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Research Paper

Physical Laws and Bond Graphs

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ABSTRACT- *This paper deals with physical laws and their representation utilizing the bond graphs. This latter is a unified strong graphical method that is appropriate to the representation of multidisciplinary physical systems which call to electrical, mechanical, hydraulic and thermic domains. The expression of physical laws can be derived from bond graph methodology. Finally, some simulation results are presented and their important characteristics are discussed.*
Keys words: physical laws, bond graphs, fundamental concepts of bond graphs

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I. INTRODUCTION

The bond graph is a graphical representation and unified modeling method of the system, it permits to view the hierarchical structure of the system, so a suitable tool for human sight. All variables could be accessed, in this way simulation or experimental results are easily interpreted.
The bond graph is based on two fundamental concepts:
-Energy is conservative in a physical system,
-The sub-systems of a multi-disciplinary physical system exchange energy mutually.

Differential equations, state-space equations, transfer function, and block diagrams could be obtained with ease from the bond graph (BG).
There are numerous analogies in different physical domains leading to analogous equations. This fact permits the construction of a unified theory to analyze physics as a whole and get a better understanding of the phenomena encountered in physics.
The BG offers an adequate tool for analysis, conception, control, and even optimization of multi-disciplinary technological systems. Building a model is an iterative procedure, as the BG highlights functional interaction between system components and energy flow, it facilitates this task.

BG was introduced by Paynter in 1959 who published this methodology in 1961 [1, 2]. Furthermore, some researchers continued the task and shed light on different concepts of this method; Rosenberg [3], Breedveld [4] [5]. Quite a lot of progress has since been achieved by other researchers.

II. BOND GRAPH METHODOLOGY

The bond graphs, based on the notion of energy exchange between system components is suitable to model a lumped parameter system. From an “energetic point of view”, one can find a
pair of conjugate power variables in each physical domain, called effort e and flow f variables, so the total power involved in the system is: $\mathrm{P} = \Sigma P_i = \Sigma e_i f_i$

To these two energy variables are associated momentum p and displacement q, defined as:

$$p = \int e \, d\tau \ ; \quad q = \int f \, d\tau$$

II.1. PHYSICAL LAWS AND THEIR REPRESENTATION BY BOND GRAPHS

The bond graph is an adequate graphical tool of modeling, common to different disciplines (production of an interface between mechanical, electrical, and control engineers), this facilitates: description, analysis, conception and control of the multidisciplinary systems.

The BGs has its proper language, common to all different physical and technical subdomains and one should pay attention to the definition of the different vocabularies and concepts and their application to the different physical subdomains. The BG is a valuable tool, facilitating the analysis of physical phenomena such as energy production, storage, and transformation. The BG permits the division of a system into subsystems that represent the energy circulation between elementary interconnected sub-systems employing simple bonds that express power continuity.

The causality upon the BG permits us to determine the cause and the effect which is resulted on. This concept permits the analysis and structuring of the physical system. From the BG, one can establish the differential equations, state-space equations and the transfer function of the physical system [6], [7].

Bond Graph elements

The main two generalized variables e: effort, f: flow and their respective integrals p: momentum and q: displacement constitute the fundamental basis of the BG.

We encounter in physical systems two active elements: effort and flow sources and three passive elements, of which two reactive (inertial and capacitive) and one dissipative (resistive) element. Inertial element materializes effort storage, whereas capacitive element materializes

capacitive or potential storage. Transformers et gyrators allow to convert the energy into the same type or into another type.

The constitutive elements of a BG are resumed as follows:

- Effort and flux sources, modulated sources,
- 0-type or 1-type junctions, indicate how the elements are associated,
- Dissipative, potential et inertial storage elements,
- Transformer, gyrator, allowing energy conversion and transfer,

Table 1 synthesizes energy conjugate variables applicable to different physical domains.

