Quest Journals
Journal of Research in Applied Mathematics
Volume 10 ~ Issue 5 (2024) pp: 66-72
ISSN (Online): 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Review Paper

New exact solutions of Schamel-Zakharov-Kuznetsov-Burgers' equation in plasma

Bingnuo Yang[a,*], Weinan Wu[a,**], Hongfeng Yu[a,***], and Peng Guo[a,****]
[a] *School of Mathematics and Physics, Lanzhou Jiaotong University, Gansu Lanzhou, 730070, China*

Abstract. *The Schamel-Zakharov-Kuznetsov-Burgers' (SZKB) equation is derived from the basic equation of long wave approximation in the dynamics of small and finite ionic acoustic shock waves in plasma. In this paper, the new exact solutions of SZKB equation, a nonlinear equation in plasma, will be solved by using Paul-Painlev′e approach (PPA) method. This study will enrich the theory and connotation of plasma research by studying such nonlinear equation in plasma.*
Keywords: *Plasma;Schamel-Zakharov-Kuznetsov-Burgers' (SZKB) equation; Paul-Painlev′e approach (PPA); nonlinear equation; exact solution.*

***R**eceived 15 May, 2024; **R**evised 26 May, 2024; **A**ccepted 28 May, 2024 © The author(s) 2024.*
P**ublished with open access at **www.questjournals.org

I. Introduction

In order to investigate the dynamics of small but finite amplitude ion-acoustic shock wave in our plasma, we derive the SZKB equation from the basic equations in the long-wave approximation [1].A lot of scholars have done a lot of research on this equation [2-4]. By introducing the stretched space and time variables, we obtain the following (3+1)- dimensional SZKB equation for the first approximation of the electrostatic wave potential [5]. The form is as follows:

$$\varphi_t + A\varphi^{\frac{1}{2}}\varphi_z + B\varphi_{zzz} + C\varphi_z\varphi_{xx} + C\varphi_z\varphi_{yy} + D\varphi_{xx} + D\varphi_{yy} + D\varphi_{zz} = 0, \quad (1)$$

where the nonlinear coefficient *A*, the dispersive coefficients *B* and *C*, the dissipative coefficient *D*.

Sort out

$$\frac{\partial\varphi}{\partial t} + A\varphi^{\frac{1}{2}}\frac{\partial\varphi}{\partial z} + B\frac{\partial^3\varphi}{\partial z^3} + C\frac{\partial\varphi}{\partial z}\frac{\partial^2\varphi}{\partial x^2} + C\frac{\partial\varphi}{\partial z}\frac{\partial^2\varphi}{\partial y^2} + D\frac{\partial^2\varphi}{\partial x^2} + D\frac{\partial^2\varphi}{\partial y^2} + D\frac{\partial^2\varphi}{\partial z^2} = 0. \quad (2)$$

Let

$$\varphi^{\frac{1}{2}} = u, \quad (3)$$

then

$$\varphi = u^2, \quad (4)$$

we can get

$$\frac{\partial u^2}{\partial t} + Au\frac{\partial u^2}{\partial z} + B\frac{\partial^3 u^2}{\partial z^3} + C\frac{\partial u^2}{\partial z}\frac{\partial^2 u^2}{\partial x^2} + C\frac{\partial u^2}{\partial z}\frac{\partial^2 u^2}{\partial y^2} + D\frac{\partial^2 u^2}{\partial x^2} + D\frac{\partial^2 u^2}{\partial y^2} + D\frac{\partial^2 u^2}{\partial z^2} = 0, \quad (5)$$

$$2uu_t + 2Au^2u_z + B(6u_zu_{zz} + 2uu_{zzz}) + 2Cuu_z(2u_x^{\,2} + 2uu_{xx} + 2u_y^{\,2} + 2uu_{yy})$$
$$+ 2D(u_x^{\,2} + uu_{xx} + u_y^{\,2} + uu_{yy} + u_z^{\,2} + uu_{zz}) = 0. \quad (6)$$

By admitting the transformation

$$\zeta = x + y + z - kt \,, \quad (7)$$

where k is constant.

Bring it in, we have

$$-2kuu' + 2Au^2u' + B(6u'u'' + 2uu''') + 8Cuu'((u')^2 + uu'') + 6D((u')^2 + uu'') = 0 \,. \quad (8)$$

Integrate Eq. (8) once, we obtain

$$-ku^2 + \frac{2}{3}Au^3 + 2B((u')^2 + uu'') + 8Cu^2(u')^2 + 6Duu' = 0 \,, \quad (9)$$

II. Analytical methods

In the following section, the analysis method is briefly introduced [6-7].

