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Research Paper

Statistical Properties of Exponentiated Burr V Distribution

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ABSTRACT: This study proposed statistical distribution called Exponentiated distribution using one of the twelve families of burr, that is, Burr V (BV) distribution. The theoretical properties of this proposed distribution are proven such as the validity of the distribution with also graphical representation of the probability density function (PDF) and the cumulative distribution function (CDF), asymptotic statistical properties, Survival function, and Hazard function, Moments, Moment Generating Function (MGF) and Characteristic Function (CF). The method of estimating its parameters; by maximum likelihood estimate (MLE) was proposed.
KEYWORDS: Burr V, Moments, quantile, MLE, Exponentiated

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I. INTRODUCTION

Many researches have been conducted in the development of new family and hybridized probability distributions. Among the many are research conducted by [1], [2], [3], [4], [5] and [6]. The quest for better compound distribution using suitable family of distribution is not exhaustive. The reason is to develop innovative flexible probability distributions to handle different characteristics of data from diverse areas of study. This paper proposes a new distribution called Exponentiated Burr-V distribution (EBVD). This distribution is an improvement on Burr-V distribution in terms of outliers and heavy-tailed data in modeling clinical and stock data that are prompt to deviation. Statistical properties for the underlying distribution are derived and the parameters are estimated using the method of maximum likelihood technique.

The probability distribution functions (PDF) and cumulative distribution function for the proposed distribution is give as follows.

- The PDF and CDF of the baseline distribution (Burr V) are given respectively (where α and β are location parameters) as:

$$g\left(x\right)=\alpha\beta e^{-\tan x}\sec^2 x\left(1+\beta e^{-\tan x}\right)^{-\alpha-1} \qquad \rightarrow (1)$$

$$G\left(x\right)=\left[1+\beta e^{-tan\,x}\right]^{-\alpha} \qquad \rightarrow (2)$$

- The PDF and CDF of the exponentiated distribution are given respectively (where α and β are location parameters and γ is the shape parameter) as:

$$f\left(x\right)=\alpha\beta\gamma e^{-tan\,x}\, sec^2\, x\left(1+\beta e^{-tan\,x}\right)^{-\alpha(\gamma-1)}\left(1+\beta e^{-tan\,x}\right)^{-\alpha-1} \qquad \rightarrow (3)$$

$$F\left(x\right)=\left(1+\beta e^{-\tan x}\right)^{-\alpha\gamma} \qquad \rightarrow (4)$$

The PDF and CDF plots for different values for each parameter are presented in Figure 1 and Figure 2 below. The plots shows the behavior of the EBVD when location and shape parameters values are in constant increment form.