II.2. ELECTRICAL DOMAIN

The first law of Kirchhoff: As the law stipulates: in an electrical node, the algebraic sum of currents into the node are null. This is materialized by a 0-junction, where the sum of currents into the node equal the sum of the output currents. A 0-junction is characterized by a common effort variable. As we note, this law is derived from the energy conservation principle.

$$\Sigma p_i = 0 \; ; \quad e_i = e \;\; \forall i \quad \rightarrow \quad \Sigma f_i = 0;$$

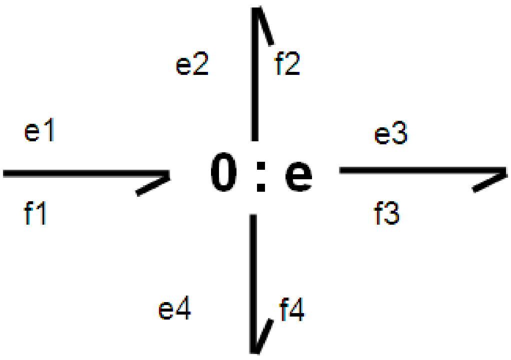

Figure 1 - 0-type junction materializes first Kirchhoff law

The second law of Kirchhoff: As the law stipulates: in an electrical loop, the algebraic sum of voltages is null. This is materialized by a 1-junction, where the sum of input efforts equals the sum of all other efforts (voltages) in the loop. A 1-type junction is characterized by a common flow variable. As we note, this law is derived from the energy conservation principle.

$$\Sigma p_i = 0 \; ; \;\; f_i = f \;\; \forall i \;\; \rightarrow \quad \Sigma e_i = 0;$$

Generalized Variables	Effort e	Flow F	Generalized Momentum p	Generalized Displacement q
Electric	Voltage U (V)	Current I (A)	Magnetic Flux ψ (Wb)	Electric Charge q (C)
Translational Mechanics	Force F (N)	Velocity v (m/s)	Impulsion H (N.s)	Displacement x (m)
Rotational Mechanics	Torque Γ(Nm)	Angular Velocity ω (rad/s)	Angular momentum H (Nm.s)	Angle θ (rad)
Hydraulics	Pressure P (Pa)	Volumetric flow rate Q (m^3/s)	Pressure Impulse H (Pa.s)	Volume V (m^3)
Magnetics	Magnetomotive force Γ(A)	Flux derivative $\dot{\psi}$ voltage (V)	-	Magnetic flux ψ (Wb)
Chemistry	Chemical potentiel (J/mol)	Molar flow (mol/s)	-	Quantity of matter N (mol)
Heat	Temperature θ (K)	Entropic Flow $\dot{S}$ (W/K)	-	Entropy S (J/K)
Acoustics	Pressure P (Pa)	Volumetric flow rate Q (m^3/s)	Pressure Impulse H (Pa.s)	Volume V (m^3)

Table 1 - Effort and flow variables in different physical domains

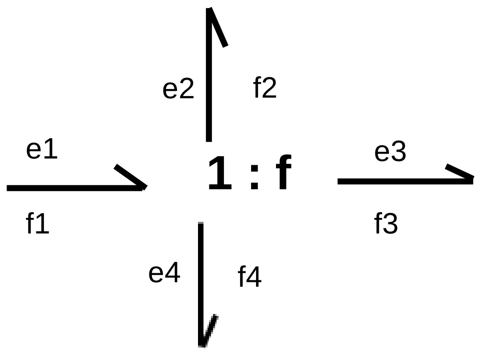

Figure 2- 1-type junction materializing second Kirchhoff law

The concept of these two junction types is more general and universal than serial and parallel connections of elements as encountered in electrical circuits and mechanical devices.

II.3. MECHANICAL DOMAIN

D'Alembert's law: This law is materialized in a mechanical system that is subject to forces, among these forces, can be enumerated: applied motive force, F_m; elastic force (in a spring), F_{sp}, inertial force F_I (in a material point of mass m); friction force, F_D. The conservation principle of energy allows us to write:

$$\Sigma p_i = 0 \ \rightarrow \Sigma e_i f_i = 0 \ \ ; \ \ f_i = f \ \forall i \quad \rightarrow \quad \Sigma e_i = 0$$

$$\Sigma F_i = 0 \quad ; \ v_i = v \ \forall i$$ where v represents velocity

$$F_m = F_{sp} + F_D + F_I$$

$$F_{sp} = \mathrm{k} \int v\, dt \ ; \quad F_D = \mathrm{b}\, v\, ; \ \ F_I = m\, \frac{dv}{dt}$$

The elastic, dissipative, and inertial elements are characterized respectively by k (stiffness), f_D (dashpot coefficient or damping), and m (mass). The above relationships are materialized by a 1-type junction. Note that here, the elements are in parallel configuration, in contrast to the electrical domain where a 1-junction corresponds to a serial connection.

