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Research Paper

Common fixed point theorems for a pair of self-maps in fuzzy metric space

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Abstract:
This paper aims to introduce the new concept of rational type fuzzy contraction mappings in fuzzy metric spaces. We prove some fixed point theorems under rational type fuzzy contraction conditions in fuzzy metric spaces with illustrative example to support the main results.
Keywords: *Fixed point, fuzzy metric space, weakly compatible mapping, (CLR) property.*
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I. Introduction and Preliminaries

The fixed theory is most demanded and interesting in solving many problems in area of research in mathematics. A lot of work was dedicated to the theory of fixed point. In 1922, Banach [2] introduced Banach Contraction Principle that is "A self-mapping in a complete metric space satisfying the contraction conditions has a unique fixed point."

In 1965, Zadeh [15] introduced the theory of partial fuzzy set. The fuzzy logic is applied in the processing of students evaluation, human behavior. In 1988, Kramosil and Michalek [9] introduced the concept of fuzzy metric spaces (FM-Space) which is the generalization of probabilistic theory. After that, the concept of metric fuzziness was given by George and Veeramani [6]. In 1988, Grabiec [7] was the first who proved the Banach Contraction Principle in fuzzy metric space.

In 1986, Jungek [8] introduced weakly compatible mappings. In 2002, Aamri *et al.* [1] generalized the concept of non-compatibility and gave the new contraction by defining E.A property.

Definition 1.1. [15] Let Xbe any set. A fuzzy set $A$of $X$is a function from domain $X$and values in$[0,1]$.

Example1.2. Consider $X = \{a,b,c,d\}$and$A: X \to [0,1]$defined as$A(a) = 0, A(b) = 0.5, A(c) = 0.2 \; and \; A(d) = 1$. Then$A$is a fuzzy set on X. This fuzzy set also can be written as follows:

$$A = \{(a,0)(b,0.5)(c,0.2)\,(d,1)\}.$$

In 1994, George and Veeramani [6] introduced the notion of fuzzy metric spaces as follows:

Definition 1.3. [6] A 3- tuple $(X, M,*)$is said to be a fuzzy metric space if $X$is an arbitrary set, $*$is a continuous t-norm and $M$is a fuzzy set on$X^2 \times (0,\infty)$ satisfying the following properties:

(FMS 1)$M(p,q,t) > 0$,
(FMS 2)$M(p,q,t) = 1$ if and only if$p = q$;
(FMS 3) $M(p,q,t) = M(q,p,t)$;
(FMS 4) $M(p,q,t) * M(q,r,s) \le M(p,r,t+s)$;
(FMS 5) $M(p,q,.): (0,\infty) \to (0,1]$iscontinuous,
for all$p,q,r \in X$ and$s,\; t > 0$.

Then M is called a fuzzy metric on X.The function $M(p,q,t)$ denote the degree of nearness between $p$and $q$with respect to t.

Definition 1.4. [5] A binary operation $*: [0,1] \times [0,1] \rightarrow [0,1]$is said to be a continuous t-norm if it satisfies the followingconditions:

(i) * is associative andcommutative,

(ii) * iscontinuous,

(iii) $a * 1 = a$ for all$a \in [0,1]$and

(iv) $a * b \leq c * d$ whenever$a \leq c$ and$b \leq d$ for all$a, b, c, d \in [0,1]$.

Example 1.5.$a * b = ab$ for$a, b \in [0,1]$is a continuous t-norm.
Definition 1.6. [9] A binary operation $\Delta : [0,1] \times [0,1] \rightarrow [0,1]$is a continuous t co-norm if it satisfies the followingconditions:

(i) $\Delta$is commutative andassociative;

(ii) Δiscontinuous;

(iii) $a\Delta\, 0 = a$ for all $a \in [0,1]$;

(iv) $a\Delta\, b = c\Delta\, d$ whenever $a \geq c$ and $b \geq d$ for each$a, b, c, d \in [0, 1]$.
Example 1.7. A binary operation$\Delta : [0,1] \times [0,1] \rightarrow [0,1]$ such that$a\Delta b = min(a + b, 1)$is a continuous t co-norm.

