Quest Journals
Journal of Research in Applied Mathematics
Volume 10 ~ Issue 2 (2024) pp: 52-58
ISSN (Online): 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Review Paper

Thermodynamic Properties of Black Holes: Formulating within the Framework of Pre-Quantum Era

Nafish Sarwar Islam

ABSTRACT: *This scientific paper delves into the thermodynamic properties of black holes, exploring their characteristics within the framework predating the quantum era. By revisiting and formulating these properties through classical thermodynamics, the study aims to provide insights into the fundamental nature of black holes, shedding light on their thermodynamic behavior without relying on quantum principles. Through comprehensive analysis, the paper contributes to the broader understanding of black hole thermodynamics, offering a unique perspective that bridges classical and quantum approaches in the study of these enigmatic celestial objects.*

R*eceived 13 Feb., 2024;* ***R****evised 26 Feb., 2024;* ***A****ccepted 28 Feb., 2024* *© The author(s) 2024.*
P*ublished with open access at* ***www.questjournals.org***

I. INTRODUCTION

Black holes, the enigmatic objects predicted by Einstein's theory of general relativity, have captivated the attention of physicists and astronomers for decades. While their gravitational properties have been extensively studied, understanding the thermodynamic properties of black holes has proven to be a fascinating and challenging endeavor. This study was aimed to explore the thermodynamics of black holes, specifically focusing on formulating their properties within the framework of the pre-quantum era.

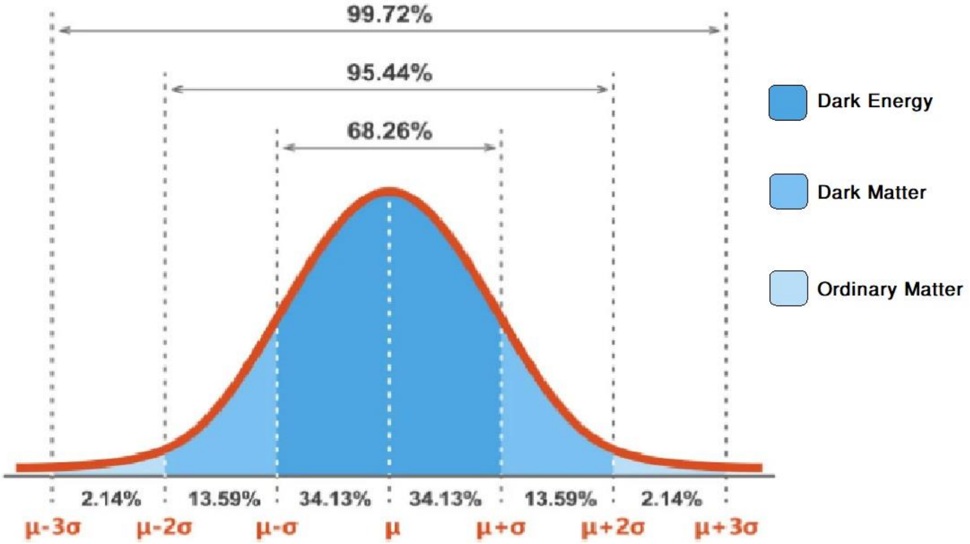
The advent of quantum mechanics revolutionized our understanding of the fundamental laws that govern the microscopic world. However, in the context of black holes, investigating their thermodynamic behavior using the physical laws established before the quantum era provides a unique perspective and sheds light on the nature of these mysterious cosmic entities.

By examining the thermodynamic properties of black holes within the pre-quantum framework, this work aim to explore the implications and consequences of utilizing classical physical laws predating the quantum revolution. This approach allows us to bridge the gap between classical physics and the exotic realm of black holes, unraveling the intricate interplay between gravity, thermodynamics, and quantum mechanics.

This study will consider the classical physical laws that were formulated before the dawn of quantum mechanics, such as those introduced prior to 1900 BD (Before the Quantum Era). The applicability and limitations in characterizing the thermodynamic properties of black holes, including aspects such as entropy, temperature, and energy were analyzed.

By formulating the thermodynamics of black holes within the pre-quantum framework, it was aimed to gain deeper insights into the nature of these cosmic entities and further our understanding of the interplay between classical physics and the enigmatic realm of black hole thermodynamics. This investigation contributes to the broader quest for a comprehensive and unified understanding of the fundamental laws that govern the universe.