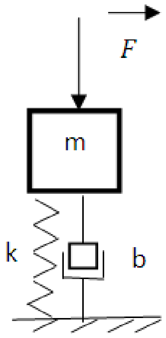

Figure 3 - mass-stiffness-damping system

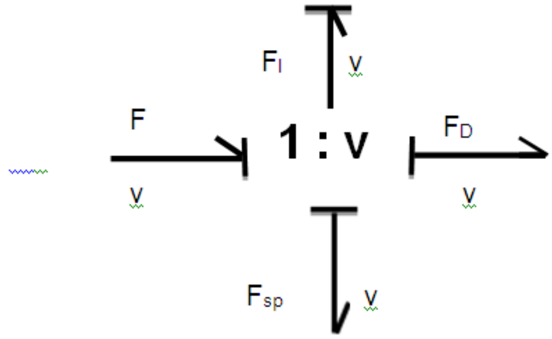

Figure 4 - Bond Graph of mass-stiffness-damping system

This relationship corresponds to the third Newton's law or fundamental principle of dynamics, known as D'Alembert's law.

In a rotational case, we substitute the rotational quantities for the translation ones as follows:

F (force) → Γ (torque) ; v (linear velocity) → Ω (angular velocity)

M (mass) → J (momentum of inertia) ; b (mechanical dashpot) → b_r (rotational dashpot

$\Sigma e_i = 0;\ f_i = f\ \ \forall i \quad \rightarrow \quad \Sigma \Gamma_i = 0;\ \Omega_i = \Omega\ \ \forall i$

$\Gamma_{sp} = \mathrm{k} \int \Omega\, dt\ ;\quad \Gamma_D = b_r\, \Omega\ ;\quad \Gamma_i = \mathrm{J}\, \frac{d\Omega}{dt}$

In a 0-type junction, all elements are subject to the same force. The following figure depicts a serial connection of the mechanical elements.

$\Sigma f_i = 0;\ e_i = e\ \ \forall i \quad \rightarrow \quad \Sigma v_i = 0;\ F_i = F\ \ \forall i$

$F_{sp} = \mathrm{k} \int v_{sp}\, dt\ ;\quad F_i = m\, \frac{dv_i}{dt}\ ;\ F_D = \mathrm{b}\, v_D;\ \mathrm{F} = F_i = F_D = \mathrm{F}\ ;\ \ \mathrm{x} = \int \sum v_j\, dx$

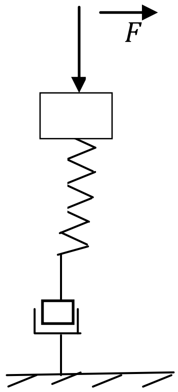

Figure 5 - Spring mass damping in serial connection

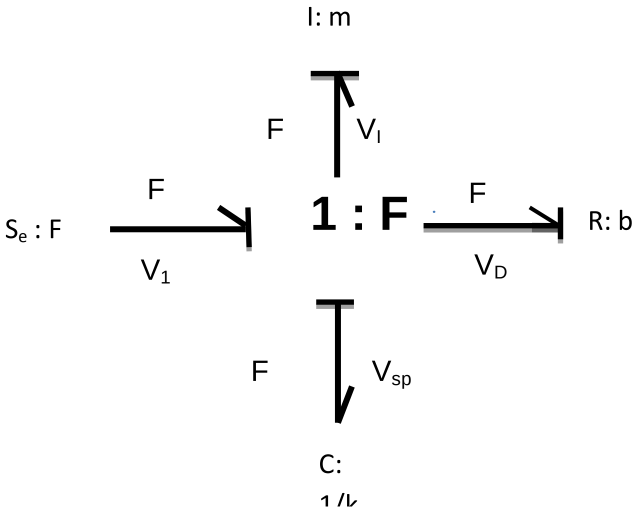

Figure 6. Bond graph of spring-mass damping in serial connection

Mobility variables
Another way to define effort and flow variables is to proceed in the inverse way of what is done classically: e.g., in mechanical systems, effort and flow variables are attributed respectively to velocity and force. This is known as mobility analogy as Firestone explains in [8].
Effort → Velocity or Angular velocity ; Flow → Force or Torque
This is advantageous for mechanical systems, where a parallel connection of elements corresponds to equality of velocities (1-type junction) and a serial connection corresponds to equality of forces or torques (0-type junction). The bond graph methodology, although being a graphical method, is based on an important physical law, i.e., principle of conservation of energy. The advantage of the BG methodology, compared to equivalent circuit methodology is that the former permits to highlight causality between constitutive elements of the system. Thus, a profound vision of physical phenomena. An equivalent circuit of a DC motor, based on mobility variables is presented in the author's paper [9]. The author dedicates [10] to the description of equivalent circuit of some multi-physic systems. [11] illustrates the BG methodology of multi-disciplinary dynamic systems.