Definition 1.8. [10] Let $(X, M,*)$ be a fuzzy metric space, $x \in X$ and$\phi \neq A \subseteq X$. We define $D(x, A, t) = sup\{M\ (x, y, t): y \in A\}(\ t > 0)$ then $D(x, A, t)$ is called a degree of closeness of x to A at t.

Definition 1.9. [10] A topological space is called a (topologically complete) fuzzy metrizable space if there exists a (topologically complete) fuzzy metric inducing the given topology on it.

Definition 1.10. [6] Let $(X, M,*)$ be a fuzzy metric space. A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ (denoted by $\lim_{n\to\infty} x_n = x$) if for $\lim_{n\to\infty} M(x_n, x, t) = 1$ for all $t > 0$.

Definition 1.11. [6] Let M be a fuzzy metric space. A sequence $\{x_n\}$ in X is called Cauchy sequence if and only if $\lim_{n\to\infty} M(x_{n+p}\ , x_n\ ,\ t) = 1$ for all $p > 0$ and $t > 0$.

Definition 1.12. [12] Two mappings S and T of a fuzzy metric space $(X, M,*)$ into itself are said to be compatible maps if $\lim_{n\to\infty} M(STx_n, TSx_n, \varepsilon) = 1$ for all $\varepsilon > 0$ where $\{x_n\} \in X$ such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = \omega \epsilon X$.

Definition 1.13. The self-mappings S and T of a fuzzy metric space $(X, M,*)$ are said to be commuting if$M(STx, TSx, t) = 1$ for all$x \in X$.

Definition 1.14. [13] Theself-mappings$S$and$T$ofafuzzymetricspace $(X, M,*)$ are said to be weakly commuting if $M(STx, TSx, t) \geq M(Sx, Tx, t)$ for each$x \in X$ and $t > 0$.
Definition 1.15. [8] The self-mappings S and T of a fuzzy metric space $(X, M,*)$
are said to be compatible if and only if$\lim_{n\to\infty} M(STx_n\ , TSx_n, t) = 1$ whenever $\{x_n\}$ is a
sequence in X such that$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = x$ for some x in X and $t > 0$.

Definition 1.16. [11] The self-mappings S and T of a fuzzy metric space $(X, M,*)$ are said to becompatible of type (K)iff $lim_{n\to\infty} M(SSx_n\ , Tx, t) = 1$ and$lim_{n\to\infty} M(TTx_n, Sx, t) = 1$,whenever $\{x_n\}$ is a sequence in X such that

$lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = x$for some $x$in X and $t > 0$.
Definition 1.17. [8] The self-mappings S and T of a fuzzy metric space $(X, M,*)$are said to be weakly compatible if they commute at their coincidence points, i.e., Sx = Tximplies STx = TSx.

Definition 1.18. [1] Let(X, d) be a metric space. Two self- mappings S, TsatisfyE.A property if there exists a sequence $\{x_n\}$in X such that$lim_{n\to\infty} Sx_n = lim_{n\to\infty} Tx_n = t$ for some $t \in X$.

Definition 1.19. [1] Let S and T be two self-maps of a metric space then they are said to satisfy (CLR_T) property if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = Tx$ for some $x \epsilon X$.

Definition 1.20. [14] Let (X, d)be a metric space and $S, T : X \rightarrow X$. Let$Y \subseteq X$. The mappings $S, T$are said to satisfy the property common limit converging in the range sub-space$Y$ if there exists a sequence $\{x_n\}$in Xsuch that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n \in Y$.

