beams were relatively close.



(a) **(b)**

(c)

Figure 11: Deflection curve of between Atena-Gid non-linear and linear vs Experimental linear analysis for the studied beams **(a)** beam 01 , **(b)** beam 02 and **(c)** beam 03

VIII. CONCLUSION

From the study, the following conclusions can be drawn;

i. The controlled RC rectangular beam shows higher strength than the RC tapered beams. This is to say that the 'geometric tapering of beam' reduces the strength of the beam to carry more loads.

ii. Beams with tapered geometry show readiness to deflect under applied load compared to beams without taper (rectangular geometry). Thus, increase in the taper, or slope of the beam increases the mid-span deflection of the beam.

iii. It can be inferred from the study that nonlinear analysis results in FEM, showed significant difference (large deflection) between classical linear FEM results and practical (theoretical) linear results.

iv. The Atena-Gid finite element simulation beam models produced close estimation of the ultimate load and mid-span deflection results to that of the experimental results analysis at the linear region while as in the nonlinear region, the results in FEM showed significant large mid-span deflection.

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