*Quest Journals*
*Journal of Research in Applied Mathematics*
*Volume 10 ~ Issue 5 (2024) pp: 24-32*
*ISSN (Online): 2394-0743 ISSN (Print): 2394-0735*
www.questjournals.org

**Review Paper**

# Generalized Eccentricity $k^{th}$ Power of Product Adjacency Energy of Graphs $(E(GE^kPA(G)))$

M.MUTHARASI [1] and M. DEVA SAROJA[2]
*[1]Research Scholar (Register Number: 19221172092014)*
*[2]Research Supervisor, Assistant Professor,*
*[1,2] PG and Research Department of Mathematics,*
*Rani Anna Government College for Women, Tirunelveli - 627 008. (India)*
*Affiliated to Manonmaniam Sundaranar University, Abishekapatti,*
*Tirunelveli - 627 012.*

***Abstract***
*Let G be a finite, simple, and undirected graph. For any integer $1 \leq k < \infty$, the generalized eccentricity $k^{th}$ power of product adjacency matrix of G is $m \times m$ matrix with its $(i,j)^{th}$ entry as $e(v_i)^k e(v_j)^k$, if $v_i$ adjacent to $v_j$ and zero otherwise, where $e(v)$ is the eccentricity of the vertex v of a graph G. In this paper, we introduce the generalized eccentricity $k^{th}$ power of product adjacency energy of some standard graphs, which is denoted by $E(GE^kPA(G))$.*

***Keywords:*** *Eccentricity, generalized eccentricity $k^{th}$ power of product adjacency matrix, generalized eccentricity $k^{th}$ power of product adjacency polynomial, eigenvalues and generalized eccentricity $k^{th}$ power of product adjacency energy.*
***2010 Mathematics Subject Classification:*** *05C50*

***R**eceived 29 Apr, 2024; **R**evised 06 May, 2024; **A**ccepted 08 May, 2024 © The author(s) 2024.*
***P**ublished with open access at **www.questjournals.org***

## I. Introduction

Let G be a finite and undirected simple graph on m vertices named by $v_1, v_2, \cdots, v_m$. Then the adjacency matrix A(G) of the graph G is a square matrix of order m, whose $(i,j)^{th}$entry is equal to 1 if the vertices $v_i$ and $v_j$ are adjacent and equal to zero otherwise. The characteristic polynomial of the adjacency matrix, ie., $\det(\eta I_m - A(G))$, where I is the unit matrix of order m, is said to be the characteristic polynomial of the graph G and will be denoted by $P(G,\eta)$. The eigenvalue of a graph G is defined as the eigenvalues of its adjacency matrix A(G), and so they are just the roots of the equation $P(G,\eta) = 0$ since A(G) is a real symmetric matrix, so its eigenvalues are all real. Denoting them by $\eta_1, \eta_2, \cdots, \eta_m$ and as a whole, they are called the spectrum of G. In 1970, I.Gutman introduced the concept of the energy of G. [6]

## II. Preliminaries

**Lemma 2.1 [2]**

Let M, N, P and Q be matrices with M invertible. Then we have $\begin{vmatrix} M & N \\ P & Q \end{vmatrix} = |M||Q - PM^{-1}N|$

**Lemma 2.2 [2]**

Let M, N, P and Q be matrices. Let $S = \begin{pmatrix} M & N \\ P & Q \end{pmatrix}$ if M and P commutes. Then $|S| = |MQ - PN|$.

**Lemma 2.3 [3]**

If $A(K_p)$ is the adjacency matrix of $K_p$, then $A^2(K_P) = (p-2)A(K_p) + (p-1)I_p$.

**Definition 2.4 [3]**

Let $K_{2p}$ be a complete graph with vertices $2p, p = 1,2,\dots,n$. We delete the edge joining the vertices $i$ and $p+i, 1 \le i \le p$. The resulting graph $D_1(K_{2p})$ has the order $2p$ and has $2p(p-1)$ edges. Further it is regular of degree $2p-2$.

**Definition 2.5 [3]**

Consider the complete graph $K_{2p}$ with $2p$ vertices. We split the vertices into two equal parts and delete the edges between that spilted parts. We obtain a disconnected graph such a graph is of order $2p$ and has $p(p-1)$ edges. Further it is regular of degree $p-1$. We denote it by $D_2(K_{2p})$.

