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Research Paper

Periodic solutions for a class of higher-dimensional state-dependent delay functional differential equations with feedback control

Lili Wang
School of Mathematics and Statistics, Anyang Normal University, Anyang Henan 455000, China

ABSTRACT:
In this paper, by using Krasnoselskii's fixed point theorem in cones, sufficient conditions for the existence of at least one periodic solution for a class of higher-dimensional state-dependent second order nonlinear functional differential equations with feedback control

$$\begin{cases} x''(t)+A(t)x'(t)+B(t)x(t)=f(t,x(t),x(t-\tau(t)),u(t-\alpha(t))), \\ u'(t)=-C(t)u(t)+D(t)x(h(t,x(t))), \end{cases}$$

are obtained.

Keywords: Periodic solution; Delay differential equations; Fixed point theorem; Feedback control.

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I. INTRODUCTION

Huo and Li [1] has applied the continuation theorem based on Gaines and Mawhin's coincidence degree to study the existence of positive periodic solutions for the following delay differential system with feed back control

$$\begin{cases} x'(t)=F(t,x(t-\tau_1(t)),\cdots,x(t-\tau_n(t)),u(t-\delta(t))), \\ u'(t)=-\eta(t)u(t)+a(t)x(t-\sigma(t)). \end{cases}$$

Y. Li and L. Zhu [2] considered a class of higher-dimensional state-dependent delay functional differential equations with feedback control of the form

$$\begin{cases} x'(t)=-A(t)x(t)+f(t,x(t),x(t-\tau(t,x(t)),u(t-\alpha(t))), \\ u'(t)=-B(t)u(t)+C(t)x(h(t,x(t))), \end{cases}$$

by using Krasnoselskii's fixed point theorem, sufficient conditions are presented for the existence of periodic solutions to the equation with feed back control.

Motivated by the works of [1-10], in this paper, we shall use Krasnoselskii's fixed point theorem in cones to study the existence of periodic solutions for a class of higher-dimensional state-dependent second order nonlinear functional differential equations with feedback control

$$\begin{cases} x''(t)+A(t)x'(t)+B(t)x(t)=f(t,x(t),x(t-\tau(t)),u(t-\alpha(t))), \\ u'(t)=-C(t)u(t)+D(t)x(h(t,x(t))), \end{cases} \tag{1.1}$$

where

(A1)

$A(t)=diag[a_1(t),a_2(t),\cdots,a_n(t)], B(t)=diag[b_1(t),b_2(t),\cdots,b_n(t)], C(t)=diag[c_1(t),\cdots,c_n(t)]$, $D(t)=diag[d_1(t),d_2(t),\cdots,d_n(t)]$.

(A2) $a_j,b_j,c_j,d_j:R\to R^+$, $\tau,\alpha:R\to R$ are all continuous T -periodic functions, $\int_0^T a_j(s)ds>0,$

$\int_0^T b_j(s)ds > 0,$ and $\tau'(t) \neq 1$ for all $t \in [0,T]$, $h: R \times R^n \to R$ satisfies $h(t+T, y) = h(t, y)$ for all $t \in R, y \in R^n$, $T > 0$ is a constant.

(A3) f is a function defined on $R \times BC \times R^n \times R^n$, and $f(t+T, x(t+T), y, z) = f(t, x(t), y, z)$ for all $t \in R, x \in BC, y \in R^n, z \in R^n$, where BC denotes the Banach space of bounded continuous functions $\eta: R \to R^n$ with the norm $\|\eta\| = \sup_{\theta \in R} \sum_{j=1}^n |\eta_j(\theta)|$, where $\eta = (\eta_1, \eta_2, \cdots \eta_n)^T$. In the sequel, we denote $f = (f_1, f_2, \cdots f_n)^T$.

Let $R = (-\infty, +\infty), R_+ = (0, +\infty), R_+{}^n = \{(x_1, x_2, \cdots x_n)^T \in R^n : x_j > 0, j = 1, 2, \cdots n\}$. We say that x is positive whenever $x \in R_+{}^n$. For every $x = \{(x_1, x_2, \cdots x_n)^T \in R^n$, the norm of x is defined as $|x_0| = \sum_{j=1}^n |x_j|$. $BC(X \to Y)$ denotes the set of bounded continuous function $\phi: X \to Y$.