II.4. ELECROMECHANICAL SYSTEM
To highlight this principle, we apply it to an electromechanical system. A DC motor is driving a mechanical load. The system parameters are resumed here:
R: armature resistance ; L: armature inductance
b1: dashpot coefficient of armature ; J_m: armature inertia
b_{load}: dashpot coefficient of mechanical load ; J_{load}: mechanical load inertia

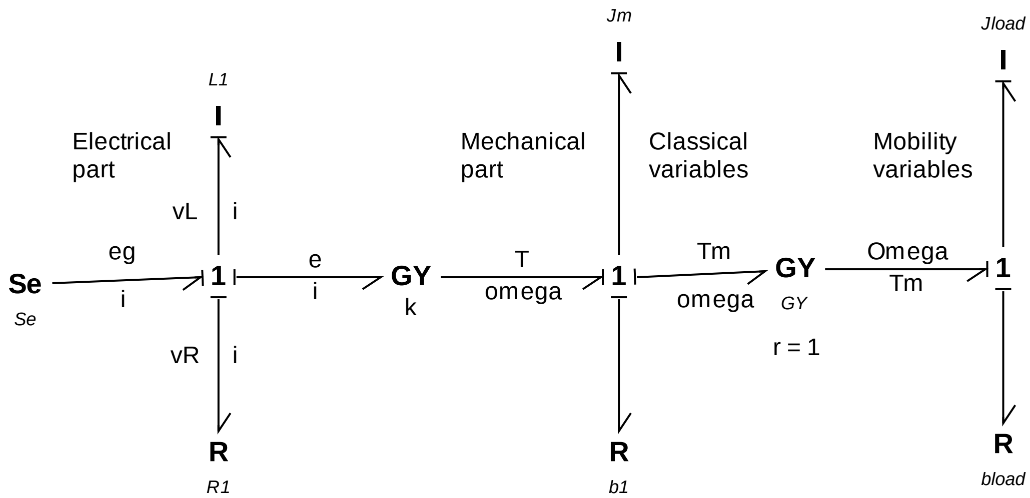

Figure 7. Bond graph of an electromechanical system using mobility variable

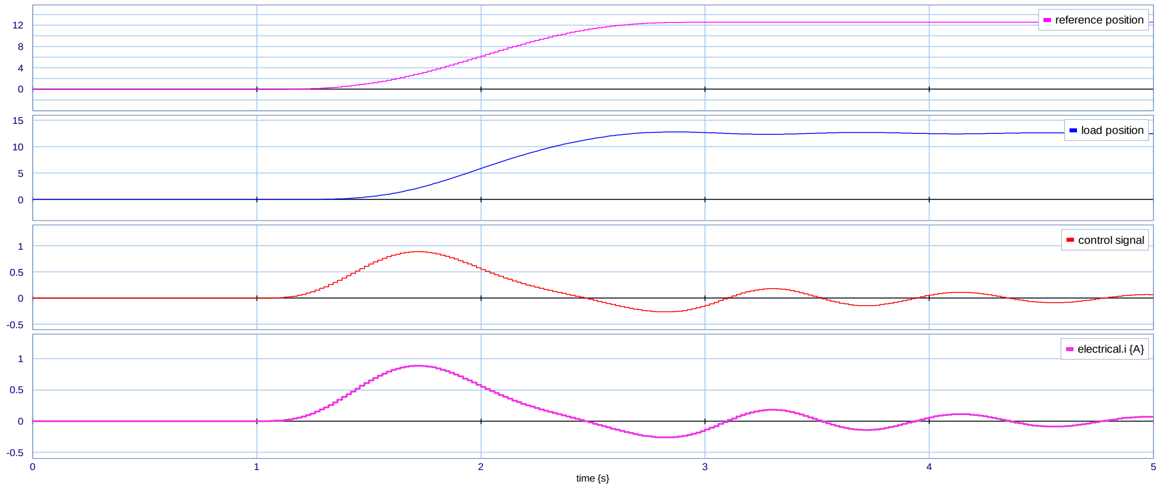

Figure 8. Results obtained by bond graph modeling

II.5. HYDRAULIC SYSTEM

In this field of physics, the pressure and fluid flow rate constitute respectively effort and flow variables. In hydraulics, the three passive elements are elaborated as follows:

Dissipative element:

The fluid flow is subject to resistance due to friction of the movement of fluid layers. This materializes the resistance element.