**Definition 2.6 [3]**

Consider the complete graph $K_{2p}$ with $2p$ vertices. We split the vertices into two equal parts such that the vertices 1 to $p$ in one part and $p+1$ to $2p$ in the other part. Now delete the edges between the vertices in the same parts also edges joining $i$ and $p+i$, $1 \le i \le p$. The resulting graph is of order $2p$ and has $p(p-1)$ edges. Further it is regular of degree $p-1$. We denote it by $D_3(K_{2p})$.

**Definition 2.7 [3]**

Consider a pair of complete graphs $K_p$ with vertex set $\{v_i, i = 1,2,3,\dots p\}$ and $\{u_j, j = 1,2,3,\dots p\}$. We obtain a graph joining $v_i$ to $u_i$, for $i = 1,2,3,\dots p$. Such a graph is of order $2p$ and $p^2$ edges. Further it is regular of degree p. We denote it by $J(K_p{}^p)$.

**Definition 2.8 [9]**

$K_{1,1,n}$ is a graph obtained by attaching root of a star $K_{1,n}$ at one end of $P_2$ and other end of $P_2$ is joined with each pendant vertex of $K_{1,n}$.

**Definition 2.9 [10]**

A Globe graph $Gl_{(n)}$ is a graph obtained from two isolated vertex are joined by n paths of length 2.

**Definition 2.10 [11]**

Let $G = (V, X)$ be a connected simple graph with $|V| = m$ vertices and $|E| = q$ edges and let $e(v_i)$ denote the eccentricity of the vertex $v_i$, for $i = 1,2,\cdots,m$. For vertices $v_i, v_j \in V(G)$, the distance $d(v_i, v_j)$ is defined as the length of the shortest path between $v_i$ and $v_j$ in G. The eccentricity of a vertex is the maximum distance from it to any other vertex. $e(v_i) = \max_{v_i \in V(G)} d(v_i, v_j)$.

## III Main Result

### 3. Generalized eccentricity $k^{th}$ power of product adjacency energy of some standard graphs

**Definition 3.1**

Let G be a graph with $m$ vertices and $q$ edges. For any integer $1 \le k < \infty$, the generalized eccentricity $k^{th}$ power of product adjacency matrix of $G$ is denoted by $GE^kPA(G) = [ge^kpa_{ij}]$ is determined as

$$[ge^kpa_{ij}] = \begin{cases} e^k(v_i)e^k(v_j), & if\ v_i\ adjacent\ to\ v_j \\ 0, & otherwise \end{cases}.$$

The generalized eccentricity $k^{th}$ power of product adjacency energy of $G$ is denoted by $E(GE^kPA(G)) = \sum_{i=1}^{m} |\eta_i|$, where $\eta_1, \eta_2, \cdots, \eta_m$ are eigenvalues of $GE^kPA(G)$.

**Theorem 3.2**

Let $K_m$ be a complete graph. Then $E(GE^kPA(K_m)) = 2(m-1)$, where $m \geq 2$.

**Proof:**

Let $K_m$ be a complete graph with m vertices for $m \geq 2$.

Since $K_m$ is connected graph with $e(v_i) = 1,\ 1 \leq k \leq m$, we get

$$[ge^kpa_{ij}](K_m) = \begin{cases} 1^{2k}, & if\ v_i\ adjacent\ to\ v_j \\ 0, & otherwise \end{cases}$$

and the generalized eccentricity $k^{th}$ power product adjacency eigenvalues of $K_m$ are $-1$ of multiplicity $(m-1)$ and $(m-1)$ of multiplicity 1 respectively. Hence $E(GE^kPA(K_m)) = 2(m-1)$.

**Theorem 3.3**

Let $K_{m,m}$ be a complete bipartite graph. Then $E(GE^kPA(K_{m,m})) = 2(2^{2k}m)$, where $m \geq 2$.

**Proof:**

Let $K_{m,m}$ be a complete bipartite graph of order $2m$ and $m^2$ edges.

Then $[ge^kpa_{ij}](K_{m,m}) = \begin{cases} 2^{2k}, & if\ v_i\ adjacent\ to\ v_j \\ 0, & otherwise \end{cases}$.