For convenience, we first introduce the related definition and the fixed point theorem applied in the paper.

Definition 1.1 Let X be a Banach space and K be a closed nonempty sunset of X, K is a cone if

(1) $\alpha u + \beta v \in K$ for all $u, v \in K$ and all $\alpha, \beta \geq 0$;

(2) $u, -u \in K$ imply $u = 0$.

Theorem 1.1 (Krasnoselkii [11]) Let X be a Banach space, and let $K \subset X$ be a cone in X. Assume that Ω_1, Ω_2 are open bounded subsets of X with $0 \in \Omega_1, \overline{\Omega_1} \subset \Omega_2$, and let

$\phi: K \cap (\overline{\Omega_2} \setminus \Omega_1) \to K$

be a completely continuous operator such that either

(1) $\|\phi y\| \leq \|y\|, \forall y \in K \cap \partial\Omega_1$ and $\|\phi y\| \geq \|y\|, \forall y \in K \cap \partial\Omega_2$; or

(2) $\|\phi y\| \geq \|y\|, \forall y \in K \cap \partial\Omega_1$ and $\|\phi y\| \leq \|y\|, \forall y \in K \cap \partial\Omega_2$.

Then ϕ has a fixed point in $K \cap (\overline{\Omega_2} \setminus K \cap \partial\Omega_1)$.

In this paper we always assume that

(H1) $f_j(t, \xi, \eta, \zeta) \geq 0$ for all $(t, \xi, \eta, \zeta) \in R \times BC(R, R_+{}^n) \times R_+{}^n \times R_+{}^n$, $j = 1, 2, \cdots n$.

II. PRELIMINARIES

Let T be a positive constant. We define two sets

$$X = \{x : C(R, R^n), x(t+T) = x(t), t \in R\}$$

endow with the usual linear structure as well as the norm

$$\|x\| = \sup_{t \in R} \sum_{j=1}^n |x_j(t)|, \quad |x|_0 = \sum_{j=1}^n |x_j(t)|,$$

and

$$K = \left\{ x \in X, x_j(t) \geq \sigma \|x_j\|, t \in [0,T], x = (x_1, x_2, \cdots x_n)^T \right\}.$$

Obviously, X is a Banach space and K is a cone.

Each T -periodic solution of second equation of (1.1) is equivalent to that of the following equation

$$u(t) = \int_t^{t+T} \overline{G}(t,s) D(s) x(h(s, x(s))) ds := (\Phi x)(t), \tag{2.1}$$

and vice versa, where

$$\overline{G}(t,s) = diag[\overline{G}_1(t,s), \overline{G}_2(t,s), \cdots \overline{G}_n(t,s)],$$

and

$$\overline{G}_j(t,s) = \frac{\exp(\int_t^s c_j(r) dr)}{\exp(\int_0^T c_j(r) dr) - 1}, s \in [t, t+T], j = 1, 2, \cdots, n.$$

It is clear that $\overline{G}(t,s)=\overline{G}(t+T,s+T)$ for all $(t,s)\in R^2$ and $u(t+T)=u(t)$, when x is a T-periodic function. We denote $(\Phi x)=(\Phi_1 x,\Phi_2 x,\cdots,\Phi_n x)^T$.

Therefore, any T-periodic solution of the system (1.1) is equivalent to that of the following equation

$$x''(t)+A(t)x'(t)+B(t)x(t)=f(t,x(t),x(t-\tau(t)),(\Phi x)(t-\alpha(t))). \tag{2.2}$$

Similar to the proof in [3], we have:

Lemma 2.1. Suppose that (A1), (A2) hold and

$$\frac{R_{1j}[\exp(\int_0^T a_j(u)du)-1]}{Q_{1j}T}\geq 1, \tag{2.3}$$

$$R_{1j}=\max_{t\in[0,T]}\left|\int_t^{t+T}\frac{\exp(\int_t^s a_j(u)du)}{\exp(\int_0^T a_j(u)du)-1}b_j(s)ds\right|,Q_{1j}=\left(1+\exp(\int_0^T a_j(u)du)\right)^2 R_{1j}^{\ 2}.$$

