



# Soft Maximal open sets and Soft Paraopen sets in Soft Generalized Topological Spaces

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## Abstract

In this paper, we introduced the notion of some new types of soft  $\mu$ -open sets namely soft maximal  $\mu$ -open sets and soft para  $\mu$ -open sets in soft generalized topological spaces. Using the basic concepts, some of the properties of soft maximal  $\mu$ -open sets and soft para  $\mu$ -open sets in soft generalized topological spaces are investigated.

**Key words:** soft generalized topology, soft  $\mu$ -open set, soft maximal  $\mu$ -open set, soft para  $\mu$ -open set and soft  $\mu$ -open neighbourhood.

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## I. Introduction:

In 1999 D. Molodtsov [10] initiated the concept of soft set theory as a mathematical tool for modelling uncertainties. A soft set is a collection of approximate descriptions of an object. Maji et al. [9] have further improved the theory of soft sets. N. Cagman et al. [1] modified the definition of soft sets which is similar to that of Molodtsov. Many researchers have worked on the topological structure of soft sets. Muhammad Shabir et al. [14] introduced soft topological spaces. In 2002 A. Csaszar [5] introduced the concept of generalized topology and also studied some of its properties. Sunil Jacob John et al. [6] introduced the concept of soft generalized topological spaces in 2014. Qays Shakir [12] introduced and studied the concept of minimal and maximal soft open sets in soft topological spaces. B. Roy and R. Sen [13] introduced the concept of maximal  $\mu$ -open and minimal  $\mu$ -closed sets via generalized topology.

Motivated by the above concepts, we extend our further study in soft maximal  $\mu$ -open sets and soft para  $\mu$ -open sets in soft generalized topological spaces and some of their properties are investigated.

## II. Preliminaries:

### Definition: 2.1 [7]

A soft set  $F_A$  on the universe  $U$  is defined by the set of ordered pairs

$$F_A = \{(e, f_A(e)) / e \in E, f_A(e) \in P(U)\}$$

where  $f_A : E \rightarrow P(U)$  such that  $f_A(e) = \emptyset$  if  $e \notin A$ . Here  $f_A$  is called an approximate function of the soft set  $F_A$ . The value of  $f_A(e)$  may be arbitrary. Some of them may be empty, some may have nonempty intersection. The set of all soft sets over  $U$  with  $E$  as the parameter set will be denoted by  $S(U)_E$  or simply  $S(U)$ .

### Definition: 2.2 [7]

Let  $F_A \in S(U)$ . If  $f_A(e) = \emptyset$ , for all  $e \in E$ , then  $F_A$  is called an empty soft set, denoted by  $F_\emptyset$ .  $f_A(e) = \emptyset$  means there is no element in  $U$  related to the parameter  $e$  in  $E$ . Therefore we do not display such elements in the soft sets as it is meaningless to consider such parameters.

**Definition: 2.3 [6]**

Let  $F_A \in S(U)$ . If  $f_A(e) = U$ , for all  $e \in A$ , then  $F_A$  is called an A-universal soft set, denoted by  $F_{\bar{A}}$ . If  $A = E$ , then the A-universal soft set is called an universal soft set, denoted by  $F_{\bar{E}}$ .

**Definition: 2.4 [6]**

Let  $F_A \in S(U)$ . Then, the soft complement of  $F_A$ , denoted by  $(F_A)$ , is defined by the approximate function  $f_{A^c}(e) = (f_A(e))^c$ , where  $(f_A(e))^c$  is the complement of the set  $f_A(e)$ , that is,  $(f_A(e))^c = U \setminus f_A(e)$  for all  $e \in E$ .

**Definition: 2.5 [6]**

Let  $F_A \in S(U)$ . A Soft Generalized Topology (SGT) on  $F_A$ , denoted by  $\mu$  (or)  $\mu_{F_A}$  is a collection of soft subsets of  $F_A$  having the following properties:

- i.  $F_\emptyset \in \mu$
- ii.  $\{F_{A_i} \subseteq F_A / i \in J \subseteq N\} \subseteq \mu \Rightarrow \bigcup_{i \in J} F_{A_i} \in \mu$

The pair  $(F_A, \mu)$  is called a Soft Generalized Topological Space (SGTS). Observe that  $F_A \in \mu$  must not hold.