$Q = k\,\Delta P$; $P_{D} = \Delta P = R\,Q$ with $R = 1/k$

R: resistance ; k: restriction, ΔP: pressure loss

Inertial element:

The relation between momentum and effort is: $p = \int e\, d\tau$

The pressure is generated by the application of force F over a section S, causing fluid flow at the speed v.

$$P_I = \frac{F}{S} = \frac{ma}{S} = \frac{m\frac{dv}{dt}}{S} = \frac{\rho LS\frac{dv}{dt}}{S} = \frac{\rho L}{S}\frac{dQ}{dt}$$

$$P_I = I\frac{dQ}{dt} \text{ witlh inertial element } I = \frac{\rho L}{S}$$

Capacitive element:

The capacitance is defined as the quotient between displacement and effort. In the mechanics of fluids, these two variables are respectively volume V and pressure P_C.

$P_C = \rho\, g\, H$; $V = SH$

$C = \frac{V}{P_C} = \frac{S}{\rho g}$; $P_C = \frac{1}{C}\int Q\, d\tau$; $V = \int Q\, d\tau$

C is denoted as hydraulic accumulation.

Bernoulli's law

The association of the three dissipative, inertial and capacitive elements in a 1-type junction with a common flow variable, i.e., flow Q, yields to Bernoulli's law, the demonstration is done as follows:

$$P = P_D + P_I + P_C = RQ + I\frac{dQ}{dt} + \frac{1}{C}\int Qdt$$

With $R = \frac{\Delta P}{Q}$; $I = \frac{\rho L}{S}$; $C = \frac{S}{\rho g}$

The Bernoulli's law is stated in terms of pressure as follows: $P = RQ + \frac{1}{2}\rho v^2 + \rho g h + P_{atm}$

P_{at} accounts for atmospheric pressure.

The term RQ reflects the effect of sink element, i.e., dissipation due to friction. This law indicates the energy density in a hydraulic system.

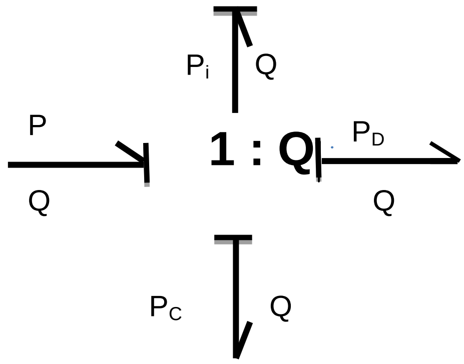

Figure 9. Bond graph illustration Bernoulli's law

II.6. ACOUSTIC SYSTEM

Both a microphone or a loudspeaker are transducers illustrating a multi-process system.

The process of conversion in a loudspeaker is depicted below:

Electrical energy → Mechanical energy → acoustic energy

More specifically, the different stages of the conversion process can be illustrated by the following sequence:

Voltage image of sound → Current through coil → Force → Vibration of voice cone → generation of a sound

In the other words, a loudspeaker materializes an electroacoustic actuator. In contrast, a microphone realizes the inverse process. We illustrate this principle through an electrodynamic loudspeaker.

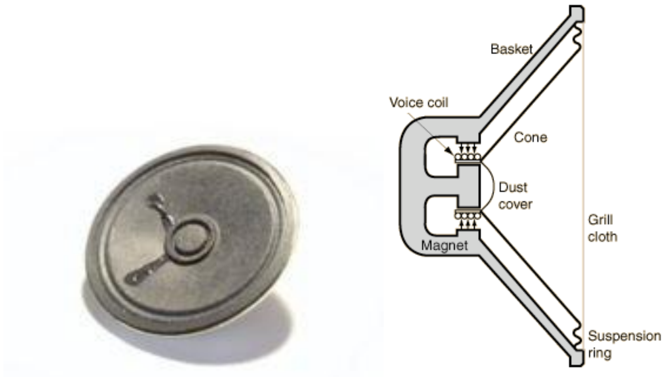

Figure 10. Loudspeaker cone

The first stage is composed of serial RLC electric circuit. This can be represented by a 1-type junction. The main element of the system is a gyrator that converts electrical energy into mechanical energy.