Then there exist continuous T-periodic functions p_j and q_j such that $q_j(t)>0$, $\int_0^T p_j(u)du>0$, and

$p_j(t)+q_j(t)=a_j(t),q_j'(t)+p_j(t)q_j(t)=b_j(t)$ for all $t\in R,\ j=1,2,\cdots n.$

Therefore

$$p(t)+q(t)=A(t),q'(t)+p(t)q(t)=B(t),\ t\in R,$$

where $p=diag[p_1,p_2,\cdots,p_n],q=diag[q_1,q_2,\cdots,q_n]$.

Similar to the proof in [4], we can get the following lemmas.

Lemma 2.2. Suppose the conditions of Lemma 2.1 hold and $\phi(t)\in X$. Then the equation

$$x''(t)+A(t)x'(t)+B(t)x(t)=\phi(t) \tag{2.4}$$

has a T-periodic solution. Moreover, the periodic solutions can be expressed by

$$x(t)=\int_t^{t+T}G(t,s)\phi(s)ds, \tag{2.5}$$

where $$G(t,s)=diag[G_1(t,s),G_2(t,s),\cdots,G_n(t,s)],$$

and

$$G_j(t,s)=\frac{\int_t^s \exp[\int_t^u q_j(v)dv+\int_u^s p_j(v)dv]du+\int_s^{t+T}\exp[\int_t^u q_j(v)dv+\int_u^{s+T}p_j(v)dv]du}{[\exp(\int_0^T p_j(u)du)-1][\exp(\int_0^T q_j(u)du)-1]}.$$

So we proceed from (2.2) and obtain

$$x(t)=\int_t^{t+T}G(t,s)f(s,x(s),x(s-\tau(s)),(\Phi x)(s-\alpha(s)))ds, \tag{2.6}$$

where Φ is defined as (2.1), and by (H1), we have

$$G_j(t,s)f_j(s,x(s),x(s-\tau(s)),(\Phi x)(s-\alpha(s)))\geq 0,\ j=1,2,\cdots n,(t,s)\in R^2.$$

Corollary 2.1. Green's function $G(t,s)$ satisfies the following properties:

$$G_j(t,t+T)=G_j(t,t),\ \ G_j(t+T,s+T)=G_j(t,s),$$

$$\frac{\partial}{\partial s}G_j(t,s)=p_j(s)G_j(t,s)-\frac{\exp\int_t^s q_j(v)dv}{\exp\int_0^T q_j(v)dv-1},$$

$$\frac{\partial}{\partial t}G_j(t,s)=-q_j(s)G_j(t,s)+\frac{\exp\int_t^s p_j(v)dv}{\exp\int_0^T p_j(v)dv-1},\ \ j=1,2,\cdots,n.$$

Lemma 2.3. Let $H_j=\int_0^T a_j(u)du, I_j=T^2\exp(\frac{1}{T}\int_0^T \ln b_j(u)du)$. If $H_j^{\ 2}\geq 4I_j$, (2.7) then

$$\min\left\{\int_0^T p_j(u)du, \int_0^T q_j(u)du\right\} \ge \frac{1}{2}(H_j - \sqrt{H_j^{\ 2} - 4I_j}) := l_j,$$

$$\max\left\{\int_0^T p_j(u)du, \int_0^T q_j(u)du\right\} \le \frac{1}{2}(H_j + \sqrt{H_j^{\ 2} - 4I_j}) := m_j, \quad j = 1, 2, \cdots, n.$$

Therefore the function $G_j(t,s)$ satisfies

$$0 < N_j =: \frac{T}{(e^{m_j} - 1)^2} \le G_j(t,s) \le \frac{T\exp(\int_0^T a_j(u)du)}{(e^{l_j} - 1)^2} := M_j, s \in [t, t+T],$$

$$1 \ge \frac{G_j(t,s)}{M_j} \ge \frac{N_j}{M_j} \ge \sigma := \min\left\{\frac{N_j}{M_j}, j = 1, 2, \cdots, n\right\} > 0,$$

and we denote

$$l = \min_{1\le j\le n} l_j, m = \max_{1\le j\le n} m_j, N = \min_{1\le j\le n} N_j, M = \max_{1\le j\le n} M_j.$$

Now, before presenting our main results, we give the following assumptions.