The generalized eccentricity $k^{th}$ power product adjacency matrix of $K_{m,m}$ is, $GE^kPA(K_{m,m}) = \begin{bmatrix} 0 & 2^{2k}J \\ 2^{2k}J & 0 \end{bmatrix}$

where $J = \begin{pmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{pmatrix}$

Therefore, $P(GE^kPA(K_{m,m}),\eta) = |\eta I_m - GE^kPA(K_{m,m})|$

$$= \begin{vmatrix} \eta I_m & -2^{2k}J \\ -2^{2k}J & \eta I_m \end{vmatrix}$$

$$= (\eta I_m - 2^{2k}J)\ (\eta I_m + 2^{2k}J)$$

$$= (\eta I_m - 2^{2k}m)\ (\eta I_m + 2^{2k}m)\eta^{2m-2}$$

Hence $S_p(GE^kPA(K_{m,m})) = \begin{pmatrix} 2^{2k}m & -2^{2k}m & 0 \\ 1 & 1 & 2m-2 \end{pmatrix}$ and

$E(GE^kPA(K_{m,m})) = 2(2^{2k}m)$.

**Theorem 3.4**

Let $K_{1,m}$ be a star graph. Then $E(GE^kPA(K_{1,m})) = 2(2^k)\sqrt{m}$ , where $m \geq 2$.

**Proof:**

Let $K_{1,m}$ be a star graph of order $m+1$ and $m$ edges.

Then $[ge^kpa_{ij}](K_{1,m}) = \begin{cases} 2^k, & if\ v_i\ adjacent\ to\ v_j \\ 0, & otherwise \end{cases}$.

The generalized eccentricity $k^{th}$ power product adjacency matrix of $K_{1,m}$ is, $GE^kPA(K_{1,m}) = \begin{bmatrix} 0 & 2^k & 2^k & \cdots & 2^k \\ 2^k & 0 & 0 & \cdots & 0 \\ 2^k & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 2^k & 0 & 0 & \cdots & 0 \end{bmatrix}$.

Therefore, $P(GE^kPA(K_{1,m}),\eta) = |\eta I_m - GE^kPA(K_{1,m})|$

$$= \begin{vmatrix} \eta I & -2^k & -2^k & \cdots & -2^k \\ -2^k & \eta I & 0 & \cdots & 0 \\ -2^k & 0 & \eta I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -2^k & 0 & 0 & \cdots & \eta I \end{vmatrix}$$

$$= \eta^{m-1}(\eta^2 - (2^k)^2 m)$$

Hence $S_p(GE^kPA(K_{1,m})) = \begin{pmatrix} 2^k\sqrt{m} & -2^k\sqrt{m} & 0 \\ 1 & 1 & m-1 \end{pmatrix}$ and

$E(GE^kPA(K_{1,m})) = 2(2^k\sqrt{m})$ .

**4. Generalized eccentricity $k^{th}$ power product adjacency energy of some regular graphs obtained by complete graph**

**Theorem 4.1**

Let $D_1(K_{2m})$ be the edge deleting graph 1 of $K_{2m}$. Then $E(GE^kPA(D_1(K_{2m}))) = 2^{2k+2}(m-1)$, where $m \geq 2$.

**Proof:**

Let $D_1(K_{2m})$ be the edge deleting graph 1 of order $2m$, $m = 1,2,\cdots,n$ and $2m\,(m-1)$ edges. Then $[ge^kpa_{ij}](D_1(K_{2m})) = \begin{cases} 2^{2k}, & if\ v_i\ adjacent\ to\ v_j \\ 0, & otherwise \end{cases}$.

The generalized eccentricity $k^{th}$ power product adjacency matrix of $D_1(K_{2m})$ is, $GE^kPA(D_1(K_{2m})) = \begin{bmatrix} 2^{2k}A(K_m) & 2^{2k}A(K_m) \\ 2^{2k}A(K_m) & 2^{2k}A(K_m) \end{bmatrix}$.