II. Main Results

In this section, we prove some common fixed point theorems for a pair of self-maps in fuzzy metric space.
Theorem 2.1: Let S, T be two weakly compatible self-mappingson fuzzy metric space $(X, M,*)$ such that
$\mathrm{M}(\mathrm{Sx},\mathrm{Sy},\mathrm{t}) < max\{\mathrm{M}(\mathrm{Tx},\mathrm{Ty},\mathrm{t}),\mathrm{M}(\mathrm{Sx},\mathrm{Tx},\mathrm{t}),\mathrm{M}(\mathrm{Ty},\mathrm{Sy},\mathrm{t}),\mathrm{M}(\mathrm{Tx},\mathrm{Sy},\mathrm{t}),\mathrm{M}(\mathrm{Sx},\mathrm{Ty},\mathrm{t})\}$(2.1)
and S, T satisfy CLR_T property then S, T have a unique common fixed point in X.
Proof: Since S and T satisfy CLR_T property, therefore there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = Tx$ for some $x \in X$.
Taking $x = x_n$ and $y = x$ in (2.1), we obtain
$\mathrm{M}(\mathrm{Sx_n},\mathrm{Sx},\mathrm{t}) < max\{M(\mathrm{Tx_n},\mathrm{Tx},\mathrm{t}),\mathrm{M}(\mathrm{Sx_n},\mathrm{Tx_n},\mathrm{t}),\mathrm{M}(\mathrm{Tx},\mathrm{Sx},\mathrm{t}),\mathrm{M}(\mathrm{Tx_n},\mathrm{Sx},\mathrm{t}),\mathrm{M}(\mathrm{Sx_n},\mathrm{Tx},\mathrm{t})\}$.
Taking limit as $n \to \infty$, we get

$$\mathrm{M}(\mathrm{Tx},\mathrm{Sx},\mathrm{t}) < \max\{1,\mathrm{M}(\mathrm{Tx},\mathrm{Sx},\mathrm{t})\}.$$

Therefore, $Tx = Sx$.
Thus x is the coincidence point of S and T.
Let $\alpha = Sx = Tx$.
Since $S,\ T$ are weakly compatible, therefore we have
$S\alpha = ST\alpha = TS\alpha = T\alpha$.
Now we prove that $S\alpha = \alpha$.
Let us suppose that $S\alpha \neq \alpha$ then

$$\begin{aligned}\mathrm{M}(\mathrm{S}\alpha,\alpha,\mathrm{t}) &< \max\{\mathrm{M}(\mathrm{T}\alpha,\mathrm{T}\alpha,\mathrm{t}),\mathrm{M}(\mathrm{S}\alpha,\mathrm{T}\alpha,\mathrm{t}),\mathrm{M}(\mathrm{T}\alpha,\alpha,\mathrm{t}),\mathrm{M}(\mathrm{S}\alpha,\alpha,\mathrm{t}),\mathrm{M}(\mathrm{S}\alpha,\mathrm{T}\alpha,\mathrm{t})\}\\ &< M(\mathrm{S}\alpha,\alpha,\mathrm{t}),\end{aligned}$$

a contradiction.
Hence, $S\alpha = \alpha = T\alpha$.
Thus α is the common fixed point of S and T.
Let α, β be two fixed points of S and T, therefore $S\alpha = T\alpha = \alpha$ and $S\beta = T\beta = \beta$.

$$\begin{aligned}\mathrm{M}(\mathrm{S}\alpha,\mathrm{S}\beta,\mathrm{t}) &< \max\{\mathrm{M}(\mathrm{T}\alpha,\mathrm{T}\beta,\mathrm{t}),\mathrm{M}(\mathrm{S}\alpha,\mathrm{T}\alpha,\mathrm{t}),\mathrm{M}(\mathrm{T}\beta,\mathrm{S}\beta,\mathrm{t}),\mathrm{M}(\mathrm{T}\alpha,\mathrm{S}\beta,\mathrm{t}),\mathrm{M}(\mathrm{S}\alpha,\mathrm{T}\beta,\mathrm{t})\}\\ &< \max\{\mathrm{M}(\alpha,\beta,\mathrm{t}),\mathrm{M}(\alpha,\alpha,\mathrm{t}),\mathrm{M}(\beta,\beta,\mathrm{t}),\mathrm{M}(\alpha,\beta,\mathrm{t}),\mathrm{M}(\alpha,\beta,\mathrm{t})\}.\end{aligned}$$