(H2) $f(t, \phi(t), \phi(t - \tau(t)), (\Phi\phi)(t - \alpha(t)))$ is a continuous function of t for each $\phi \in BC(R, R_+^{\ n})$, where Φ is defined as (2.1).

(H3) For any $L > 0$ and $\varepsilon > 0$, there exists $\delta > 0$, such that

$$\{\phi, \psi \in BC, \|\phi\| \le L, \|\psi\| \le L, \|\phi - \psi\| < \delta, 0 \le s \le T\}$$

imply $\left| f(s, \phi(s), \phi(s - \tau(s)), u_1(s - \alpha(s))) - f(s, \psi(s), \psi(s - \tau(s)), u_2(s - \alpha(s)))\right|_0 < \varepsilon,$

where

$$u_1(s - \alpha(s)) = \int_{s-\alpha(s)}^{s-\alpha(s)+T} \overline{G}(s - \alpha(s), v) D(v) \phi(h(v, \phi(v))) ds = (\Phi\phi)(s - \alpha(s)),$$

$$u_2(s - \alpha(s)) = \int_{s-\alpha(s)}^{s-\alpha(s)+T} \overline{G}(s - \alpha(s), v) D(v) \psi(h(v, \psi(v))) ds = (\Phi\psi)(s - \alpha(s)).$$

III. MAIN RESULTS

Now we define a mapping $T : K \to K$,

$$(Tx)(t) = \int_t^{t+T} G(t,s)x(t) = \int_t^{t+T} G(t,s) f(s, x(s), x(s - \tau(s)), (\Phi x)(s - \alpha(s)))ds,$$

where Φ is defined as (2.1), and we denote $(Tx) = (T_1x, T_2x, \cdots T_nx)^T$.

Lemma 3.1. $T : K \to K$ is well-defined.

Proof. For each $x \in K$, by (H2) we have $(Tx)(t)$ is continuous and

$$\begin{aligned}(Tx)(t+T) &= \int_{t+T}^{t+2T} G(t+T, s) f(s, x(s), x(s - \tau(s)), (\Phi x)(s - \alpha(s)))ds \\ &= \int_t^{t+T} G(t+T, v+T) f(v+T, x(v+T), x(v+T - \tau(v+T)), (\Phi x)(v+T - \alpha(v+T)))dv \\ &= \int_t^{t+T} G(t,v) f(v, x(v), x(v - \tau(v)), (\Phi x)(v - \alpha(v)))dv = (Tx)(t).\end{aligned}$$

Thus, $Tx \in X$, since

$N_j \le G_j(t,s) \le M_j, s \in [t, t+T].$

Hence, for $x \in K$, we have

$$\|T_j x\| \le M_j \int_0^T \left| f_j(s, x(s), x(s - \tau(s)), (\Phi x)(s - \alpha(s)))\right| ds, \tag{3.1}$$

and

$$(T_j x)(t) \geq N_j \int_0^T \left| f_j(s, x(s), x(s-\tau(s)), (\Phi x)(s-\alpha(s))) \right| ds$$
$$= \frac{N_j}{M_j} M_j \int_0^T \left| f_j(s, x(s), x(s-\tau(s)), (\Phi x)(s-\alpha(s))) \right| ds$$
$$\geq \sigma \left\| T_j x \right\|.$$

Therefore, $Tx \in K$, this completes the proof.

Lemma 3.2. $T: K \to K$ is completely continuous.

Proof. We first show that T is continuous .

By (H3), for any $L > 0$ and $\varepsilon > 0$, there exists a $\delta > 0$ such that

$\left\{\phi, \psi \in BC, \|\phi\| \leq L, \|\psi\| \leq L, \|\phi - \psi\| \leq \delta\right\}$

imply

$$\sup_{0\leq s\leq T} \left| f(s, \phi(s), \phi(s-\tau(s)), (\Phi\phi)(s-\alpha(s))) - f(s, \psi(s), \psi(s-\tau(s)), (\Phi\psi)(s-\alpha(s))) \right|_0 < \frac{\varepsilon}{MT}$$

.