**Definition: 2.6 [6]**

Let  $(F_A, \mu)$  be a SGTS. Then every element of  $\mu$  is called a soft  $\mu$ -open set.

**Definition 2.7 [7]**

Let  $(F_A, \mu)$  be a SGTS and  $\alpha \in F_A$ . If there is a soft  $\mu$ -open set  $F_B$  such that  $\alpha \in F_B$ , then  $F_B$  is called a soft  $\mu$ -open neighbourhood (or) soft  $\mu$ -nbd of  $\alpha$ . The set of all soft  $\mu$ -nbds of  $\alpha$ , denoted by  $\psi(\alpha)$ , is called the family of soft  $\mu$ -nbds of  $\alpha$ . i.e.  $\psi(\alpha) = \{F_B / F_B \in \mu, \alpha \in F_B\}$ .

**Definition: 2.8 [7]**

Let  $(F_A, \mu)$  be a SGTS and  $F_B \subseteq F_A$ . Then the soft  $\mu$ -interior of  $F_B$ , denoted by  $i_\mu(F_B)$  is defined as the soft union of all soft  $\mu$ -open subsets of  $F_B$ . Note that  $i_\mu(F_B)$  is the largest soft  $\mu$ -open set that is contained in  $F_B$ .

**Definition: 2.9 [7]**

Let  $(F_A, \mu)$  be a SGTS and  $F_B \subseteq F_A$ . Then the soft  $\mu$ -closure of  $F_B$ , denoted by  $c_\mu(F_B)$  is defined as the soft intersection of all soft  $\mu$ -closed super sets of  $F_B$ . Note that  $c_\mu(F_B)$  is the smallest soft  $\mu$ -closed superset of  $F_B$ .

**Definition 2.10 [11]**

A proper nonempty open subset U of X is said to be a maximal open set if any open set which contains U is X or U.

**Definition 2.11 [12]**

A proper nonempty soft open subset  $F_K$  of a soft topological space  $(F_A, \tilde{\tau})$  is said to be maximal soft open set if any soft open set which contains  $F_K$  is  $F_A$  or  $F_K$ .

**Definition: 2.12 [6]**

A soft generalized topology  $\mu$  on  $F_A$  is said to be strong if  $F_A \in \mu$ .

**Definition: 2.13 [16]**

A proper non-empty soft  $\mu$ -open subset  $F_K$  of a soft generalized topological space  $(F_A, \mu)$  is said to be soft minimal  $\mu$ -open set if any soft  $\mu$ -open set which is contained in  $F_K$  is  $F_\emptyset$  or  $F_K$ . The family of all soft minimal  $\mu$ -open sets in a soft generalized topological space  $(F_A, \mu)$  is denoted by  $\mathcal{SM}_\mu O(F_A)$ .

**Definition: 2.14 [2]**

Any open subset  $U$  of a topological space  $X$  is said to be a paraopen set if it is neither minimal open nor maximal open set.

**■ Soft Maximal  $\mu$ -open sets:**

**Definition: 3.1**

A proper non-empty soft  $\mu$ -open subset  $F_K$  of a soft generalized topological space  $(F_A, \mu)$  is said to be soft maximal  $\mu$ -open set if any soft  $\mu$ -open set which contains  $F_K$  is  $F_A$  or  $F_K$ . The family of all soft maximal  $\mu$ -open sets in a soft generalized topological space  $(F_A, \mu)$  is denoted by  $\mathcal{SM}_\mu \mathcal{A}(F_A)$ .

**Example: 3.2**

Let  $\mathcal{K} = \{\kappa_1, \kappa_2, \kappa_3, \kappa_4\}$ ,  $E = \{x'_1, x'_2, x'_3\}$ ,  $A = \{x'_1, x'_2\}$ , then  $(F_A, \mu) = \{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}\}$  is a soft generalized topological space where

$$\begin{aligned} F_\emptyset &= \{(x'_1, \phi), (x'_2, \phi)\} \\ F_A &= \{(x'_1, \{\kappa_1, \kappa_2, \kappa_3, \kappa_4\}), (x'_2, \{\kappa_2, \kappa_3, \kappa_4\})\} \\ F_{A_1} &= \{(x'_1, \{\kappa_1, \kappa_2, \kappa_3\}), (x'_2, \{\kappa_2, \kappa_3\})\} \\ F_{A_2} &= \{(x'_1, \{\kappa_1, \kappa_2\}), (x'_2, \{\kappa_2, \kappa_3\})\} \\ F_{A_3} &= \{(x'_1, \{\kappa_3\}), (x'_2, \{\kappa_2\})\} \\ F_{A_4} &= \{(x'_1, \{\kappa_1, \kappa_3\}), (x'_2, \{\kappa_2\})\} \end{aligned}$$

Here  $F_{A_1}$  is a soft maximal  $\mu$ -open set.