2.1. PPA method

In order to present the general form of the nonlinear development equation, introduce T as u function, the general form of a nonlinear development equation is obtained by introducing its partial derivative. The form is as follows:

$$T(u, u_x, u_t, u_{xx}, u_{tt,\ldots\ldots}) = 0 \,. \quad (10)$$

By admitting the transformation

$$u(x,t) = u(\zeta) \,, \quad (11)$$

$$\zeta = x - kt \,. \quad (12)$$

We get S as a function of u

$$S(u', u'', u''', \ldots\ldots) = 0 \,. \quad (13)$$

The solution of the nonlinear development equation is written as follows

$$u(\zeta) = A_0 + W(X)\mathrm{e}^{-N\zeta} \,, \quad (14)$$

or

$$u(\zeta) = A_0 + A_1W(x)\mathrm{e}^{-N\zeta} + A_2W^2(x)\mathrm{e}^{-2N\zeta} \,. \quad (15)$$

Including

$$X = T(\zeta) = E - \frac{\mathrm{e}^{-N\zeta}}{N} \,. \quad (16)$$

Using Riccati-equation

$$W_X + FW^2 = 0 \text{ ,(17)}$$

we have

$$W(X) = \frac{1}{FX + X_0} . \quad (18)$$

Where A_0,A_1,A_2,N, E, FandX_0 are constants.

III. Application

In this part, we will apply PPA method as the new idea to solve the SZKB equation. When these variables take a specific value, we can get new traveling wave solution.

3.1 *Application of PPA method*

Calculation Eq. (14) get the partial derivative with respect to $u(\zeta)$ or the square,we obtain

$$u^2 = A_0^{\ 2} + 2A_0 e^{-N\zeta} W + e^{-2N\zeta} W^2 , \quad (19)$$

$$u^3 = A_0^{\ 3} + 3A_0^{\ 2} e^{-N\zeta} W + 3A_0 e^{-2N\zeta} W^2 + e^{-3N\zeta} W^3 , \quad (20)$$

$$u' = -Ne^{-N\zeta} W - Fe^{-2N\zeta} W^2 , \quad (21)$$

$$uu' = \left(A_0 + e^{-N\zeta} W\right)\left(-Ne^{-N\zeta} W - Fe^{-2N\zeta} W^2\right), (22)$$

$$u'' = N^2 e^{-N\zeta} W + 3FNe^{-2N\zeta} W^2 + 2F^2 e^{-3N\zeta} W^3 , \quad (23)$$

$$\left(u'\right)^2 = N^2 e^{-2N\zeta} W^2 + F^2 e^{-4N\zeta} W^4 + 2FNe^{-3N\zeta} W^3 , \quad (24)$$

$$uu'' = \left(A_0 + e^{-N\zeta} W\right)\left(N^2 e^{-N\zeta} W + 3FNe^{-2N\zeta} W^2 + 2F^2 e^{-3N\zeta} W^3\right), (25)$$

$$u^2\left(u'\right)^2 = \left(A_0^{\ 2} + 2A_0 e^{-N\zeta} W + e^{-2N\zeta} W^2\right)\left(N^2 e^{-2N\zeta} W^2 + F^2 e^{-4N\zeta} W^4 + 2FNe^{-3N\zeta} W^3\right). \quad (26)$$

Plugging Eqs. (19-26) into Eq. (9) and set the coefficient on constant, $e^{-N\zeta}W$, $e^{-2N\zeta}W$, $e^{-3N\zeta}W$, $e^{-4N\zeta}W$ to zero, we get

$$\text{constant} : -kA_0^{\ 2} + \frac{2}{3} AA_0^{\ 3} = 0 , \ (27)$$

$$e^{-N\zeta} W : -2kA_0 + 2AA_0^{\ 2} + 2BA_0 N^2 - 6DA_0 N = 0 , (28)$$

$$e^{-2N\zeta} : -k + 2AA_0 + 2B\left(2N^2 + 3A_0 FN\right) + 8CA_0^{\ 2} N^2 + 6D\left(-A_0 F - N\right) = 0 , \quad (29)$$

$$e^{-3N\zeta} : \frac{2}{3} A + 2B\left(5FN + 2A_0 F^2\right) + 8C\left(2A_0^{\ 2} FN + 2A_0 N^2\right) - 6DF = 0 , \quad (30)$$

$$e^{-4N\zeta} : 6BF^2 + 8C\left(A_0^{\ 2} F^2 + 4A_0 FN + N^2\right) = 0 , \quad (31)$$

$$e^{-5N\zeta}:8C\left(2A_0F^2+2FN\right)=0\,,\quad (32)$$

$$e^{-6N\zeta}:8CF^2=0\,.\quad (33)$$

Using Mathematical, we obtain

$$N\neq 0, F=\frac{2AN}{3k}\,.\quad (34)$$

The form of the solution to the equation is as follows

$$u(x,y,z,t)=A_0+\frac{1}{FX+X_0}\mathrm{e}^{-\mathrm{N}(x+y+z-kt)}\,.\quad (35)$$