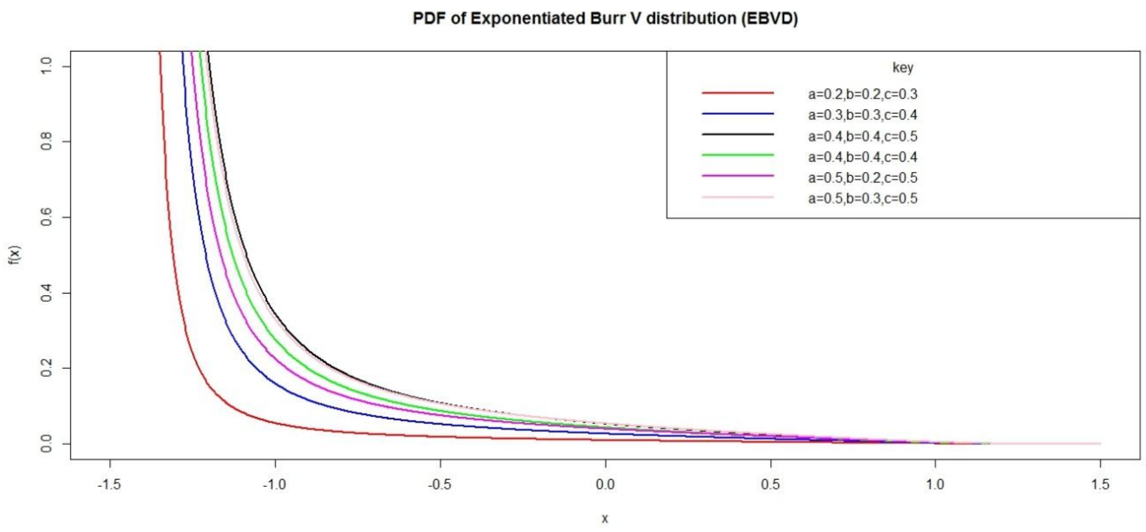

Figure 1: PDF of EBVD

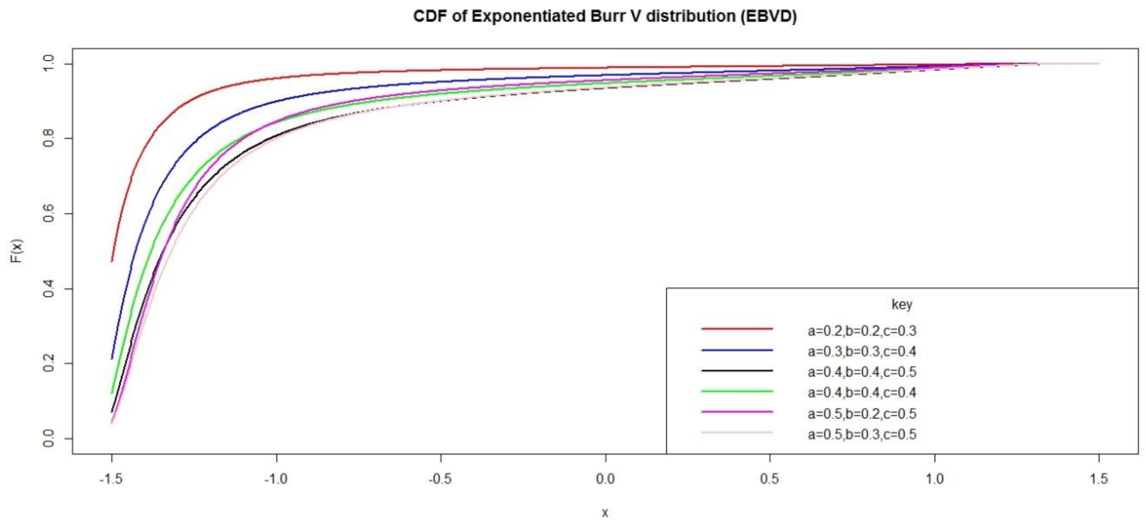

Figure 2: CDF of EBVD

II. INVESTIGATION OF THE PROPOSED DENSITY FUNCTION

We verify the proposed density function that is a proper probability density function. This is achieved by integrating *f*(*x*) with respect to *x* to see if the integral is equal to unity or otherwise.

Theorem 1: Let *x* be a random variable of a density function given as

$$f(x) = \alpha\beta\gamma e^{-\tan x}\sec^2 x\left(1+\beta e^{-\tan x}\right)^{-\alpha(\gamma-1)}\left(1+\beta e^{-\tan x}\right)^{-\alpha-1} \qquad \rightarrow (5)$$

where, the limit of *x* is taking the limit from negative infinity to positive infinity

Proof:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\,dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \alpha\beta\gamma e^{-\tan x}\sec^2 x\left(1+\beta e^{-\tan x}\right)^{-\alpha(\gamma-1)}\left(1+\beta e^{-\tan x}\right)^{-\alpha-1}dx \qquad \rightarrow (6)$$

$$= \alpha\beta\gamma\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{-\tan x}\sec^2 x\left(1+\beta e^{-\tan x}\right)^{-\alpha(\gamma-1)}\left(1+\beta e^{-\tan x}\right)^{-\alpha-1}dx \qquad \rightarrow (7)$$

Let $p = \beta e^{-\tan x}$ then we have,

$$\frac{dp}{dx} = -\sec^2 x\beta e^{-\tan x} \Rightarrow dp = dx\sec^2 x\beta e^{-\tan x} \Rightarrow dx = \frac{dp}{\sec^2 x\beta e^{-\tan x}}$$

as $x \to \frac{\pi}{2}$, $p \to \infty$ and as $x \to -\frac{\pi}{2}$, $p \to 0$

so, we have equation (7) to be,

$$= \alpha\beta\gamma \int_0^\infty e^{-\tan x} \sec^2 x \left(1+p\right)^{-\alpha(\gamma-1)} \left(1+p\right)^{-\alpha-1} \frac{dp}{\sec^2 x \beta e^{-\tan x}} \qquad \to (8)$$

$$= \int_0^\infty \alpha\gamma \left(1+p\right)^{-\alpha(\gamma-1)} \left(1+p\right)^{-\alpha-1} dp$$

let $w = \left(1+p\right)^{-\alpha}$

$$\frac{dw}{dp} = -\alpha\left(1+p\right)^{-\alpha-1} \Rightarrow dw = -\alpha\left(1+p\right)^{-\alpha-1} dp \Rightarrow dp = \frac{dw}{\alpha\left(1+p\right)^{-\alpha-1}}$$