Therefore, $P(GE^kPA(D_1(K_{2m})),\eta) = |\eta I_m - GE^kPA(D_1(K_{2m}))|$

$$= \begin{vmatrix} \eta I_m - 2^{2k}A(K_m) & -2^{2k}A(K_m) \\ -2^{2k}A(K_m) & \eta I_m - 2^{2k}A(K_m) \end{vmatrix}$$

$$= |(\eta I_m - 2^{2k}A(K_m))^2 - (2^{2k}A(K_m))^2|$$

$$= |\eta^2 I_m - 2\eta(2^{2k}A(K_m))|$$

$$= (2\eta)^m \left|\frac{\eta^2}{2\eta} I_m - 2^{2k}A(K_m)\right|$$

$$= (2\eta)^m (\frac{\eta}{2} - 2^{2k}(m-1))(\frac{\eta}{2} + 2^{2k})^{m-1}$$

$$= \eta^m(\eta - 2^{2k+1}(m-1))(\eta + 2^{2k+1})^{m-1}$$

Hence $S_p(GE^kPA(D_1(K_{2m}))) = \begin{pmatrix} 2^{2k+1}(m-1) & -2^{2k+1} & 0 \\ 1 & m-1 & m \end{pmatrix}$ and

$E(GE^kPA(D_1(K_{2m}))) = 2^{2k+2}(m-1)$.

**Theorem 4.2**

Let $D_3(K_{2m})$ be the edge deleting graph 3 of $K_{2m}$. Then $E\,(GE^kPA(D_3(K_{2m}))) = 4(3^{2k})(m-1)$, where $m \geq 3$.

**Proof:**

Let $D_3(K_{2m})$ be the edge deleting graph 3 of $K_{2m}$ order $2m$, $m = 3,4,\cdots,n$ and $m(m-1)$ edges. Then $[ge^kpa_{ij}](D_3(K_{2m})) = \begin{cases} 3^{2k}, & if\ v_i\ adjacent\ to\ v_j \\ 0, & otherwise \end{cases}$.

The generalized eccentricity $k^{th}$ power product adjacency matrix of $D_3(K_{2m})$ is, $GE^kPA(D_3(K_{2m})) = \begin{bmatrix} 0 & 3^{2k}A(K_m) \\ 3^{2k}A(K_m) & 0 \end{bmatrix}$.

Therefore, $P(GE^kPA(D_3(K_{2m})),\eta) = |\eta I_m - GE^kPA(D_3(K_{2m}))|$

$$= \begin{vmatrix} \eta I_m & -3^{2k}A(K_m) \\ -3^{2k}A(K_m) & \eta I_m \end{vmatrix}$$

$$= |\eta I_m| \left|\eta I_m - \frac{(3^{2k}A(K_m))^2}{\eta}\right|$$

$$= \eta^m \left|\eta I_m - (3^{4k})(\frac{(m-2)A(K_m)+(m-1)I_m}{\eta})\right|$$

$$= |\eta^2 I_m - (3^{4k})(m-2)A(K_m) - (3^{4k})(m-1)I_m|$$

$$= (m-2)^m \left|\left(\frac{\eta^2-(3^{4k})(m-1)}{m-2}\right) I_m - (3^{4k})A(K_m)\right|$$

$$=(m-2)^m \left(\frac{\eta^2-(3^{4k})(m-1)}{m-2} - (3^{4k})(m-1)\right)$$

$$(\frac{\eta^2-(3^{4k})(m-1)}{m-2} + (3^{4k}))^{m-1}$$

$$= (\eta^2 - (3^{4k})(m-1)^2)(\eta^2 - (3^{4k}))^{m-1}$$

Hence $S_p(\mathrm{GE^kPA}(D_3(K_{2m})) = \begin{pmatrix} -(3^{2k})(m-1) & (3^{2k})(m-1) & -3^{2k} & 3^{2k} \\ 1 & 1 & m-1 & m-1 \end{pmatrix}$

and $E(GE^kPA(D_3(K_{2m}))) = 4(3^{2k})(m-1)$.

**Theorem 4.3**

Let $J({K_m}^m)$ be the join of a complete graph. Then $E(GE^kPA\,(J({K_m}^m))) = 2\,(2^{2k+1})\,(m-1)$, where $m \geq 3$.

**Proof:**

Let $J({K_m}^m)$ be the join of a complete graph of order $2m$ and $m^2$ edges.

Then $[ge^kpa_{ij}](J({K_m}^m)) = \begin{cases} 2^{2k}, & if\ v_i\ adjacent\ to\ v_j \\ 0, & otherwise \end{cases}$.