Then  $\mathcal{SM}_\mu \mathcal{A}(F_A) = \{(x'_1, \{\kappa_1, \kappa_2, \kappa_3\}), (x'_2, \{\kappa_2, \kappa_3\})\}$

**Proposition: 3.3**

Let  $F_M$  and  $F_N$  be soft  $\mu$ -open subsets of a soft generalized topological space  $(F_A, \mu)$ . If  $F_M$  is a soft maximal  $\mu$ -open set, then  $F_M \tilde{\cup} F_N = F_A$  or  $F_N \tilde{\subseteq} F_M$ .

**Proof:**

Suppose  $F_M \tilde{\cup} F_N \neq F_A$ . Then  $F_N \subseteq F_M \tilde{\cup} F_N$  and  $F_M$  is soft maximal  $\mu$ -open set  $F_M \tilde{\cup} F_N = F_M$ . Hence  $F_N \subseteq F_M$ .

**Proposition: 3.4**

Let  $F_M$  and  $F_N$  be soft maximal  $\mu$ -open subsets of a soft generalized topological space  $(F_A, \mu)$ . If  $F_M$  is a soft maximal  $\mu$ -open set, then  $F_M \tilde{\cup} F_N = F_A$  or  $F_M = F_N$ .

**Proof:**

Suppose  $F_M \tilde{\cup} F_N \neq F_A$ . Then  $F_M \subseteq F_M \tilde{\cup} F_N$  and  $F_N$  is soft maximal  $\mu$ -open set  $F_M \tilde{\cup} F_N = F_M$ . Hence  $F_M \subseteq F_N$ . Similarly,  $F_M$  is soft maximal  $\mu$ -open set, we get  $F_N \subseteq F_M$ . Therefore  $F_M = F_N$ .

**Proposition: 3.5**

Let  $F_M$  be a soft maximal  $\mu$ -open set of a soft generalized topological space  $(F_A, \mu)$ . If  $\alpha \tilde{\in} F_M$ , then  $F_M \tilde{\cup} F_N = F_A$  or  $F_N \subseteq F_M$ , for any soft  $\mu$ -open neighbourhood  $F_N$  of  $\alpha$ .

**Proof:**

Let  $F_M$  be a soft maximal  $\mu$ -open set and  $F_N$  be a soft  $\mu$ -open neighbourhood of  $\alpha$ . If  $F_M \tilde{\cup} F_N = F_A$ , then there is nothing to prove. Suppose  $F_M \tilde{\cup} F_N \neq F_A$ . Then  $F_N \subseteq F_M \tilde{\cup} F_N$  and  $F_M \tilde{\cup} F_N$  is a soft  $\mu$ -open set, we have  $F_M \tilde{\cup} F_N = F_A$  or  $F_M \tilde{\cup} F_N = F_M$ . But by our assumption  $F_M \tilde{\cup} F_N \neq F_A$ . Hence  $F_M \tilde{\cup} F_N = F_M$  which implies  $F_N \subseteq F_M$ .

**Proposition: 3.6**

Let  $F_Z$  be a soft maximal  $\mu$ -open set of a soft generalized topological space  $(F_A, \mu)$ . Then

$F_Z = \tilde{\cup} \{F_X / F_X \tilde{\cup} F_Z \neq F_A \text{ where } F_X \text{ is a soft } \mu\text{-open neighbourhood of } \alpha\}$   
for any element  $\alpha \tilde{\in} F_Z$ .