$\mathrm{M}(\alpha,\beta,\mathrm{t}) < max\{1, M(\alpha,\beta,\mathrm{t})\}$.
So, $\alpha = \beta$.
Thus S and T have a unique common fixed point.
Theorem 2.2: let S, T be two weakly compatible self mappings on fuzzy metric space $(X, M,*)$ and $K: [0,\infty) \to (0,1)$ be a function satisfying the followings:
$\phi(\mathrm{M}(\mathrm{Sx},\mathrm{Sy},\mathrm{t})) \leq \mathrm{K}(\mathrm{t}).\phi(\mathrm{M}(\mathrm{x},\mathrm{y},\mathrm{t}))$, (2.2)
where $\phi: [0,1] \to [0,1]$ is a function and
$\mathrm{M}(\mathrm{x},\mathrm{y},\mathrm{t}) = \min\{\mathrm{M}(\mathrm{Tx},\mathrm{Ty},\mathrm{t}),\mathrm{M}(\mathrm{Sx},\mathrm{Tx},\mathrm{t}),\mathrm{M}(\mathrm{Ty},\mathrm{Sy},\mathrm{t}),\mathrm{M}(\mathrm{Sx},\mathrm{Ty},\mathrm{t}),\frac{\mathrm{M}(\mathrm{Tx},\mathrm{Sy},\mathrm{t})\mathrm{M}(\mathrm{Sx},\mathrm{Ty},\mathrm{t})}{\mathrm{M}(\mathrm{Sx},\mathrm{Sy},\mathrm{t})}\}$. (2.3)
Either S and T satisfy CLR_S property or S and T satisfy CLR_T property.
$SX \supset TX$.
Then S and T have a unique common fixed point in X.
Proof: Since S and T satisfy CLR_T property, so there exists a sequence $\{x_n\}$ in X such that
$\lim_{n\to\infty} S\,x_n = \lim_{n\to\infty} T\,x_n = Tx$ for some $x \in X$. (2.4)
Now $\lim_{n\to\infty} T\,x_n = T\alpha$ for some α.
Therefore, $\lim_{n\to\infty} S\,x_n = \lim_{n\to\infty} T\,x_n = T\alpha$.
Now we will prove that $T\alpha = S\alpha$.
Let us suppose that $T\alpha \neq S\alpha$ and $\mathrm{M}(\mathrm{S}\alpha,\mathrm{T}\alpha,\mathrm{t}) > 1$.
Taking $x = x_n$ and $y = \alpha$ in (2.2) and (2.3), we get
$\phi(\mathrm{M}(\mathrm{Sx_n},\mathrm{S}\alpha,\mathrm{t})) \leq \mathrm{K}(\mathrm{t}).\phi(\mathrm{M}(\mathrm{x_n},\alpha,\mathrm{t}))$ and
$\mathrm{M}(\mathrm{x_n},\alpha,\mathrm{t}) = \min\{\mathrm{M}(\mathrm{Tx_n},\mathrm{T}\alpha,\mathrm{t}),\mathrm{M}(\mathrm{Sx_n},\mathrm{Tx_n},\mathrm{t}),\mathrm{M}(\mathrm{T}\alpha,\mathrm{S}\alpha,\mathrm{t}),\mathrm{M}(\mathrm{Sx_n},\mathrm{T}\alpha,\mathrm{t}),\frac{\mathrm{M}(\mathrm{Tx_n},\mathrm{S}\alpha,\mathrm{t})\mathrm{M}(\mathrm{Sx_n},\mathrm{T}\alpha,\mathrm{t})}{\mathrm{M}(\mathrm{Sx_n},\mathrm{S}\alpha,\mathrm{t})}\}$.
Taking limit as $n \to \infty$ we get,