$$= \int_0^\infty \alpha\gamma w^{\gamma-1} \left(1+p\right)^{-\alpha-1} \frac{dw}{\alpha\left(1+p\right)^{-\alpha-1}} \qquad \to (9)$$

as $p \to \infty$, w = 1 and as $p \to 0$, w = 0

$$= \int_0^1 \gamma w^{\gamma-1} dw \qquad \to (10)$$

Integrate the above expression, we obtained

$$\left.\frac{\gamma w^\gamma}{\gamma}\right|_0^1 = \left. w^\gamma \right|_0^1 = \left[1-0\right]^\gamma = 1 \qquad \to (11)$$

if $\gamma > 0$

therefore, $f(x)$ is indeed a proper probability density function of a continuous distribution as given in equation (11).

2.1 Asymptotic Behavior Statistics of EBVD

In order to investigate the behavior of the proposed EBVD, we proceed thus,

Theorem 2: The limit of beta Burr V density function as $x \to \frac{\pi}{2}$ is 0.

Proof:

This can be shown by getting the limit of the EBVD density in equation (3)

For $x \to \frac{\pi}{2}$:

$$\lim_{x\to\frac{\pi}{2}} g\left(x\right) = \lim_{x\to\frac{\pi}{2}} \alpha\beta\gamma e^{-\tan x} \sec^2 x \left(1+\beta e^{-\tan x}\right)^{-\alpha(\gamma-1)} \left(1+\beta e^{-\tan x}\right)^{-\alpha-1} \qquad \to (12)$$

$$= \alpha\beta\gamma \lim_{x\to\frac{\pi}{2}} e^{-\tan x} \sec^2 x \lim_{x\to\frac{\pi}{2}} \left(1+\beta e^{-\tan x}\right)^{-\alpha(\gamma-1)} \lim_{x\to\frac{\pi}{2}} \left(1+\beta e^{-\tan x}\right)^{-\alpha-1} \qquad \to (13)$$

$$= \alpha\beta\gamma \lim_{x\to\frac{\pi}{2}} e^{-\tan x} \sec^2 x \lim_{x\to\frac{\pi}{2}} \left(1+\beta e^{-\tan x}\right)^{-\alpha(\gamma-1)} \lim_{x\to\frac{\pi}{2}} \frac{1}{\left(1+\beta e^{-\tan x}\right)^{-\alpha-1}} = 0 \qquad \to (14)$$

Since $\lim_{x\to-\frac{\pi}{2}} \frac{1}{\left(1+\beta e^{-\tan x}\right)^{\alpha a+1}} = 0$.

Theorem 3: The Limit of EBV density functions as $x \to -\frac{\pi}{2}$ is 0.

Proof:

This can be shown by getting the limit of the EBVD density in equation (3)

For $x \to -\frac{\pi}{2}$

$$\lim_{x\to-\frac{\pi}{2}} g\left(x\right) = \lim_{x\to-\frac{\pi}{2}} \alpha\beta\gamma e^{-\tan x} \sec^2 x \left(1+\beta e^{-\tan x}\right)^{-\alpha(\gamma-1)} \left(1+\beta e^{-\tan x}\right)^{-\alpha-1} \qquad \to (15)$$

$$= \alpha\beta\gamma \lim_{x \to -\frac{\pi}{2}} e^{-\tan x} \sec^2 x \lim_{x \to -\frac{\pi}{2}} \left(1+\beta e^{-\tan x}\right)^{-\alpha(\gamma-1)} \lim_{x \to -\frac{\pi}{2}} \left(1+\beta e^{-\tan x}\right)^{-\alpha-1} \rightarrow (16)$$

$$= \alpha\beta\gamma \lim_{x \to -\frac{\pi}{2}} e^{-\tan x} \sec^2 x \lim_{x \to -\frac{\pi}{2}} \left(1+\beta e^{-\tan x}\right)^{-\alpha(\gamma-1)} \lim_{x \to -\frac{\pi}{2}} \frac{1}{\left(1+\beta e^{-\tan x}\right)^{-\alpha-1}} = 0 \rightarrow (17)$$

Since, $\lim_{x \to -\frac{\pi}{2}} e^{-\tan x} = 0$.

This is an indication that the proposed distribution has at least a mode.

III. RELIABILITY STATISTICS

In this section, some reliability statistics for reliability engineering and disease progression study are presented. These statistics include survival function, hazard function and quantile function.

3.1 Survival Function

Given the CDF of a probability distribution, the survival function is the probability of survival beyond time x and is defined by;

$$S(x) = 1 - F(x) \rightarrow (18)$$

For the EBVD, the survival function is given as:

$$S(x) = 1 - \left(1+\beta e^{-\tan x}\right)^{-\alpha\gamma};\ x > 0,\ \alpha > 0,\ \beta > 0,\ \gamma > 0 \rightarrow (19)$$

where α and β parameter of location, and γ is the shape parameter of the Exponentiated distribution. Figure 3 shows the survival function of the EBVD.