The generalized eccentricity $k^{th}$ power product adjacency matrix of $J({K_m}^m)$ is, $GE^kPA(J({K_m}^m)) = \begin{bmatrix} 2^{2k}A(K_m) & 2^{2k}(I_m) \\ 2^{2k}(I_m) & 2^{2k}A(K_m) \end{bmatrix}$.

Therefore, $P(GE^kPA(J(K_m{}^m)),\eta) = |\eta I_m - GE^kPA(J(K_m{}^m))|$

$$= \begin{vmatrix} \eta I_m - 2^{2k}A(K_m) & -2^{2k}(I_m) \\ -2^{2k}(I_m) & \eta I_m - 2^{2k}A(K_m) \end{vmatrix}$$

$$= (\eta I_m - 2^{2k}A(K_m))^2 - (2^{2k}(I_m))^2$$

$$= \big((\eta - 2^{2k})I_m - 2^{2k}(m-1)\big)\big((\eta - 2^{2k})I_m + 2^{2k}\big)^{m-1}$$

$$((\eta + 2^{2k})I_m - 2^{2k}(m-1))((\eta + 2^{2k})I_m + 2^{2k})^{m-1}$$

$$= \eta^{m-1}\big(\eta - 2^{2k}(m)\big)(\eta + 2^{2k}(2-m)(\eta + 2^{2k+1})^{m-1}$$

Hence $S_p(GE^KPA(J(K_m{}^m))) = \begin{pmatrix} 2^{2k}(m) & 2^{2k}(m-2) & -2^{2k+1} & 0 \\ 1 & 1 & m-1 & m-1 \end{pmatrix}$

and $E(GE^kPA(J(K_m{}^m))) = 2(2^{2k+1})(m-1)$.

## 5. Generalized eccentricity $k^{th}$ power of product adjacency energy of complement of regular graph obtained from complete graph

The complement graphs of $D_1(K_{2m})$, $D_2(K_{2m})$, $D_3(K_{2m})$ and $J(K_m{}^m)$ are denoted by $\overline{D_1(K_{2m})}$, $\overline{D_2(K_{2m})}$,$\overline{D_3(K_{2m})}$ and $\overline{J(K_m{}^m)}$. In [4], $\bar{A} = J - I - A$, where $\bar{A}$ is the adjacency matrix of complement graph.

**Theorem 5.1**

Let $\overline{D_2(K_{2m})}$ be the complement of edge deleting graph 2 of $K_{2m}$. Then $E(GE^kPA(\overline{D_2(K_{2m})})) = 2^{2k+1}(m)$, where $m \geq 2$.

**Proof:**

Let $\overline{D_2(K_{2m})}$ be the complement of edge deleting graph 2 of $K_{2m}$. Then the generalized eccentricity $k^{th}$ power product adjacency matrix of $\overline{D_2(K_{2m})}$ is, $GE^kPA(\overline{D_2(K_{2m})}) = \begin{bmatrix} 0 & 2^{k+1}(J) \\ 2^{k+1}(J) & 0 \end{bmatrix}$, where $J = \begin{pmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{pmatrix}$

Therefore, $P(GE^kPA(\overline{D_2(K_{2m})}),\eta) = |\eta I_m - GE^kPA(\overline{D_2(K_{2m})})|$

$$= \begin{vmatrix} \eta I_m & -2^{2k}(J) \\ -2^{2k}(J) & \eta I_m \end{vmatrix}$$

Hence $S_p\left(GE^kPA(\overline{D_2(K_{2m})})\right) = \begin{pmatrix} -2^{2k}(m) & 2^{2k}(m) & 0 \\ 1 & 1 & 2m-2 \end{pmatrix}$

and $E(GE^kPA(\overline{D_2(K_{2m})})) = 2^{2k+1}(m)$.

**Theorem 5.2**

Let $\overline{D_3(K_{2m})}$ be the complement of edge deleting graph 3 of $K_{2m}$. Then $E(GE^kPA(\overline{D_3(K_{2m})})) = 2(2^{2k+1})(m-1)$.

**Proof:**

Let $\overline{D_3(K_{2m})}$ be the complement of edge deleting graph 3 of $K_{2m}$. Then the generalized eccentricity $k^{th}$ power product adjacency matrix of $\overline{D_3(K_{2m})}$ is, $GE^kPA(\overline{D_3(K_{2m})}) = \begin{bmatrix} 2^{2k}A(K_m) & 2^{2k}I_m \\ 2^{2k}I_m & 2^{2k}A(K_m) \end{bmatrix}$

$= GE^kPA(J(K_m{}^m))$ (by theorem 4.3)

Since $E(GE^kPA(J(K_m{}^m))) = 2(2^{2k+1})(m-1)$.