$$\mathrm{M}(\alpha,\alpha,\mathrm{t}) = \min\left\{\mathrm{M}(\mathrm{T}\alpha,\mathrm{T}\alpha,\mathrm{t}),\mathrm{M}(\mathrm{S}\alpha,\mathrm{T}\alpha,\mathrm{t}),\mathrm{M}(\mathrm{T}\alpha,\mathrm{S}\alpha,\mathrm{t}),\mathrm{M}(\mathrm{S}\alpha,\mathrm{T}\alpha,\mathrm{t}),\frac{\mathrm{M}(\mathrm{T}\alpha,\mathrm{S}\alpha,\mathrm{t})\mathrm{M}(\mathrm{S}\alpha,\mathrm{T}\alpha,\mathrm{t})}{\mathrm{M}(\mathrm{S}\alpha,\mathrm{S}\alpha,\mathrm{t})}\right\}.$$

$\mathrm{M}(\alpha,\alpha,\mathrm{t}) \geq \min\{1,\mathrm{M}(\mathrm{S}\alpha,\mathrm{T}\alpha,\mathrm{t})\}$.
Therefore, $M(\alpha,\alpha,t) \geq 1$.
Now $\phi: [0,1] \to [0,1]$ is a map and $K(t).\phi(M(S\alpha,T\alpha,t)) \geq \phi(t) = 1$,
a contradiction.
Hence, $S\alpha = T\alpha$.

Let α, β be two fixed points of S and T, therefore $S\alpha = T\alpha = \alpha$ and $S\beta = T\beta = \beta$.
$\phi(M(S\alpha, S\beta, t)) \le K(t).\phi(M(\alpha, \beta, t))$ and
$M(\alpha, \beta, t) = \min\{M(T\alpha, T\beta, t), M(S\alpha, T\alpha, t), M(T\beta, S\beta, t), M(S\alpha, T\beta, t), \frac{M(T\alpha,S\beta,t)M(S\alpha,T\beta,t)}{M(S\alpha,S\beta,t)}\}$.
$\phi(M(\alpha, \beta, t)) \le K(t).\phi(M(\alpha, \beta, t))$ and
$M(\alpha, \beta, t) = \min\{M(\alpha, \beta, t), M(\alpha, \alpha, t), M(\beta, \beta, t), M(\alpha, \beta, t), \frac{M(\alpha,\beta,t)M(\alpha,\beta,t)}{M(\alpha,\beta,t)}\} = \min\{1, M(\alpha, \beta, t)\}$.
Therefore $\alpha = \beta$.
Thus S and T have a unique common fixed point in X.

Theorem 2.3: Let S, T be self-mappings of a fuzzy metric space $(X, M,*)$ satisfying
$M(Sx, Ty, t) \ge \phi(\min\{M(Sx, Ty, t), M(Sx, Tx, t), M(Sx, Sy, t), M(Ty, Sy, t)\})$ for all $x, y \in X$ and $t > 0$, (2.5)
where $\phi: [0,1] \to [0,1]$ is a continuous function with $\phi(s) > s$,
where $0 < s < 1$.
S and T satisfy CLR_S property such that $TX \subset SX$. Then S and T have a unique common fixed point.