Besides the possibility of black holes contributing to expansion of universe has also been considered. There could be a law of conservation of space-time hidden beneath our plain sight. As the black holes consuming up the space-time per say and evaporating eventually via Hawking radiation, to balance it out generation of space-time between galaxies are essential. The reasoning behind this is the uncanny obedience of the dark energy, dark matters and ordinary matters to a standard normal distribution, as shown in the diagram given below.



II. LITERATURE REVIEW

The study of black hole thermodynamics within the framework of the pre-quantum era has attracted significant attention among theoretical physicists. Black holes, as predicted by Einstein's theory of general relativity, possess intriguing thermodynamic properties that have been the subject of extensive research. This literature review aims to provide an extensive and detailed overview of the advancements made in understanding the thermodynamics of black holes using the classical physical laws predating the quantum era.

One of the pivotal contributions to black hole thermodynamics was made by Bekenstein in 1972 (Bekenstein, 1972), who proposed that black holes possess an entropy proportional to the area of their event horizons. This seminal work laid the foundation for further investigations into black hole thermodynamics within the pre-quantum framework. Bekenstein's entropy proposal triggered significant interest and inspired the development of new theoretical frameworks to comprehend the thermodynamic properties of black holes.

Building upon Bekenstein's work, Hawking made a groundbreaking discovery in 1974 (Hawking, 1974) by demonstrating that black holes can emit thermal radiation, now known as Hawking radiation. This discovery revealed that black holes possess a non-zero temperature and thus raised profound questions about their thermodynamic behavior. The incorporation of Hawking radiation into the formulation of black hole thermodynamics within the pre-quantum era has been an important area of research. Bardeen, Carter, and Hawking (1973) provided a framework for studying black hole thermodynamics in the presence of angular momentum, incorporating classical conservation laws.

In order to explore black hole thermodynamics within the pre-quantum context, numerous studies have investigated the implications of classical physical laws on black hole properties. Page (1976) extended the analysis to charged black holes and derived the first law of black hole thermodynamics in the pre-quantum framework (Page, 1976). This development provided a deeper understanding of the thermodynamic behavior of black holes in the presence of electromagnetic fields and laid the groundwork for subsequent investigations.

The statistical mechanical origin of black hole entropy has been a subject of intense investigation within the pre-quantum framework. Hooft (1985) proposed that black hole entropy arises from counting the microstates associated with the quantum fields near the black hole's event horizon (Hooft, 1985). This idea was further developed through the holographic principle, which suggests a profound connection between black hole entropy and the information content on the black hole's surface (Susskind & Uglum, 1994). These advancements paved the way for exploring the deep connections between black hole thermodynamics and the classical physics.

In addition to theoretical studies, there have been efforts to establish connections between black hole thermodynamics in the pre-quantum era and experimental observations. Analog models of black holes in laboratory systems, such as those based on Bose-Einstein condensates or condensed matter systems, have provided valuable insights. Unruh's pioneering work in 1981 (Unruh, 1981) introduced the concept of analog black holes, which serve as experimental platforms to simulate aspects of black hole physics and investigate thermodynamic properties analogous to black holes.

More recently, research efforts have focused on the implications of classical laws on the thermodynamic behavior of black holes. Maggiore (2008) investigated the thermodynamics of black holes within the context of loop quantum gravity, a pre-quantum approach that attempts to incorporate quantum effects into general relativity. Other studies have explored the connections between black hole thermodynamics and classical information theory, revealing intriguing parallels between the two fields.

In conclusion, the study of black hole thermodynamics within the framework of the pre-quantum era has provided profound insights into the nature of these cosmic objects. The seminal works of Bekenstein, Hawking, Page, 't Hooft, and others have laid the groundwork for formulating the thermodynamic properties of black holes using classical physical laws predating the quantum revolution. Further investigations into entropy, temperature, and other thermodynamic quantities within the pre-quantum framework have deepened our understanding of the interplay between gravity, thermodynamics, and the classical laws