**Proof:**

Let  $F_X$  be a soft  $\mu$ -open neighbourhood of  $\alpha$ . Then by proposition 3.5,  
 $F_Z \subseteq \tilde{\cup} \{F_X / F_X \tilde{\cup} F_Z \neq F_A \text{ where } F_X \text{ is a soft } \mu\text{-open neighbourhood of } \alpha\} \subseteq F_Z$ .  
Hence  $F_Z = \tilde{\cup} \{F_X / F_X \tilde{\cup} F_Z \neq F_A \text{ where } F_X \text{ is a soft } \mu\text{-open neighbourhood of } \alpha\}$   
for any element  $\alpha \tilde{\in} F_Z$ .

**Proposition: 3.7**

Every soft maximal  $\mu$ -open set is a soft  $\mu$ -open set.

**Proof:**

By definition 3.1, every soft maximal  $\mu$ -open set is a soft  $\mu$ -open set.

**Proposition: 3.8**

Let  $F_Y$  be a soft maximal  $\mu$ -open set of a strong soft generalized topological space  $(F_A, \mu)$  and  $\alpha \notin F_Y$ . Then  $F_Y^c \cong F_V$  for any soft  $\mu$ -open set  $F_V$  containing  $\alpha$ .

**Proof:**

Let  $F_V$  be a soft  $\mu$ -open set containing  $\alpha$  and  $\alpha \notin F_Y$  which implies  $F_V \not\subseteq F_Y$ . By proposition 3.3,  $F_Y \cup F_V = F_A$  which means  $F_Y^c \cong F_V$ .

**Proposition: 3.9**

Let  $F_L$  be a soft maximal  $\mu$ -open set of a strong soft generalized topological space  $(F_A, \mu)$ . Then either of the following holds:

- (i). For each  $\alpha \in F_L^c$  and any soft  $\mu$ -open set  $F_Q$  containing  $\alpha$ , we have  $F_Q = F_A$ .
- (ii). There exists a soft  $\mu$ -open set  $F_Q$  such that  $F_L^c \cong F_Q$  and  $F_Q \cong F_A$ .

**Proof:**

Suppose (i) does not hold. Then there exists  $\alpha \in F_L^c$  and a soft  $\mu$ -open set  $F_Q$  containing  $\alpha$ , such that  $F_Q \cong F_A$  and by proposition 3.8, we get  $F_L^c \cong F_Q$ .

**Proposition 3.10**

Let  $F_L$  be a soft maximal  $\mu$ -open set of a strong soft generalized topological space  $(F_A, \mu)$ . Then either of the following holds:

- (i). For each  $\alpha \in F_L^c$  and any soft  $\mu$ -neighbourhood  $F_Q$  containing  $\alpha$ , we have  $F_L^c \cong F_Q$ .
- (ii). There exists a proper soft  $\mu$ -open set  $F_Q$  such that  $F_L^c = F_Q$ .

**Proof:**

Suppose (ii) does not hold. Then there exists  $\alpha \in F_L^c$  and a soft  $\mu$ -neighbourhood  $F_Q$  containing  $\alpha$  and by proposition 3.9, we get  $F_L^c \cong F_Q$ .

**4. Soft Para  $\mu$ -open sets:**

**Definition: 4.1**

A proper non-empty soft  $\mu$ -open subset  $F_K$  of a soft generalized topological space  $(F_A, \mu)$  is said to be soft para  $\mu$ -open set if it is neither a soft minimal  $\mu$ -open set nor a soft maximal  $\mu$ -open set. The family of all soft para  $\mu$ -open sets in a soft generalized topological space  $(F_A, \mu)$  is denoted by  $SP_{\mu}O(F_A)$ .

**Example: 4.2**

Consider  $\mathcal{K}' = \{\mathcal{k}'_1, \mathcal{k}'_2, \mathcal{k}'_3, \mathcal{k}'_4\}$ ,  $\mathfrak{N}' = \{r'_1, r'_2, r'_3\}$ ,  $\mathfrak{A} = \{r'_1, r'_2\}$ , then  $(F_{\mathfrak{A}}, \mu) = \{F_{\emptyset}, F_{\mathfrak{A}_1}, F_{\mathfrak{A}_2}, F_{\mathfrak{A}_3}\}$  is a soft generalized topological space where