Hence we get $E(GE^kPA(\overline{D_3(K_{2m})})) = 2(2^{2k+1})(m-1)$.

**Theorem 5.3**

Let $\overline{J(K_m{}^m)}$ be the complement of join of a complete graph. Then $E(GE^kPA(\overline{J(K_m{}^m)})) = 4(3^k)(m-1)$, where $m \geq 3$.

**Proof:**

Let $\overline{J(K_m{}^m)}$ be the complement of join of a complete graph. Then the generalized eccentricity $k^{th}$ power product adjacency matrix of $\overline{J(K_m{}^m)}$ is, $GE^kPA(\overline{J(K_m{}^m)}) = \begin{bmatrix} 0 & 3^{2k}A(K_m) \\ 3^{2k}A(K_m) & 0 \end{bmatrix}$

$= GE^kPA(D_3(K_{2m}))$ (by theorem 4.2)

Since $E(GE^kPA(D_3(K_{2m}))) = 4(3^{2k})(m-1)$.

Hence we get $E(GE^kPA(\overline{J(K_m{}^m)})) = 4(3^{2k})(m-1)$.

## 6. Generalized eccentricity $k^{th}$ power product adjacency energy of some irregular graphs

**Theorem 6.1**

Let $F_m$ be a friendship graph. Then $E(GE^kPA(F_m)) = 4^k(2m)$, where $m \geq 2$.

**Proof:**

Let $F_m$ be a friendship graph with $2m+1$ vertices.Then the generalized eccentricity $k^{th}$ power product adjacency matrix is,

$$GE^KPA(F_m) = \begin{bmatrix} 0 & 2^k & 2^k & \cdots & 2^k & 2^k \\ 2^k & 0 & 2^{2k} & \cdots & 0 & 0 \\ 2^k & 2^{2k} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2^k & 0 & 0 & \cdots & 0 & 2^{2k} \\ 2^k & 0 & 0 & \cdots & 2^{2k} & 0 \end{bmatrix}.$$

Therefore, $P(GE^kPA(F_m),\eta) = |\eta I_m - GE^kPA(F_m)|$

$$= (\eta^2 - 4^k\eta - 4^k(2m))(\eta - 4^k)^{m-1}(\eta + 4^k)^m.$$

Hence $S_p(GE^KPA(F_m)) = \begin{pmatrix} \frac{4^k - \sqrt{4^k(4^k+8m)}}{2} & \frac{4^k + \sqrt{4^k(4^k+8m)}}{2} & 4^k & -4^k \\ 1 & 1 & m-1 & m \end{pmatrix}$

and $E(GE^KPA(F_m)) = 4^k(2m)$.

**Theorem 6.2**

Let $Gl_m$ be a globe graph. Then $E(GE^kPA(Gl_m)) = 2\sqrt{16^k(2m)}$.

**Proof:**

Let $Gl_m$ be a globe graph with $m+2$ vertices. Then the generalized eccentricity $k^{th}$ power product adjacency matrix is,

$$GE^kPA(Gl_m) = \begin{bmatrix} 0 & 0 & 2^{2k} & 2^{2k} & \cdots & 2^{2k} & 2^{2k} \\ 0 & 0 & 2^{2k} & 2^{2k} & \cdots & 2^{2k} & 2^{2k} \\ 2^{2k} & 2^{2k} & 0 & 0 & \cdots & 0 & 0 \\ 2^{2k} & 2^{2k} & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2^{2k} & 2^{2k} & 0 & 0 & \cdots & 0 & 0 \\ 2^{2k} & 2^{2k} & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}.$$

Therefore, $P(GE^kPA(Gl_m),\eta) = |\eta I - GE^kPA(Gl_m)|$

$$= (\eta^2 - 16^k(2m))(\eta)^m$$

Hence $S_p(GE^KPA(Gl_m)) = \begin{pmatrix} -\sqrt{16^k(2m)} & \sqrt{16^k(2m)} & 0 \\ 1 & 1 & m \end{pmatrix}$

and $E\,(GE^KPA(Gl_m)) = 2\sqrt{16^k(2m)}$ .