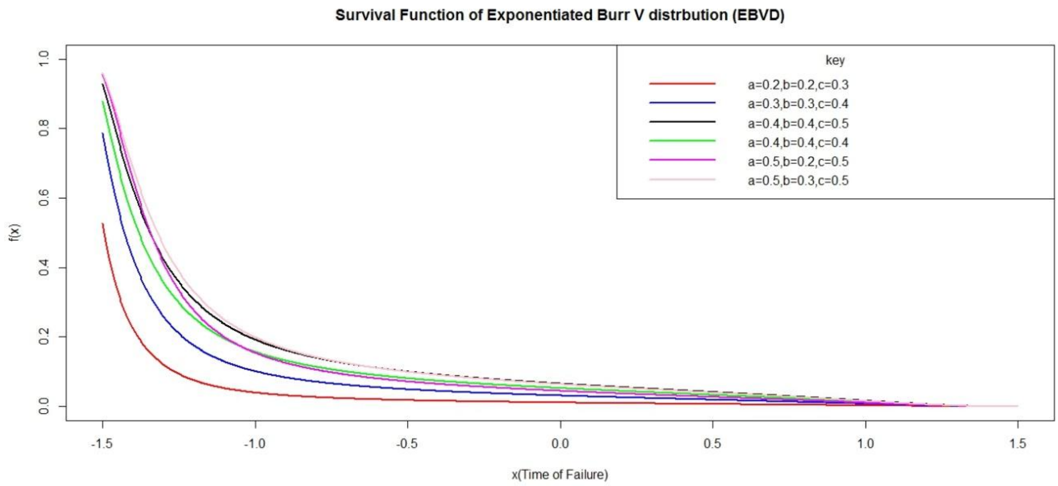

Figure 3: Survival function of EBVD

3.2 Hazard Function

The hazard rate which is also known as the failure rate is defined as the conditional probability of failure of an item/device given that the item has survived time x. The hazard rate function of a random variable X is given as:

$$H(x) = \frac{f(x)}{S(x)} \rightarrow (20)$$

Therefore, the hazard function for our new proposed distribution (EBV) is given as:

$$H(x) = \frac{\alpha\beta\gamma e^{-\tan x} \sec^2 x \left(1+\beta e^{-\tan x}\right)^{-\alpha(\gamma-1)} \left(1+\beta e^{-\tan x}\right)^{-\alpha-1}}{1-\left(1+\beta e^{-\tan x}\right)^{-\alpha\gamma}};\ x > 0,\ \alpha > 0,\ \beta > 0,\ \gamma > 0 \rightarrow (21)$$

$$H(x) = \frac{\alpha\beta\gamma e^{-\tan x} \sec^2 x \left(1+\beta e^{-\tan x}\right)^{-\alpha(\gamma-1)}}{1-\left(1+\beta e^{-\tan x}\right)^{-\alpha\gamma} \left(1+\beta e^{-\tan x}\right)^{\alpha-1}}; x > 0, \alpha > 0, \beta > 0, \gamma > 0 \rightarrow (22)$$

where α and β parameter of location, and γ is the shape parameter of the Exponentiated distribution. Figure 4 shows the hazard function of the EBVD.