Proof: Since S and T satisfy CLR_S property, so there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} S x_n = \lim_{n\to\infty} T x_n = Sx$ for some $x \in X$.
$\lim_{n\to\infty} S x_n = Sk$ for some k.
Therefore, $\lim_{n\to\infty} S x_n = \lim_{n\to\infty} T x_n = Sk$.
Now we prove that $Sk = Tk$.
Let us suppose that $Sk \ne Tk$ and $M(Sk, Tk, t) > 1$.
Taking $x = x_n$ and $y = k$ in (2.5), we get
$M(Sx_n, Tk, t) \ge \phi(\min\{M(Sx_n, Tk, t), M(Sx_n, Tx_n, t), M(Sx_n, Sk, t), M(Tk, Sk, t)\})$.
Taking limit as $n \to \infty$, we have
$M(Sk, Tk, t) \ge \phi(\min\{M(Sk, Tk, t), M(Sk, Tk, t), M(Sk, Sk, t), M(Tk, Sk, t)\})$.
$M(Sk, Tk, t) \ge \phi(\min\{1, M(Sk, Tk, t)\})$ and $\phi(s) > s$,
where $0 < s < 1$.
$M(Sk, Tk, t) > M(Sk, Tk, t)$,
a contradiction.
Therefore, $Sk = Tk$.
Now we prove uniqueness of common fixed point.
Let α, β be two fixed points of S and T, therefore $S\alpha = T\alpha = \alpha$ and $S\beta = T\beta = \beta$.
$M(S\alpha, T\beta, t) \ge \phi(\min\{M(S\alpha, T\beta, t), M(S\alpha, T\alpha, t), M(S\alpha, S\beta, t), M(T\beta, S\beta, t)\})$.
$M(\alpha, \beta, t) \ge \phi(\min\{M(\alpha, \beta, t), M(\alpha, \alpha, t), M(\alpha, \beta, t), M(\beta, \beta, t)\})$.
$M(\alpha, \beta, t) \ge \phi(\min\{1, M(\alpha, \beta, t)\})$ and $\phi(s) > s$,
where $0 < s < 1$.
$M(\alpha, \beta, t) \ge M(\alpha, \beta, t)$.
So, $\alpha = \beta$.
Thus, S and T have a unique common fixed point.

Example 2.4: Let $X = [0, \infty)$. Define $\phi: [0,1] \to [0,1]$ by $\phi(k) = k + 1$.
And define $S, T : X \to X$ by $S(x) = 2x$ and $T(x) = x^2$ for all $x \in X$.
Define $M(x, y, t) = \frac{t}{t+|x-y|}$
Then S, T have a unique common fixed point.

Solution: S and T satisfy
$M(Sx, Sy, t) \le \phi(M(x, y, t))$ and
$M(x, y, t) = \min\{M(Sx, Sy, t), M(Sx, Ty, t), M(Sy, Ty, t), M(Ty, Sx, t)\}$ for all $x \ne y$ and $t > 0$.
S and T satisfy CLR_S property for sequence $\{x_n\} = \{\frac{1}{2n}\}$.
Since, $S(0) = T(0) = 0$.
Therefore, 0 is a unique common fixed point of S and T.

Example 2.5: Let $X = [0, \infty)$ and define $S, T: X \to X$ by $S(x) = 2x - 1$ and $T(x) = x^2$ for all $x \in X$.
Define $M(x, y, t) = \frac{t}{t+\left|\frac{1}{x}-\frac{1}{y}\right|}$.
And $\phi: [0,1] \to [0,1]$ by $\phi(s) = \frac{s}{2}$ is continuous function with $\phi(s) > s$, $0 < s < 1$.
S and T satisfy $M(Sx, Ty, t) \ge \phi(\min\{M(Sx, Ty, t), M(Sx, Tx, t), M(Sx, Sy, t), M(Ty, Sy, t)\})$ for all $x, y \in X$ and $t > 0$.
Also $SX \supset TX$.
Also, S and T satisfy $(CLR)_T$ property for sequence $\{x_n\} = \{\frac{1}{n}\}$.

$\lim_{n\to\infty} ST(x_n) = \lim_{n\to\infty} ST(\frac{1}{n}) = \lim_{n\to\infty} S\left(\frac{1}{n^2}\right) = \lim_{n\to\infty}\left(\frac{2}{n^2} - 1\right) = -1.$
$\lim_{n\to\infty} TS(x_n) = \lim_{n\to\infty} TS(\frac{1}{n}) = \lim_{n\to\infty} T\left(\frac{2}{n} - 1\right) = \lim_{n\to\infty}(\frac{2}{n} - 1)^2 = 1.$
$\lim_{n\to\infty} ST(x_n) \neq \lim_{n\to\infty} TS(x_n).$
Hence, (S, T) is not compatible mapping but (S, T) is weakly compatible mapping as $S(x) = T(x)$ when $x = 1$.
Thus S and T have a unique common fixed point at $x = 1$.

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