**Theorem 6.3**

Let $K_{1,1,m}$ be a graph. Then $E(GE^kPA\left(K_{1,1,m}\right)) = 2 \pm \frac{1}{2}(\sqrt{1+4^{k+1}(2m)})$ .

**Proof:**

Let $K_{1,1,m}$ be a graph with $m+2$ vertices. Then the generalized eccentricity $k^{th}$ power product adjacency matrix is,

$$GE^kPA\left(K_{1,1,m}\right) = \begin{bmatrix} 0 & 1 & 2^{k} & 2^{k} & \cdots & 2^{k} & 2^{k} \\ 1 & 0 & 2^{k} & 2^{k} & \cdots & 2^{k} & 2^{k} \\ 2^{k} & 2^{k} & 0 & 0 & \cdots & 0 & 0 \\ 2^{k} & 2^{k} & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2^{k} & 2^{k} & 0 & 0 & \cdots & 0 & 0 \\ 2^{k} & 2^{k} & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}.$$

Therefore, $P(GE^kPA\left(K_{1,1,m}\right),\eta) = \left|\eta I - GE^kPA\left(K_{1,1,m}\right)\right|$

$$= (\eta)^{m-1}(\eta+1)(\eta^2 - \eta - 4^k(2m)).$$

Hence $S_p(GE^KPA(Gl_m)) = \begin{pmatrix} \frac{1}{2}(1-\sqrt{1+4^{k+1}(2m)}) & \frac{1}{2}(1+\sqrt{1+4^{k+1}(2m)}) & -1 & 0 \\ 1 & 1 & 1 & m-1 \end{pmatrix}$

and $E(GE^kPA\left(K_{1,1,m}\right)) = 2 \pm \frac{1}{2}(\sqrt{1+4^{k+1}(2m)})$ .

## Reference


[1]. Bukley F, Harary F istance in Graphs. Addison Wesley, Redwood (1990).

[2]. Cvetkovic D, Rowlinson P and Simic S. An Introduction to the theory of Graph Spectra. Cambridge University Press, Cambridge (2010).

[3]. M. Deva Saroja, M.S Paulraj, "Equienergetic regular graphs", International Journal of Algorithms, Computing and Mathematics Vol. 3, No.3, (2010) 21 - 25.

[4]. M. Deva Saroja, M.S Paulraj, "Energy of complement graphs of some equienergetic regular graphs", Journal of Computer and Mathematical Sciences Vol. 1, Issue 6, (2010) 754 - 757.

[5]. Fathima, "Generalized Eccentricity K[th] power sum energy of graphs", Mathematical Statistician and Engineering Applications, Vol. 72, No. 1, (2023) 1062 - 1069.

[6]. Fathima, "Generalized Eccentricity K$^{th}$ power Product energy of graphs", South East Asian Journal of Mathematics and Mathematical Sciences, Vol. 19, Proceeding (2010), 35 - 42.
[7]. Gutman I. The Energy of a Graph. Ber. Math. Statist. Sekt. Forschungsz. Graz, (1978) 103, 1 - 22.
[8]. M.Mutharasi, M. Deva Saroja, "Combined Sum Eccentricity Adjacency Energy", NeuroQuantology, Vol. 20, Issue 19, (2022) 5155 – 5163.
[9]. M.Mutharasi, M. Deva Saroja, "Generalized Eccentricity K$^{th}$ Power Sum Adjacency Energy of Graphs", Vol. 44, No. 2, (2023) 800 – 805.
[10]. A.Nellai Murugan, S.F.M. Robina, "Tree Related Analytic mean cordial graphs", International Journal of Applied Science Engineering and Management, Vol. 2, Issue 10, (2016) 34 - 51.
[11]. Shanti S. Khunti, Mehul A. Chaursiya and Mehul P. Rupani, "Maximum Eccentricity of globe graph, bistar graph and some graph of its related to bistar graph", Malaya Journal of Mathematik, Vol. 8, No. 4, (2020) 1521-1526.
[12]. N. Prabhavathy, " A new concept of energy from eccentricity matrix of graphs", Malaya Journal of Matematik, Vol. S, No. 1, (2019) 400 - 402.