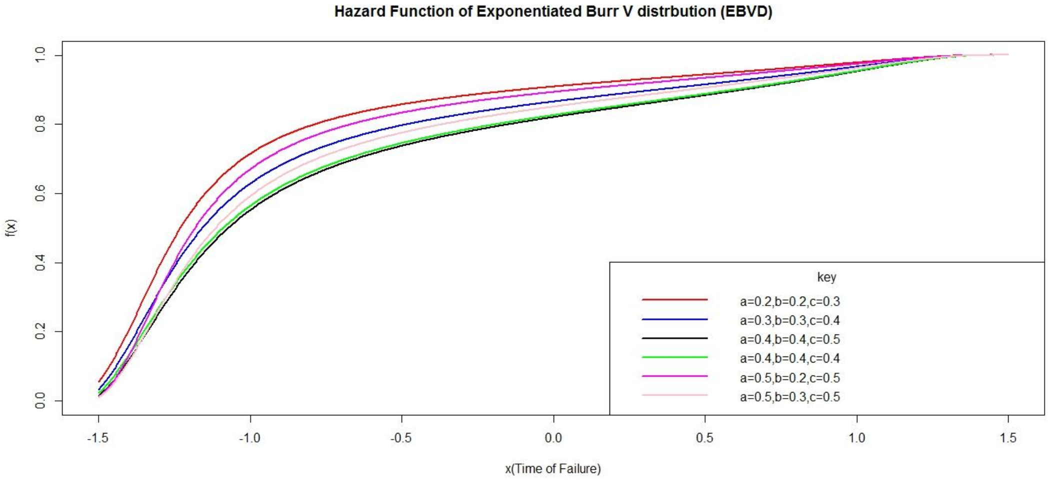

Figure 4: Hazard function of EBVD

3.3 The Quantile Function and the Median

The quantile function of the EBVD is obtained using the CDF of the proposed distribution by equating it to a variable's and making *x* the subject of the formula

$$F(x) = p \qquad \rightarrow (23)$$

$$\Rightarrow \left(1+\beta e^{-\tan x}\right)^{\alpha\gamma} = p$$

$$1+\beta e^{-\tan x} = p^{-\frac{1}{\alpha\gamma}}$$

$$\beta e^{-\tan x} = p^{-\frac{1}{\alpha\gamma}} - 1$$

$$e^{-\tan x} = \frac{p^{-\frac{1}{\alpha\gamma}} - 1}{\beta}$$

$$-\tan x = \ln\left(\frac{p^{-\frac{1}{\alpha\gamma}} - 1}{\beta}\right)$$

$$\tan x = -\ln\left(\frac{p^{-\frac{1}{\alpha\gamma}} - 1}{\beta}\right)$$

Therefore the quantile function is given by:

$$x_p = \tan^{-1} - \ln\left(\frac{p^{-\frac{1}{\alpha\gamma}} - 1}{\beta}\right) \qquad \rightarrow (24)$$

The median is derived when $p = 0.5$

$$x_{0.5} = \tan^{-1} - \ln\left(\frac{0.5^{-\frac{1}{\alpha\gamma}} - 1}{\beta}\right) \qquad \rightarrow (25)$$

IV. DENSITY FUNCTION EXPANSION AND PARAMETER ESTIMATION

The probability density function of EBVD can be expanded using the binomial series expansion technique given as:

$$(1-x)^n = 1 + {}^nC_1(-x) + {}^nC_2(-x)^2 + {}^nC_3(-x)^3 + \ldots \quad \rightarrow (26)$$

$$= 1 - {}^nC_1 x + {}^nC_2 x^2 - {}^nC_3 x^3 + \ldots \quad \rightarrow (27)$$

$$= \sum_{r=0}^{\infty} (-1)^r \, {}^nC_r x^r \quad \rightarrow (28)$$

The density function and distribution function of the proposed distribution is given as

$$f(x) = \alpha\beta\gamma e^{-\tan x} \sec^2 x \left(1+\beta e^{-\tan x}\right)^{-\alpha(\gamma-1)} \left(1+\beta e^{-\tan x}\right)^{-\alpha-1} \quad \rightarrow (29)$$

$$F(x) = \left(1+\beta e^{-\tan x}\right)^{-\alpha\gamma} \quad \rightarrow (30)$$

The above expression can also be written as

$$F(x) = \left(\left(1+\beta e^{-\tan x}\right)^{-\alpha}\right)^{\gamma} \quad \rightarrow (31)$$

$$F(x) = \left(\left(1+\beta e^{-\tan x}\right)^{-\alpha}\right)^{\gamma} = \sum_{i=1}^{\infty} \binom{\gamma}{i} \left(1+\beta e^{-\tan x}\right)^{-\alpha i} \quad \rightarrow (32)$$

The density function binomial expansion is given as;

$$f(x) = \alpha\beta\gamma e^{-\tan x} \sec^2 x \left(1+\beta e^{-\tan x}\right)^{-\alpha-1} \sum_{i=1}^{\infty} \binom{\gamma}{i} \left(1+\beta e^{-\tan x}\right)^{-\alpha i} \quad \rightarrow (33)$$

$$= \alpha\beta\gamma e^{-\tan x} \sec^2 x \sum_{i=1}^{\infty} \binom{\gamma}{i} \left(1+\beta e^{-\tan x}\right)^{-(\alpha i+1)} \quad \rightarrow (34)$$

$$\left(1+\beta e^{-\tan x}\right)^{-(\alpha i+1)} = \sum_{j=0}^{\infty} \binom{-\alpha i + j}{j} \left(\beta e^{-\tan x}\right)^{j} \quad \rightarrow (35)$$

$$\left(1+\beta e^{-\tan x}\right)^{-(\alpha i+1)} = \sum_{j=0}^{\infty} \binom{-\alpha i + j}{j} \beta^{j} e^{-j\tan x} \quad \rightarrow (36)$$

$$= \alpha\beta\gamma e^{-\tan x} \sec^2 x \sum_{i=1}^{\infty} \binom{\gamma}{i} \binom{-\alpha i + j}{j} \beta^{j} e^{-j\tan x} \quad \rightarrow (37)$$

$$= \alpha\gamma\beta^{j+1} \sum_{i=1}^{\infty} \binom{\gamma}{i} \binom{-\alpha i + j}{j} \sec^2 x e^{-(j+1)\tan x} \quad \rightarrow (38)$$