A coil carrying a current I, the image of sound is plunged in a fixed radial magnetic field B. Let:
N: number of coil turns
R_c: coil radius
L_C: total length of the coil conductor; $L_C = 2\pi R_c N$
B: magnetic flux density produced by a cylindrical fixed permanent magnet
The magnetic flux is distributed radially around the magnet, crossing the coil radially but returning axially.
The coil is attached to the speaker cone to generate acoustic vibration.
The generated Lorentz force acting on the coil is:

$$\vec{F} = -\int i\,\overrightarrow{dl} \times \vec{B} = F = 2\pi R_c N B\, i \qquad (a)$$

The back emf induced in the coil is: $e = \frac{d\Psi}{dt} = \frac{d(NBA)}{dt} = \frac{d(2\pi N R_c l B)}{dt} = 2\pi N R_c B\,\frac{dl}{dt} = 2\pi R_c B v$
Where v represents the coil velocity. The electric field strength: $\vec{E} = \vec{v} \times \vec{B}$
The total back emf induced on the coil is:

$$e = \int \vec{E}.\overrightarrow{dl} = 2\pi N R_c B v \qquad (b)$$

The equations a and b can be materialized by a gyrator whose modulus is: $r = 2\pi R_c N B$.
F = r i ; e = r v

Mechanical part:
The mechanical part is characterized by a mass (mass of mobile coil) M_c, damping b, and stiffness k due to suspension device. These three elements, subjected to the same velocity, are represented by a 1-type junction. From, the relative BG, the differential equation can be derived that represents the well-known fundamental principle of dynamics:
$F - b\frac{dx}{dt} - k\,x = M_m\,\frac{d^2x}{d\,t^2}$
Where M_m denotes the moving mass.
Acoustic part:

A loudspeaker is an electro-acoustic transducer that converts an electric signal, an image of the sound, provided by the amplifier stage, into mechanical energy, then acoustic energy.

Electric energy → mechanical energy→ acoustic energy

A coil wherein flows a current and plunged in a fixed magnetic field permits to generate a force. The magnetic circuit presents low reluctance iron routes to decrease flux leakage. The process of sound generation is embodied by a transformer of modulus S whose secondary acoustical circuit is composed of air mass M_a and a radiation impedance R_M. The acoustical equivalent circuit is composed of air mass M_A and radiation impedance r_A. [12]; [13].

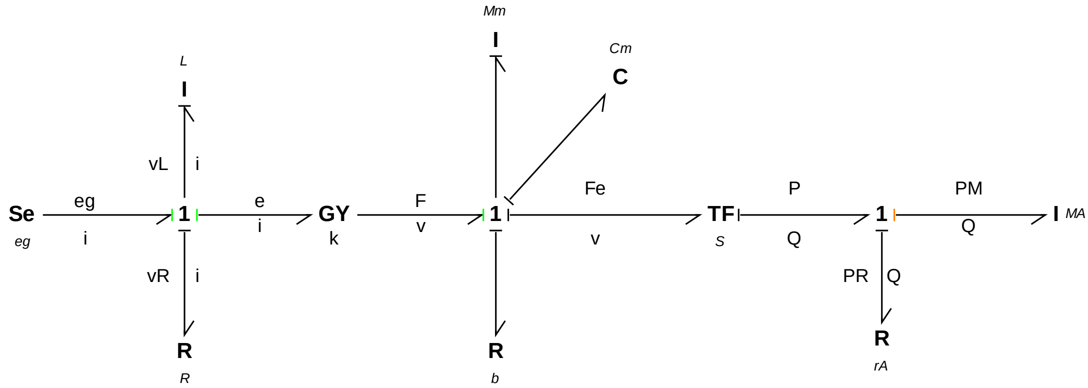

Figure 11. Bond graph of a loudspeaker

III. CONCLUSIONS

In this article we dealt with bond graphs, a powerful graphical and rigorous tool, adequate to the modeling of multibody physical systems. Physical laws and theorems could be derived from their respective bond graphs. A bond graph permits a deep insight to functional, energetical, and hierarchical structure of the whole system, allowing thus, a better analysis, calculation, design, and control of the system. Some simulation results are presented to highlight the strength of this tool.

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