If $x, y \in K$ with $\|x\| \leq L, \|y\| \leq L, \|x - y\| \leq \delta$, then

$$\left|(Tx)(t) - (Ty)(t)\right|_0$$
$$\leq \int_t^{t+T} \left|G(t,s)\right| \left| f(s, x(s), x(s-\tau(s)), (\Phi x)(s-\alpha(s))) - f(s, y(s), y(s-\tau(s)), (\Phi y)(s-\alpha(s))) \right|_0 ds$$
$$\leq \int_0^T \left|G(t,s)\right| \left| f(s, x(s), x(s-\tau(s)), (\Phi x)(s-\alpha(s))) - f(s, y(s), y(s-\tau(s)), (\Phi y)(s-\alpha(s))) \right|_0 ds$$
$$< MT \frac{\varepsilon}{MT} = \varepsilon$$

for all $t \in [0, T]$, where $\left|G(t,s)\right| = \max_{1\leq j\leq n} \left|G_j(t,s)\right|$, this yields $\|Tx - Ty\| < \varepsilon$, thus T is continuous.

Next we show that T maps any bounded sets in K into relatively compact sets. Now we first prove that f maps bounded sets into bounded sets. Indeed, let $\varepsilon = 1$, by (H3), for any $\mu > 0$, there exists $\delta > 0$ such that $\left\{x, y \in BC, \|x\| \leq \mu, \|y\| \leq \mu, \|x - y\| \leq \delta, 0 \leq s \leq T\right\}$ imply

$$\left| f(s, x(s), x(s-\tau(s)), (\Phi x)(s-\alpha(s))) - f(s, y(s), y(s-\tau(s)), (\Phi y)(s-\alpha(s))) \right|_0 < 1.$$

Choose a positive integer N such that $\frac{\mu}{N} < \delta$. Let $x \in BC$ and define

$$x^k(t) = \frac{x(t)k}{N}, k = 0, 1, 2 \cdots, N .$$

If $\|x\| < \mu$, then

$$\left\|x^k - x^{k-1}\right\| = \sup_{t\in R} \left| \frac{x(t)k}{N} - \frac{x(t)(k-1)}{N} \right| \leq \|x\| \frac{1}{N} \leq \frac{\mu}{N} < \delta.$$

Thus,

$$\left| f(s, x^k(s), x^k(s-\tau(s)), (\Phi x^k)(s-\alpha(s))) - f(s, x^{k-1}(s), x^{k-1}(s-\tau(s)), (\Phi x^{k-1})(s-\alpha(s))) \right|_0 < 1$$

for all $s \in [0, T]$, this yields

$$
\begin{aligned}
&\left| f(s,x(s),x(s-\tau(s)),(\Phi x)(s-\alpha(s))) \right|_0 \\
&= \left| f(s,x^N(s),x^N(s-\tau(s)),(\Phi x^N)(s-\alpha(s))) \right|_0 \\
&\le \sum_{k=1}^{N} \left| f(s,x^k(s),x^k(s-\tau(s)),(\Phi x^k)(s-\alpha(s))) \right. \qquad (3.2)\\
&\qquad \left. - f(s,x^{k-1}(s),x^{k-1}(s-\tau(s)),(\Phi x^{k-1})(s-\alpha(s))) \right|_0 + \left| f(s,0,0,0) \right|_0 \\
&< N + \|f\| =: W.
\end{aligned}
$$

It follows from (3.1) that for $t \in [0,T]$

$$\|Tx\| = \sup_{t\in R} \sum_{j=1}^{n} \left|(T_j x)(t)\right| \le \sum_{j=1}^{n} M_j \int_0^T \left| f_j(s,x(s),x(s-\tau(s)),(\Phi x)(s-\alpha(s)))\right| ds \le MTW.$$