4.1 Moments

The *r*th moment μ_r is given as:

$$\mu_s = E\left[X^s\right] = \int_{-\infty}^{\infty} X^s f(x)dx \quad \rightarrow (39)$$

where *f*(*x*) is the PDF of the proposed distribution in equation (1).

From the binomial expansion of the proposed distribution, the pdf is rewritten as:

$$f(x) = \alpha\gamma\beta^{j+1} \sum_{i=1}^{\infty} \binom{\gamma}{i} \binom{-\alpha i + j}{j} \sec^2 x e^{-(j+1)\tan x} \quad \rightarrow (40)$$

Let $W_{\alpha,\gamma,\beta} = \alpha\gamma\beta^{j+1} \sum_{i=1}^{\infty} \binom{\gamma}{i} \binom{-\alpha i + j}{j}$

So,

$$f(x) = W_{\alpha,\gamma,\beta} Sec^2 x e^{-(j+1)\tan x} \quad \rightarrow (41)$$

$$\mu_s = \int_{-\infty}^{\infty} x^s f(x)dx = \int_{-\pi/2}^{\pi/2} x^s W_{\alpha,\gamma,\beta} Sec^2 xe^{-(j+1)\tan x} dx \qquad \to (42)$$

$$\mu_s = W_{\alpha,\gamma,\beta} \int_{-\pi/2}^{\pi/2} x^s \, Sec^2 xe^{-(j+1)\tan x} dx \qquad \to (43)$$

Let $u = (j+1)\tan(x)$

$$\frac{du}{dx} = \sec^2 x(j+1)\tan(x) \qquad \to (44)$$

$$dx = \frac{du}{\sec^2 x(j+1)\tan(x)} \qquad \to (45)$$

Making x the subject of the formular from (45)

$$\frac{u}{j+1} = \tan(x) \Rightarrow x = \tan^{-1}\left(\frac{u}{j+1}\right) \qquad \to (46)$$

$$\mu_s = W_{\alpha,\gamma,\beta} \int_{-\pi/2}^{\pi/2} \left[\tan^{-1}\left(\frac{u}{j+1}\right)\right]^s \frac{1}{j+1} e^{-u} du \qquad \to (47)$$

Recall that

$$\tan^{-1}(x) = \frac{1}{1+x^2}$$

So,

$$\mu_s = W_{\alpha,\gamma,\beta} \int_{-\pi/2}^{\pi/2} \frac{1}{\left(1+\frac{u^2}{(j+1)^2}\right)^s} \frac{1}{j+1} e^{-u} du \qquad \to (48)$$

$$\mu_s = W_{\alpha,\gamma,\beta} \frac{1}{j+1} \int_{-\pi/2}^{\pi/2} \left(1+\frac{u^2}{(j+1)^2}\right)^{-s} e^{-u} du \qquad \to (49)$$

Using expansion series for the integral expression

$$\sum_{w=0}^{\infty} \binom{s}{w} \frac{u^{2s}}{(j+1)^{2s}}$$

$$\mu_s = W_{\alpha,\gamma,\beta} \frac{1}{j+1} \sum_{w=0}^{\infty} \binom{s}{w} \frac{1}{(j+1)^{2s}} \int_{-\pi/2}^{\pi/2} u^{2s} e^{-u} du \qquad \to (50)$$

4.2 Moment Generating Function

Suppose X is a continuous random variable with PDF $f(x)$. The moment generating function of X is defined as:

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} f(x) dx \qquad \to (51)$$

where t is a parameter and $f(x)$ is the proposed distribution (EBVD) rewritten as:

$$f(x) = W_{\alpha,\gamma,\beta} Sec^2 xe^{-(j+1)\tan x} \qquad \to (52)$$

$$M_x(t) = \int_{-\infty}^{\infty} e^{tx} W_{\alpha,\gamma,\beta} Sec^2 xe^{-(j+1)\tan x} dx \qquad \to (53)$$

$$M_x(t) = W_{\alpha,\gamma,\beta} \int_{-\infty}^{\infty} e^{tx} Sec^2 xe^{-(j+1)\tan x} dx \qquad \to (54)$$

$$M_x(t) = W_{\alpha,\gamma,\beta} \int_{-\infty}^{\infty} Sec^2 xe^{tx} e^{-(j+1)\tan x} dx \qquad \to (55)$$

$$M_x(t) = W_{\alpha,\gamma,\beta} \int_{-\infty}^{\infty} Sec^2 xe^{tx-(j+1)\tan x} dx \qquad \to (56)$$