$$\begin{aligned} F_{\emptyset} &= \{(r'_1, \emptyset), (r'_2, \emptyset)\} \\ F_{\mathfrak{A}} &= \{(r'_1, \{\mathcal{k}'_1, \mathcal{k}'_2, \mathcal{k}'_3, \mathcal{k}'_4\}), (r'_2, \{\mathcal{k}'_1, \mathcal{k}'_3, \mathcal{k}'_4\})\} \\ F_{\mathfrak{A}_1} &= \{(r'_1, \{\mathcal{k}'_1, \mathcal{k}'_2, \mathcal{k}'_4\}), (r'_2, \{\mathcal{k}'_1, \mathcal{k}'_4\})\} \\ F_{\mathfrak{A}_2} &= \{(r'_1, \{\mathcal{k}'_1, \mathcal{k}'_2\}), (r'_2, \{\mathcal{k}'_4\})\} \\ F_{\mathfrak{A}_3} &= \{(r'_1, \{\mathcal{k}'_2, \mathcal{k}'_4\}), (r'_2, \{\mathcal{k}'_1, \mathcal{k}'_4\})\} \end{aligned}$$

Here  $F_{\mathfrak{A}_2}$  is a soft para  $\mu$ -open set.

$$\text{Then } SP_{\mu}O(F_{\mathfrak{A}}) = \{(r'_1, \{\mathcal{k}'_1, \mathcal{k}'_2\}), (r'_2, \{\mathcal{k}'_4\})\}$$

**Proposition: 4.3**

Every soft para  $\mu$ -open set is a soft  $\mu$ -open set. But the converse need not be true and is shown by the following illustration.

**Example: 4.4**

Let  $\mathfrak{U}' = \{u'_1, u'_2, u'_3, u'_4, u'_5\}$ ,  $\mathcal{E} = \{\widehat{e}_1, \widehat{e}_2, \widehat{e}_3\}$ ,  $\mathcal{A} = \{\widehat{e}_1, \widehat{e}_2\}$ , then  $(F_{\mathcal{A}}, \mu) = \{F_{\emptyset}, F_{\mathcal{A}_1}, F_{\mathcal{A}_2}, F_{\mathcal{A}_3}, F_{\mathcal{A}_4}\}$  is a SGTS where

$$\begin{aligned} F_{\emptyset} &= \{(\widehat{e}_1, \emptyset), (\widehat{e}_2, \emptyset)\} \\ F_{\mathcal{A}} &= \{(\widehat{e}_1, \{u'_1, u'_2, u'_3, u'_4\}), (\widehat{e}_2, \{u'_1, u'_2, u'_4\})\} \\ F_{\mathcal{A}_1} &= \{(\widehat{e}_1, \{u'_1, u'_2, u'_3\}), (\widehat{e}_2, \{u'_1, u'_2\})\} \\ F_{\mathcal{A}_2} &= \{(\widehat{e}_1, \{u'_2, u'_3\}), (\widehat{e}_2, \{u'_1, u'_2\})\} \\ F_{\mathcal{A}_3} &= \{(\widehat{e}_1, \{u'_3\}), (\widehat{e}_2, \{u'_2\})\} \\ F_{\mathcal{A}_4} &= \{(\widehat{e}_1, \{u'_1, u'_3\}), (\widehat{e}_2, \{u'_2\})\} \end{aligned}$$

**Proposition: 4.5**

Soft union and soft intersection of a soft para  $\mu$ -open set need not be a soft para  $\mu$ -open set.

**Example: 4.6**

Let  $\mathcal{N}' = \{n'_1, n'_2, n'_3, n'_4, n'_5\}$ ,  $P' = \{p'_1, p'_2, p'_3\}$ ,  $\mathbb{A} = \{p'_1, p'_2\}$ , then  $(F_{\mathbb{A}}, \mu) = \{F_{\emptyset}, F_{\mathbb{A}_1}, F_{\mathbb{A}_2}, F_{\mathbb{A}_3}, F_{\mathbb{A}_4}\}$  is a soft generalized topological space where

$$\begin{aligned} F_{\emptyset} &= \{(p'_1, \emptyset), (p'_2, \emptyset)\} \\ F_{\mathbb{A}} &= \{(p'_1, \{n'_1, n'_2, n'_3, n'_4\}), (p'_2, \{n'_2, n'_3, n'_4\})\} \\ F_{\mathbb{A}_1} &= \{(p'_1, \{n'_1, n'_2, n'_3\}), (p'_2, \{n'_2, n'_3\})\} \\ F_{\mathbb{A}_2} &= \{(p'_1, \{n'_1, n'_2\}), (p'_2, \{n'_2, n'_3\})\} \\ F_{\mathbb{A}_3} &= \{(p'_1, \{n'_3\}), (p'_2, \{n'_2\})\} \\ F_{\mathbb{A}_4} &= \{(p'_1, \{n'_1, n'_3\}), (p'_2, \{n'_2\})\} \end{aligned}$$