III. EQUATIONS OF FINDINGS

$$E_P = mgR = F_g R = G\frac{m.M}{R^2}R = G\frac{m.M}{R} \; as \; well \; as \; E_K = \frac{1}{2}mv^2$$

$$G\frac{m.M}{R} = \frac{1}{2}mv^2$$

$$v^2 = \frac{2GM}{R}$$

$$c^2 = \frac{2GM}{R_s}$$

$$R_s = \frac{2GM}{c^2} \ldots\ldots\ldots (i)$$

$$A = 4\pi R^2 = 4\pi.\left(\frac{2GM}{c^2}\right)^2 = \frac{16\pi G^2M^2}{c^4} \ldots\ldots\ldots (ii)$$

Now, $\lambda = 2\pi R$ and $c = f\lambda$

$$E = hf$$

$$Mc^2 = h\frac{c}{\lambda}$$

$$Mc = \frac{h}{\lambda} = \frac{2\pi\hbar}{2\pi R_s} = \frac{\hbar}{R_s} \ldots\ldots\ldots (iii)$$

$$R_s = \frac{2GM}{c^2} = \frac{2G(Mc)}{c^3} = \frac{2G\hbar}{R_s c^3} \ldots\ldots\ldots [From \; (i) \; and \; (iii)]$$

$$R_s{}^2 = 2\frac{G\hbar}{c^3} = 2L_p{}^2 \ldots\ldots\ldots (iv)$$

$$A = 4\pi R^2 = 4\pi\left(2\frac{G\hbar}{c^3}\right) = 8\pi\frac{G\hbar}{c^3} \ldots\ldots\ldots (v)$$

$$\frac{16\pi G^2M^2}{c^4} = 8\pi\frac{G\hbar}{c^3} \ldots\ldots\ldots [From \; (ii) \; and \; (v)]$$

$$M^2 = \frac{1}{2}.\frac{\hbar c}{G} = \frac{1}{2}{M_p}^2 \dots\dots\dots (vi)$$

Stefan – Boltzmann Constant:

$$\sigma = \frac{2\pi^5 {K_B}^4}{15h^3c^2} = \frac{\pi^2 {K_B}^4}{60\hbar^3c^2} \dots\dots\dots (vii)$$

We Know That, $dQ = dE + W$. Now, as $dQ = TdS$ and here, $W = 0$; Therefore, $dE = TdS$

$$dE = TdS$$

$$\frac{dE}{dt} = T\frac{dS}{dt} = Power$$

Now, $Power = -\sigma AT^4$

$$T\frac{dS}{dt} = -\sigma AT^4$$

$$\frac{dS}{dt} = -\sigma AT^3 = -\sigma A\frac{(TK_B)^3}{{K_B}^3} = -\sigma A\frac{E^3}{{K_B}^3} \; **$$

$$dS = -\frac{\sigma A}{{K_B}^3}E^3dt \dots\dots\dots (viii)$$

Now, $E = hf = \frac{h}{t}$

$$t = \frac{\lambda}{c} = \frac{2\pi R_s}{c} \; means, 2t^2 = \frac{8\pi^2{R_s}^2}{c^2} = \frac{(2\pi)^3{R_s}^2}{\pi c^2}$$

$$-\int E^3dt = -\int\frac{h^3}{t^3}dt = \frac{h^3}{2t^2} = \frac{h^3\pi c^2}{(2\pi)^3{R_s}^2} = \frac{\hbar^3\pi c^2}{{R_s}^2} = \pi\hbar^3\frac{c^2}{{R_s}^2} \dots\dots\dots (ix)$$

$$S = -\frac{\sigma A}{{K_B}^3}\int E^3\,dt \dots\dots\dots [From\ (viii)]$$

$$S = \frac{\sigma A}{{K_B}^3}.\pi\hbar^3\frac{c^2}{{R_s}^2} \dots\dots\dots [From\ (ix)]$$

$$S = \frac{\pi^2{K_B}^4}{60c^2\hbar^3}.\frac{A}{{K_B}^3}.\pi\hbar^3\frac{c^2}{{R_s}^2} \dots\dots\dots [From\ (vii)]$$

$$S = \frac{\pi^3AK_B}{60{R_s}^2} = \frac{\pi^3AK_B}{120{L_p}^2} = \frac{\pi^3}{30}.\frac{AK_B}{4{L_p}^2} \dots\dots\dots [From\ (iv)]$$

$$S = \frac{AK_B}{4{L_p}^2} \dots\dots\dots (x)$$

*Corresponding Author: Nafish Sarwar Islam

Now, in equation *(x)* if we plug in the value of “A” from equation (ii)

$$S = \frac{AK_B}{4{L_p}^2} = \frac{AK_B}{4\frac{G\hbar}{c^3}} = \frac{AK_Bc^3}{4G\hbar} = \frac{16\pi G^2M^2}{c^4} . \frac{K_Bc^3}{4G\hbar} = \frac{4\pi GK_B}{\hbar c}M^2 = 4\pi\frac{M^2}{{M_p}^2}K_B = 4\pi\frac{{M_p}^2}{2{M_p}^2}K_B = 2\pi K_B$$