Let $y = tx \Rightarrow x = \frac{y}{t} \Rightarrow \frac{dx}{dy} = \frac{1}{t}$

$$M_x(t) = W_{\alpha,\gamma,\beta} \int_{-\infty}^{\infty} \frac{Sec^2(y/t)e^{y-(j+1)\tan x}}{t} dy \qquad \rightarrow (57)$$

Therefore,

$$M_x(t) = E\left(e^{tx}\right) = W_{\alpha,\gamma,\beta} \int_{-\infty}^{\infty} \frac{Sec^2(y/t)e^{y-(j+1)\tan x}}{t} dy \qquad \rightarrow (58)$$

4.3 Characteristic function

Suppose X is a continuous random variable with PDF *f*(*x*). The characteristic function of X is defined as:

$$M_X(t) = E(e^{itX}) = \int_{-\infty}^{\infty} e^{itx} f(x) dx \qquad \rightarrow (59)$$

where *it* is a parameter and *f*(*x*) is the proposed distribution (EBV) rewritten as:

$$f(x) = W_{\alpha,\gamma,\beta} Sec^2 x e^{-(j+1)\tan x} \qquad \rightarrow (60)$$

$$M_x(it) = \int_{-\infty}^{\infty} e^{itx} W_{\alpha,\gamma,\beta} Sec^2 x e^{-(j+1)\tan x} dx \qquad \rightarrow (61)$$

$$M_x(t) = W_{\alpha,\gamma,\beta} \int_{-\infty}^{\infty} e^{itx} Sec^2 x e^{-(j+1)\tan x} dx \qquad \rightarrow (62)$$

$$M_x(t) = W_{\alpha,\gamma,\beta} \int_{-\infty}^{\infty} Sec^2 x e^{itx} e^{-(j+1)\tan x} dx \qquad \rightarrow (63)$$

$$M_x(t) = W_{\alpha,\gamma,\beta} \int_{-\infty}^{\infty} Sec^2 x e^{itx-(j+1)\tan x} dx \qquad \rightarrow (64)$$

Let $y = itx \Rightarrow x = \frac{y}{it} \Rightarrow \frac{dx}{dy} = \frac{1}{it}$

$$M_x(t) = W_{\alpha,\gamma,\beta} \int_{-\infty}^{\infty} \frac{Sec^2(y/it)e^{y-(j+1)\tan x}}{it} dy \qquad \rightarrow (65)$$

Therefore,

$$M_x(t) = E\left(e^{itx}\right) = W_{\alpha,\gamma,\beta} \int_{-\infty}^{\infty} \frac{Sec^2(y/it)e^{y-(j+1)\tan x}}{it} dy \qquad \rightarrow (66)$$

4.4 Order statistic

The order statistics of a random sample $X_1,...,X_n$ are the sample values placed in ascending order. They are denoted by $X_{(1)},...,X_{(n)}$. The order statistics are random variables that satisfy $X_{(1)} \le X_{(2)} \le ... \le X_{(n)}$. The following are some statistics that are easily defined in terms of the order statistics. Let $X_1, X_{2,}..., X_n$ be an independent random sample from a PDF *f*(*x*) with an associated CDF *F*(*x*). The PDF of the i^{th} order statistics $x_{(i)}$ is defined by:

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-i)!} f(x)\left(F(x)\right)^{i-1}\left[1-F(x)\right]^{n-i} \qquad \rightarrow (67)$$

Using binomial expansion by simplifying the above expression $\left[1-F(x)\right]^{n-i}$ can be rewritten as

$$\left[1-F(x)\right]^{n-i} = \sum_{k=0}^{n-i} {}^{n-i}C_k \left[F(x)\right]^k \qquad \rightarrow (68)$$

Then

$$f_{i:n}(x) = \sum_{k=0}^{n-i} {}^{n-i}C_k f(x)\left[F(x)\right]^{k+i-1} \qquad \rightarrow (69)$$