Plugging Eq. (34) into Eq. (35), we get

$$u(x,y,z,t)=A_0+\frac{1}{\frac{2AN}{3k}X+X_0}\mathrm{e}^{-\mathrm{N}(x+y+z-kt)}\,.\quad (36)$$

Case 1. When A=3, N=1, k=1, we get F=2. By setting X_0=1, A_0=1, E = 1, we obtain

$$u(x,y,z,t)=\frac{3-\mathrm{e}^{-(x+y+z-t)}}{3-2\mathrm{e}^{-(x+y+z-t)}}\,.\quad (37)$$

When y = 1, z = 1, we have

$$u(x,t)=\frac{3-\mathrm{e}^{-(x-t+2)}}{3-2\mathrm{e}^{-(x-t+2)}}\,.\quad (38)$$

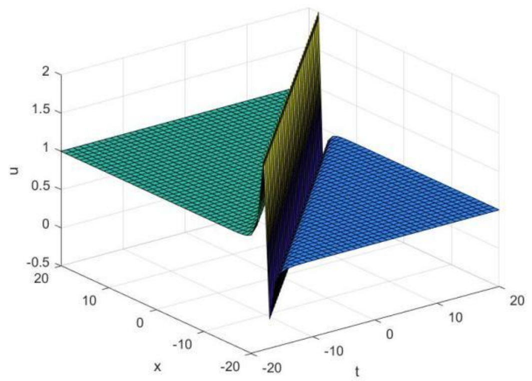

Fig.1. The plot of the soliton solution of Eq. (38) in 3D with values: A = 3, N = 1, k = 1, F = 2, X_0 = 1, A_0 = 1, E = 1, y = 1, z = 1.

When y = 1, z = 1, t = 1, we have

$$u(x)=\frac{3-\mathrm{e}^{-(x+1)}}{3-2\mathrm{e}^{-(x+1)}}\,.\quad (39)$$

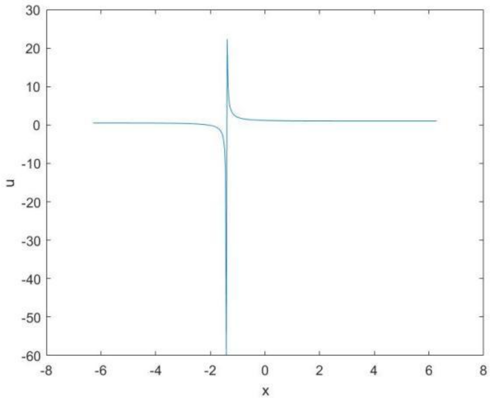

Fig.2. The plot of the soliton solution of Eq. (39) in 2D with values:A = 3, N = 1, k = 1, F = 2, X_0 = 1, A_0 = 1, E = 1, y = 1, z = 1, t = 1.

When y = 1, z = 1, x = 1,we have

$$u(t)=\frac{3-e^{-(-t+3)}}{3-2e^{-(-t+3)}}. \quad (40)$$

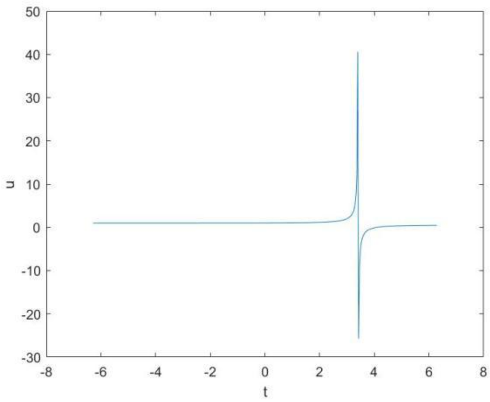

Fig.3. The plot of the soliton solution of Eq. (40) in 2D with values:A = 3, N = 1, k = 1, F = 2, X_0 = 1, A_0 = 1, E = 1, y = 1, z = 1, x = 1.

Case 2. When A=3,N= 2,k= 2, we get F= 2.By settingX_0=1,A_0=1, E = 1, we obtain

$$u(x,y,z,t)=\frac{3}{3-e^{-2(x+y+z-2t)}}, \quad (41)$$

When y = 1, z = 1,we have

$$u(x,t)=\frac{3}{3-e^{-2(x-2t+2)}}. \quad (42)$$

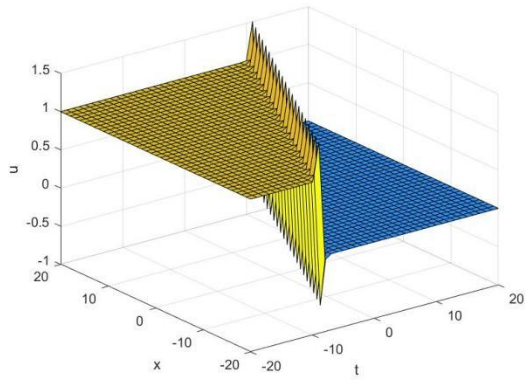

Fig.4. The plot of the soliton solution of Eq. (42) in 3D with values: $A = 3$, $N = 2$, $k = 2$, $F = 2$, $X_0 = 1$, $A_0 = 1$, $E = 1$, $y = 1$, $z = 1$.