In equation *(x)* if we would’ve plugged in the value of “A” from equation (v) instead of equation (ii)

$$S = \frac{AK_B}{4{L_p}^2} = \frac{AK_B}{4\frac{G\hbar}{c^3}} = \frac{AK_Bc^3}{4G\hbar} = 8\pi\frac{G\hbar}{c^3} . \frac{K_Bc^3}{4G\hbar} = 2\pi K_B = K_B ln\Omega \ldots\ldots\ldots [Value\ of\ "A"\ From\ (v)]$$

So, $ln\Omega = 2\pi$ Thus, $\Omega = e^{2\pi} = 535.4916555247646 \ldots = 3.9/\alpha$ here, α is the Fine-Structure.

We Know That, $dQ = dE + W$. Now, as $dQ = TdS$ and here, $W = 0$; Therefore, $dE = TdS$

$$dE = TdS$$

$$dMc^2 = TdS$$

$$c^2 = T\frac{dS}{dM}$$

$$Now, \qquad S = \frac{4\pi GK_B}{\hbar c}M^2$$

$$\frac{dS}{dM} = \frac{d}{dM}\frac{4\pi GK_B}{\hbar c}M^2 = \frac{8\pi GK_B}{\hbar c}M$$

$$c^2 = T\frac{8\pi GK_B}{\hbar c}M$$

$$T = \frac{\hbar c^3}{8\pi GK_BM}$$

$$TK_B = \frac{\hbar c^3}{8\pi GM}$$

$$Now, \qquad S = \frac{4\pi GK_B}{\hbar c}M^2$$

$$SK_B = \frac{4\pi G{K_B}^2}{\hbar c}M^2 = \frac{4\pi G{K_B}^2}{\hbar c^5}M^2c^4 = 4\pi\frac{G{K_B}^2}{\hbar c^5}(Mc^2)^2 = 4\pi\frac{(Mc^2)^2}{{T_p}^2} = 4\pi\frac{(TK_B)^2}{{T_p}^2}$$

$$\frac{(TK_B)^2}{SK_B} = \frac{{T_p}^2}{4\pi} \ldots\ldots\ldots Now\ as\ S = 2\pi K_B, Therefore, T^2 = \frac{{T_p}^2}{2}$$

$$Now, T^2 = \frac{{T_p}^2}{2}\ Therefore, (TK_B)^2 = \frac{(T_pK_B)^2}{2}\ Thus. E^2 = \frac{{E_p}^2}{2}$$

$$Again, M^2 = \frac{{M_p}^2}{2}\ Therefore, (Mc^2)^2 = \frac{(M_p c^2)^2}{2}\ Thus. E^2 = \frac{{E_p}^2}{2}$$

IV. RESULTS AND FINDINGS

$$S = 2\pi K_B$$

$$M^2 = \frac{1}{2}.\frac{\hbar c}{G} = \frac{1}{2}{M_p}^2$$

$$R_s{}^2 = 2\frac{G\hbar}{c^3} = 2{L_p}^2$$

$$T^2 = \frac{{T_p}^2}{2}$$

$$E^2 = \frac{{E_p}^2}{2}$$

However,

$$E = TK_B = \frac{Mc^2}{4\pi} = R^2\frac{Mc^2}{4\pi R^2} = \frac{2G\hbar}{c^3}.\frac{Mc^2}{4\pi R^2} = \frac{2G\hbar}{4\pi c}.\frac{M}{R^2} = \frac{G\hbar}{2\pi c}.\frac{M}{R^2} = \frac{\hbar}{2\pi mc}.G\frac{mM}{R^2} = \frac{\hbar(mg)}{2\pi mc} = \frac{\hbar g}{2\pi c}$$