Hence, the i^{th} order statistic for the EBV distribution is given as:

$$f_{i:n}(x) = \sum_{p=0}^{n-i} {}^{n-i}C_p \alpha\beta\gamma e^{-\tan x} \sec^2 x\left(1+\beta e^{-\tan x}\right)^{-\alpha(\gamma-1)}\left(1+\beta e^{-\tan x}\right)^{-\alpha-1}\left(1+\beta e^{-\tan x}\right)^{-\alpha\gamma(p+i-1)} \qquad \rightarrow (70)$$

4.5 Maximum Likelihood Estimation of the Parameters of the Proposed Model

Let $X_1, X_2, ...X_n$ be a random sample from a population X with probability density function $f(x:\theta)$ where θ the parameters with unknown. The likelihood function $L(\theta)$ is defined as the product of the joint density of the random variables $X_1, X_2, ...X_n$ given as

$$L(\theta) = \prod_{i=1}^{n} f(x;\theta) \quad \rightarrow (71)$$

The value of θ that maximizes the likelihood function $L(\theta)$ is called the maximum likelihood estimators of θ which is denoted by $\hat{\theta}$.

$$L(x:\alpha,\beta,\gamma) = \prod_{i=1}^{n} g(x:\alpha,\beta,\gamma) \quad \rightarrow (72)$$

$$= \prod_{i=1}^{n} \alpha\beta\gamma e^{-\tan x} \sec^2 x \left(1+\beta e^{-\tan x}\right)^{-\alpha(\gamma-1)} \left(1+\beta e^{-\tan x}\right)^{-\alpha-1} \quad \rightarrow (73)$$

$$= \alpha^n \beta^n \gamma^n \prod_{i=1}^{n} e^{-\tan x} \sec^2 x \left(1+\beta e^{-\tan x}\right)^{-\alpha(\gamma-1)} \left(1+\beta e^{-\tan x}\right)^{-\alpha-1} \quad \rightarrow (74)$$

$$= \alpha^n \beta^n \gamma^n \prod_{i=1}^{n} e^{-\tan x_i} \sec^2 x_i \left(1+\beta e^{-\tan x_i}\right)^{-\alpha(\gamma-1)} \left(1+\beta e^{-\tan x_i}\right)^{-\alpha-1} \quad \rightarrow (75)$$

Taking the log of the above equation, we have the log-likelihood function given as:

$$l(x;\alpha,\beta,\gamma) = n\log(\alpha) + n\log(\beta) + n\log(\gamma) - \sum_{i=1}^{n} \tan x_i + \sum_{i=1}^{n} \log\sec^2 x_i - \alpha(\gamma-1)\sum_{i=1}^{n} \log\left(1+\beta e^{-\tan x}\right)$$
$$-(\alpha-1)\sum_{i=1}^{n}\log\left(1+\beta e^{-\tan x}\right)$$

Differentiating the above log-likelihood function partially with respect to the parameters and equating to zero;

$$\frac{dl}{\alpha} = \frac{n}{\alpha} - (\gamma-1)\sum_{i=1}^{n}\log\left(1+\beta e^{-\tan x_i}\right) - \sum_{i=1}^{n}\log\left(1+\beta e^{-\tan x_i}\right) = 0 \quad \rightarrow (76)$$

$$\frac{dl}{\beta} = \frac{n}{\beta} - \alpha(\gamma-1)\sum_{i=1}^{n}\frac{e^{-\tan x_i}}{1+\beta e^{-\tan x_i}} - (\alpha-1)\sum_{i=1}^{n}\frac{e^{-\tan x_i}}{1+\beta e^{-\tan x_i}} = 0 \quad \rightarrow (77)$$

$$\frac{dl}{\gamma} = \frac{n}{\gamma} - \alpha\sum_{i=1}^{n}\log\left(1+\beta e^{-\tan x_i}\right) = 0 \quad \rightarrow (78)$$

V. FINDINGS AND DISCUSSION

Exponentiated distributions have been studied extensively in statistics especially with the work of [7]. Many classes of these distributions have been developed by various authors such as the work of [7], [8], [9]. The generalized distribution of the Exponentiated distribution was computed [10] and [11]. Also, the burr family [1] is special type of continuous distribution with wide application in the area of household income modeling. However, due to the outlier experienced in different countries of the world, this paper developed a distribution with the properties to capture outlier and heavy tailed distribution present in such data set to regress in clustering analysis of settlement and location using the Burr V distribution. In this paper, a new model was proposed called Exponentiated Burr V distribution to serve as a better alternative. Explicit expressions for survival, hazard and quantile functions were estimated. Also other statistical properties including moments, moment generating function, characteristics function and order statistics were derived. The parameters for the new distribution were estimated theoretically using the maximum likelihood estimation method. It is expected that the proposed model will application in various data set with heavy tailed pattern or outlier(s), especially in clinical trial research, hydrology and related areas.

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