When $y = 1$, $z = 1$, $t = 1$,we have

$$u(x) = \frac{3}{3 - e^{-2x}} . \quad (43)$$

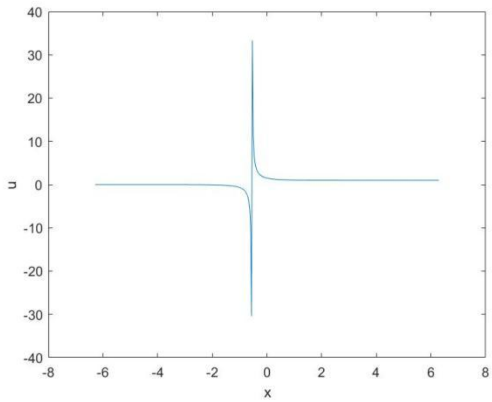

Fig.5. The plot of the soliton solution of Eq. (43) in 2D with values: $A = 3$, $N = 2$, $k = 2$, $F = 2$, $X_0 = 1$, $A_0 = 1$, $E = 1$, $y = 1$, $z = 1$, $t = 1$.

When y = 1, z = 1, x = 1,we have

$$u(t) = \frac{3}{3 - e^{-2(-2t+3)}} . \quad (44)$$

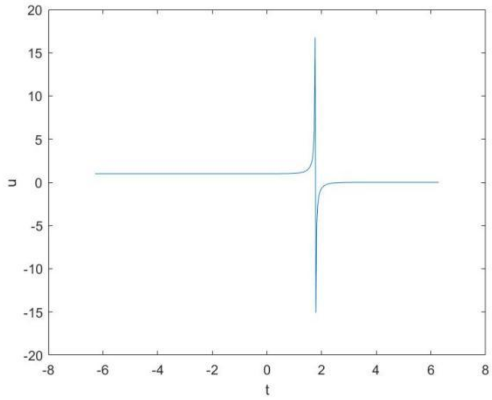

Fig.6. The plot of the soliton solution of Eq. (44) in 2D with values:$A = 3$, $N = 2$, $k = 2$, $F = 2$, $X_0 = 1$, $A_0 = 1$, $E = 1$, $y = 1$, $z = 1$, $x = 1$.

IV. Summary

In this work, the exact solution of SZKB equation was obtained by using PPA method. The exact solution obtained is new compared with the previous one, and the mathematical software is used to simulate the new exact solution. In summary, the above one method extend the solution of SZKB equation and carry out numerical simulation.

References

[1]. Tosiya Taniuti,Nobuo Yajima. Perturbation method for a nonlinear wave modulation. I. Journal of Mathematical Physics,1969,10(8): 1369-1372.

[2]. EL-Kalaawy O. H,Moawad S. M,Tharwat M. M, et al. Conservation laws, analytical solutions and stability analysis for the time-fractionalSchamel–Zakharov–Kuznetsov–Burgers equation. Advances in Difference Equations,2019,2019(1): 1-23.

[3]. Idir Hadjaz,Mouloud Tribeche. Alternative dust-ion acoustic waves in a magnetized charge varying dusty plasma with nonthermal electrons having a vortex-like velocity distribution. Astrophysics and Space Science,2014,351(2): 591-598.

[4]. EL-Kalaawy O. H, Ahmed E. A. Shock Waves, Variational Principle and conservation laws of a Schamel–Zakharov–Kuznetsov–Burgers equation in a magnetised dust plasma. Zeitschrift für Naturforschung A,2018,73(8): 693-704.

[5]. Shimin Guo,Liquan Mei,Yaling He, et al. The effect of trapped electrons on the three-dimensional ion-acoustic shock wave in magnetized ionic-pair plasma.Europhysics Letters,2016,114(2): 25002-25002.

[6]. Bekir A,Emad H. M and Güner O, Soliton solutions of the(3+1)-dimensional Yu–Toda–Sassa–Fukuyama equation by the new approach and its numerical solutions. Int J. Mod Phys B,2021, 35(2): 1-8.

[7]. Mihai Anitescu,Florian A. Potra,David E. Stewart. Time-stepping for three-dimensional rigid body dynamics. Computer Methods in Applied Mechanics and Engineering,1999,177(3): 183-197.