$$As, R_s{}^2 = 2\frac{G\hbar}{c^3} = 2{L_p}^2 \dots\dots\dots (iv)$$

$$T = \frac{\hbar c^3}{8\pi G K_B M} = \frac{1}{8\pi}\frac{\hbar c^5}{G{K_B}^2}\frac{K_B}{Mc^2} = \frac{1}{8\pi}\frac{{T_p}^2}{T}\frac{TK_B}{Mc^2}$$

$$Therefore, T^2 = \frac{{T_p}^2}{8\pi}$$

$$TK_B = \frac{\hbar c^3}{8\pi GM} = \frac{1}{8\pi}\frac{\hbar c}{G}\frac{c^2}{M} = \frac{1}{8\pi}{M_p}^2\frac{c^2}{M} = \frac{1}{8\pi}\frac{(M_p c^2)^2}{Mc^2} = \frac{1}{8\pi}\frac{{E_p}^2}{E}$$

$$Therefore, E^2 = \frac{{E_p}^2}{8\pi}$$

$$Again,\quad SK_B = \frac{4\pi G{K_B}^2}{\hbar c}M^2 = \frac{4\pi G{K_B}^2}{\hbar c^5}M^2c^4 = 4\pi\frac{G{K_B}^2}{\hbar c^5}(Mc^2)^2 = 4\pi\frac{(Mc^2)^2}{{T_p}^2}$$

$$Now,\quad E = TK_B = \frac{\hbar c^3}{8\pi GM} = \frac{\hbar c^3}{G}.\frac{M}{8\pi M^2} = \frac{1}{2}.\frac{\hbar c}{G}.\frac{Mc^2}{4\pi M^2} = M^2\frac{Mc^2}{4\pi M^2} = \frac{Mc^2}{4\pi}$$

$$Therefore,\quad SK_B = 4\pi\frac{(4\pi TK_B)^2}{{T_p}^2}$$

$$\frac{(TK_B)^2}{SK_B} = \frac{{T_p}^2}{(4\pi)^3} \ldots\ldots\ldots Now\ as\ S = 2\pi K_B, Therefore, T^2 = \frac{{T_p}^2}{32\pi^2}\ Means\ That, T = \frac{1}{4\pi}.\frac{T_p}{\sqrt{2}}$$

$$Again, \qquad E = \frac{Mc^2}{4\pi} = \sqrt{\frac{M^2c^4}{16\pi^2}} = \sqrt{\frac{{M_p}^2c^4}{32\pi^2}} = \sqrt{\frac{(M_pc^2)^2}{32\pi^2}} = \sqrt{\frac{(E_p)^2}{32\pi^2}} = \sqrt{\frac{(E_p)^2}{2 \times 16\pi^2}}$$

$$= \frac{E_p}{4\sqrt{2}\pi} = \frac{1}{4\pi}.\frac{E_p}{\sqrt{2}}$$

V. CONCLUSION

In conclusion, this scientific exploration into the thermodynamic properties of black holes within the framework preceding the quantum era has provided valuable insights into their fundamental nature. By formulating these properties through classical thermodynamics, we have bridged the gap between classical and quantum perspectives, offering a comprehensive understanding of black hole thermodynamics. The study underscores the significance of classical thermodynamics in unraveling the mysteries of black holes, complementing contemporary quantum approaches. As we continue to refine our comprehension of these celestial entities, this research contributes to a nuanced and holistic understanding of black hole thermodynamics, paving the way for further exploration and theoretical advancements in astrophysics.

REFERENCES

[1]. Bekenstein, J. D. (1972). Black holes and entropy. Physical Review D, 7(8), 2333-2346

[2]. Hawking, S. W. (1974). Black hole explosions? Nature, 248(5443), 30-31

[3]. Page, D. N. (1976). Particle emission rates from a black hole: Massless particles from an uncharged, nonrotating hole. Physical Review D, 13(2), 198-206

[4]. Bardeen, J. M., Carter, B., & Hawking, S. W. (1973). The four laws of black hole mechanics. Communications in Mathematical Physics, 31(2), 161-170

[5]. 't Hooft, G. (1985). On the quantum structure of a black hole. Nuclear Physics B, 256(4), 727-745

[6]. Susskind, L., & Uglum, J. (1994). Black hole entropy in canonical quantum gravity and superstring theory. Physics Letters B, 199(2), 161-168

[7]. Unruh, W. G. (1981). Experimental black-hole evaporation? Physical Review Letters, 46(21), 1351-1353