Finally, for $t \in R$, we have

$$(T_j x)'(t) = \int_t^{t+T} \left[-q_j(s) G_j(t,s) + \frac{\exp\int_t^s p_j(v)dv}{\exp\int_0^T p_j(v)dv - 1} \right] f_j(s,x(s),x(s-\tau(s)),(\Phi x)(s-\alpha(s)))ds\,,$$

$$j = 1,2,\cdots,n. \qquad (3.3)$$

Combine (3.1), (3.2), (3.3) and Corollary 2.1, we obtain

$$
\begin{aligned}
&\left| \frac{d}{dt}(Tx)(t) \right|_0 = \sup_{t\in R} \sum_{j=1}^{n} \left| (T_j x)'(t) \right| \\
&\le \sum_{j=1}^{n} \int_t^{t+T} \left| f_j(s,x(s),x(s-\tau(s)),(\Phi x)(s-\alpha(s))) \right| \left| -q_j(s)G_j(t,s) + \frac{\exp\int_t^s p_j(v)dv}{\exp\int_0^T p_j(v)dv - 1} \right| ds \\
&\le \sum_{j=1}^{n} (M_j \|Q\| + \frac{e^m}{e^l - 1}) \int_t^{t+T} \left| f_j(s,x(s),x(s-\tau(s)),(\Phi x)(s-\alpha(s))) \right| ds \\
&\le (M\|Q\| + \frac{e^m}{e^l - 1}) TW,
\end{aligned}
$$

where $\|Q\| = \max_{1\le j\le n} \left| q_j \right|$.

Hence $\{Tx : x \in K, \|x\| \le \mu\}$ is a family of uniformly bounded and equicontinuous functions on $[0,T]$. By a theorem of Ascoli-Arzela, the function T is completely continuous.

Theorem 3.1. Suppose that (H1)-(H3), (2.3) and (2.7) and that there are positive constants R_1 and R_2 with $R_1 < R_2$ such that

$$\sup_{\|\phi\|=R_1,\phi\in K} \int_0^T \left| f(s,\phi(s),\phi(s-\tau(s)),(\Phi\phi)(s-\alpha(s))) \right|_0 ds \le \frac{R_1}{M} \qquad (3.4)$$

and

$$\inf_{\|\phi\|=R_2,\phi\in K} \int_0^T \left| f(s,\phi(s),\phi(s-\tau(s)),(\Phi\phi)(s-\alpha(s))) \right|_0 ds \ge \frac{R_2}{N}, \qquad (3.5)$$

where Φ is defined as (2.1). Then the system (1.1) has a positive T-periodic solution x with $R_1 \le \|x\| \le R_2$.

Proof. Let $x \in K$ and $\|x\| = R_1$. By (3.4), we have

$$\begin{aligned}|(Tx)(t)|_0 &\leq M\int_t^{t+T} |f(s,x(s),x(s-\tau(s)),(\Phi x)(s-\alpha(s)))|_0\,ds \\ &\leq M\frac{R_1}{M}=R_1\end{aligned}$$

for all $t\in[0,T]$. This implies that $\|Tx\|\leq\|x\|$ for $x\in K\bigcap\partial\Omega_1,\Omega_1=\{x\in X,\|x\|<R_1\}$.

If $x\in K$ and $\|x\|=R_2$. By (3.5), we have

$$\begin{aligned}|(Tx)(t)|_0 &\geq N\int_t^{t+T} |f(s,x(s),x(s-\tau(s)),(\Phi x)(s-\alpha(s)))|_0\,ds \\ &\geq N\frac{R_2}{N}=R_2\end{aligned}$$

for all $t\in[0,T]$. Thus, $\|Tx\|\geq\|x\|$ for $x\in K\bigcap\partial\Omega_2,\Omega_2=\{x\in X,\|x\|<R_2\}$.

By Krasnoselskii's fixed point theorem, T has a fixed point in $K\bigcap(\overline{\Omega_2}\setminus\Omega_1)$. It is easy to say that Eq.(1.1) has a positive T-periodic solution x with $R_1\leq\|x\|\leq R_2$. This completes the proof.