**Proposition: 4.7**

Let  $(F_A, \mu)$  be a soft generalized topological space with a non-empty soft para  $\mu$ -open subset  $F_P$  of  $(F_A, \mu)$ . Then there exists a soft minimal  $\mu$ -open set  $F_R$  such that  $F_R \subseteq F_P$ .

**Proof:**

By definition of soft minimal  $\mu$ -open set, it is clear that  $F_R \subseteq F_P$ .

**Proposition 4.8**

Let  $(F_A, \mu)$  be a soft generalized topological space with a non-empty soft para  $\mu$ -open subset  $F_D$  of  $(F_A, \mu)$ . Then there exists a soft maximal  $\mu$ -open set  $F_S$  such that  $F_D \subseteq F_S$ .

**Proof:**

By definition of soft maximal  $\mu$ -open set, it is clear that  $F_D \subseteq F_S$ .

**Proposition 4.9**

Let  $(F_A, \mu)$  be a soft generalized topological space. Then the following holds:

- (i). If  $F_W$  is a soft para  $\mu$ -open set and  $F_C$  is a soft minimal  $\mu$ -open set then  $F_W \cap F_C = F_\emptyset$  or  $F_C \subseteq F_W$ .
- (ii). If  $F_V$  is a soft para  $\mu$ -open set and  $F_U$  is a soft maximal  $\mu$ -open set then  $F_U \cup F_V = F_A$  or  $F_V \subseteq F_U$ .
- (iii). If  $F_M$  and  $F_T$  are soft para  $\mu$ -open sets in  $(F_A, \mu)$ , then their intersection is either a soft para  $\mu$ -open set or a soft minimal  $\mu$ -open set.

**Proof:**

(i) Let  $F_W$  be a soft para  $\mu$ -open set and  $F_C$  be a soft minimal  $\mu$ -open set in  $(F_A, \mu)$ . If  $F_W \cap F_C = F_\emptyset$ , then there is nothing to prove. Suppose  $F_W \cap F_C \neq F_\emptyset$ , then  $F_W \cap F_C$  is a soft  $\mu$ -open set and  $F_W \cap F_C \subseteq F_C$  which implies  $F_C \subseteq F_W$ .

(ii) Let  $F_V$  be a soft para  $\mu$ -open set and  $F_U$  be a soft maximal  $\mu$ -open set in  $(F_A, \mu)$ . If  $F_U \cup F_V = F_A$ , then the result follows. Suppose  $F_U \cup F_V \neq F_A$ , then  $F_U \cup F_V$  is a soft  $\mu$ -open set and  $F_V \subseteq F_U \cup F_V$ . Since  $F_U$  is a soft maximal  $\mu$ -open set,  $F_U \cup F_V = F_U$  which implies  $F_V \subseteq F_U$ .

(iii) If  $F_M$  and  $F_T$  are soft para  $\mu$ -open sets in  $(F_A, \mu)$ . If  $F_M \cap F_T$  is a soft para  $\mu$ -open set then the proof is obvious. Suppose  $F_M \cap F_T$  is not a soft para  $\mu$ -open set. Then by definition,  $F_M \cap F_T$  is a soft minimal  $\mu$ -open set or a soft maximal  $\mu$ -open set. If  $F_M \cap F_T$  is a soft minimal  $\mu$ -open set then the result is true. Suppose  $F_M \cap F_T$  is not a soft minimal  $\mu$ -open set, then  $F_M \cap F_T \subseteq F_M$  and  $F_M \cap F_T \subseteq F_T$  which is a contradiction to  $F_M$  and  $F_T$  are soft para  $\mu$ -open sets (By Proposition 4.7). Therefore  $F_M \cap F_T$  is a soft minimal  $\mu$ -open set.

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