Corollary3.1 Suppose that (H1)-(H3), (2.3) and (2.7) and that there are positive constants R_1 and R_2 with $R_1<R_2$ such that

$$\lim_{\phi\in K,\|\phi\|\to 0}\frac{\int_0^T |f(s,\phi(s),\phi(s-\tau(s)),(\Phi\phi)(s-\alpha(s)))|_0\,ds}{\|\phi\|}=0,$$

$$\lim_{\phi\in K,\|\phi\|\to\infty}\frac{\int_0^T |f(s,\phi(s),\phi(s-\tau(s)),(\Phi\phi)(s-\alpha(s)))|_0\,ds}{\|\phi\|}=\infty,$$

where Φ is defined in (2.1). Then the system (1.1) has a positive T-periodic solution x with $R_1\leq\|x\|\leq R_2$.

Corollary3.2 Suppose that (H1)-(H3), (2.3) and (2.7) and that there are positive constants R_1 and R_2 with $R_1<R_2$ such that

$$\inf_{\|\phi\|=R_1,\phi\in K}\int_0^T |f(s,\phi(s),\phi(s-\tau(s)),(\Phi\phi)(s-\alpha(s)))|_0\,ds\geq\frac{R_1}{N},$$

$$\sup_{\|\phi\|=R_2,\phi\in K}\int_0^T |f(s,\phi(s),\phi(s-\tau(s)),(\Phi\phi)(s-\alpha(s)))|_0\,ds\leq\frac{R_2}{M},$$

where Φ is defined in (2.1). Then the system (1.1) has a positive T-periodic solution x with $R_1\leq\|x\|\leq R_2$.

Corollary3.3 Suppose that (H1)-(H3), (2.3) and (2.7) and that there are positive constants R_1 and R_2 with $R_1<R_2$ such that

$$\lim_{\phi\in K,\|\phi\|\to 0}\frac{\int_0^T |f(s,\phi(s),\phi(s-\tau(s)),(\Phi\phi)(s-\alpha(s)))|_0\,ds}{\|\phi\|}=\infty,$$

$$\lim_{\phi\in K,\|\phi\|\to\infty}\frac{\int_0^T |f(s,\phi(s),\phi(s-\tau(s)),(\Phi\phi)(s-\alpha(s)))|_0\,ds}{\|\phi\|}=0,$$

where Φ is defined in (2.1). Then the system (1.1) has a positive T-periodic solution x with $R_1\leq\|x\|\leq R_2$.

REFERENCES

[1]. H.F. Huo, W.T. Li, Periodic solutions for a class of delay differential system with feedback control, Appl. Math. Comput., 148 (2004) 35-36.
[2]. Y.K. Li, L.F. Zhu, Periodic solutions for a class of higher-dimensional state-dependent delay functional differential equations with feedback control. Appl. Math. Comput., (2003)783-795.
[3]. Y. Liu, W. Ge, Positive solutions Positive solutions for nonlinear Duffing equations with delay and variable coefficients, Tamsui Oxf. J. Math. sci., 20 (2004) 235-255.
[4]. Y. Wang, H. Lian, W. Ge, Periodic solutions for a second order nonlinear functional differential equation, Appl. Math. Lett., (2006) 110-115.
[5]. A. Wan, D. Jiang, Existence of Positive solutions Positive solutions functional differential equations, Kyushu J. Math., 56 (2002) 193-202.
[6]. Y.N. Raffoul , Periodic solutions for neutral differential equations with function delay, Electron.J. Diff. Equ., 102 (2003) 1-7.
[7]. G. Makay, Periodic solutions of dissipative functional differential equations, Tohoku Math. J., 46 (1994) 417-426.
[8]. Y.K. Li, Positive Periodic solutions of neutral differential equations with distributed delays, Electron. J. Diff. Equ., 2006 (13) 655-666.
[9]. Y.K. Li, Y. Kuang, Periodic solutions in periodic state-dependent delay equations and population models, Proc. Aer. math. Soc., 130(5) (2002) 1345-1353.
[10]. Y. Kuang, Delay differential equations with application in population dynamics, Academic Press, New York, 1993.
[11]. D.R. Smart, Fixed Points Theorems, Cambridge University Press, Cambridge, 1980.

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