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**Research Paper**

# Quarter Symmetric Non-metric Connection on Lorentzian $\alpha$-Sasakian Manifolds

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***ABSTRACT:*** *The object of the present paper is to study a quarter symmetric non-metric connection on a Lorentzian α-Sasakian manifold. We study the concircular curvature tensor, projective curvature tensor, and conformal curvature tensor on a Lorentzian α-Sasakian manifold with respect to quarter symmetric non-metric connection and also we studied the Second-order parallel tensor with respect to the quarter symmetric non-metric connection.*

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***KEY WORDS:*** *Lorentzian α -Sasakian manifold, concircular curvature tensor, pro- jective curvature tensor, conformal curvature tensor, Second-order parallel tensor.*

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## I. INTRODUCTION

As a generalization of semisymmetric connection[6]. Golab introduced the notion of quarter symmetric linear connection, and is defined as follows: A linear connection $\tilde{\nabla}$ on a $n$-dimensional Riemannian manifold is said to be quarter symmetric connection [6], if its torsion tensor $T$ satisfies

(1.1) $$T(W, Y) = \tilde{\nabla}_W Y - \tilde{\nabla}_Y W - [W, Y]$$

(1.2) $$T(W, Y) = \eta(Y)\varphi W - \eta(W)\varphi Y.$$

In the above equation $\eta$ stands for 1-form and $\varphi$ is a (1, 1) tensor field. On the other hand, Hayden [7] introduced the notion of metric connection on a Riemannian manifold. A connection $\tilde{\nabla}$ on a Riemannian manifold is said to be metric connection if

(1.3) $$(\tilde{\nabla}_W g)(Y, Z) = 0,$$

other wise it is non-metric.

for all $W, Y, Z \in T_pM$, where T(M) is the lie algebra of the vector field on $M$, then $\tilde{\nabla}$ is said to be a quarter symmetric metric connection. In particular if $\varphi W = W$, then the quarter symmetric metric connection reduces to a Semi-symmetric connection [5], otherwise it is said to be a quartersymmetric nonmetric connection. .

Later Rastogi [11, 12] continued to the study of quarter symmetric metric connection on the same way in 1980, [8] studied the quarter symmetric metric connection on Riemannian, sasakian and Kaehlerian manifolds on the same way so many authors [1, 2, 9, 14, 17] studied various types of quarter symmetric metric connection and their properties.

Motivated by the above studies, in the present paper we study quarter symmetric non-metric connections on Lorentzian $\alpha$ -Sasakian manifold and is organized as follows: The followed section is preliminary in nature. In section 3, we exhibit a relation between Riemannian connection and quarter symmetric non-metric connection. Section 4 is devoted to the study of curvature tensor, Ricci tensor, scalar curvature and the first Binachi identity with respect to quarter symmetric non- metric connection. Sections 5 and 6 deal with the study of concircular, conformal and projective curvature tensors on a Lorentzian $\alpha$-Sasakian manifold admitting quarter symmetric non-metric connection. Ultimately, in last section we study second-order symmetric parallel tensor with respect to quarter symmetric non-metric connection on a Lorentzian $\alpha$ -Sasakian manifold.

## II. Preliminaries

An $n(=2m+1)$-dimensional differentiable manifold $M$ is said to be an Lorentzian $\alpha$ -Sasakian manifold, if it admits a (1, 1) tensor field $\varphi$, a contravariant vector field $\xi$, a covariant vector field $\eta$ and a Lorentzian metric $g$ which satisfies

(2.1) $$\eta(\xi) = -1, \quad g(W, \xi) = \eta(W), \; \varphi^2 W = W + \eta(W)\xi, \; \varphi\xi = 0,$$

(2.2) $$g(\varphi W, \varphi Y) = g(W, Y) + \eta(W)\eta(Y), \quad \nabla_W \xi = -\alpha\varphi W,$$

(2.3) $$(\nabla_W \varphi)(Y) = \alpha\, g(W, Y)\xi + \eta(Y)W,$$

for any $W, Y \in T_pM$, and for a smooth function $\alpha$ on $M$, where $\nabla$ denotes the operator of covariant differentiation with respect to the Lorentzian metric $g$. Further on a Lorentzian $\alpha$-Sasakian manifold the following relations hold [19]:

(2.4) $$g(R(W, Y)Z, \xi) = \eta(R(W, Y)Z) = \alpha^2[g(Y, Z)\eta(W) - g(W, Z)\eta(Y)],$$

(2.5) $$R(\xi, W)Y = \alpha^2[g(W, Y)\xi - \eta(Y)W,$$

(2.6) $$R(W, Y)\xi = \alpha^2[\eta(Y)W - \eta(W)Y,$$

(2.7) $$R(\xi, W)\xi = \alpha^2[W + \eta(W)\xi],$$

(2.8) $$S(W, \xi) = S(\xi, W) = (n-1)\alpha^2\eta(W).$$

(2.9) $$S(\xi, \xi) = -(n-1)\alpha^2,$$

(2.10) $$Q\xi = (n-1)\alpha^2\xi,$$

(2.11) $$g(QW, Y) = S(W, Y),$$

(2.12) $$S(\varphi W, \varphi Y) = S(W, Y) + (n-1)\alpha^2 g(W, Y).$$

Let $(M, g)$ be an $n$-dimensional Riemannian manifold. Then the concircular curvature tensor $C^*$ [18], the Weyl conformal curvature tensor $C$ [16] and projective curvature tensor $P$[3] are defined by

(2.13) $$C^*(W, Y)Z = R(W, Y)Z - \frac{r}{n(n-1)}\{g(Y, Z)W - g(W, Z)Y\},$$

(2.14) $$C(W, Y)Z = R(W, Y)Z - \frac{1}{n-2}\{S(Y, Z)W - S(W, Z)Y + g(Y, Z)QW - g(W, Z)QY\} + \frac{r}{(n-1)(n-2)}\{g(Y, Z)W - g(W, Z)Y\},$$

(2.15) $$P(W, Y)Z = R(W, Y)Z - \frac{1}{n-1}\{S(Y, Z)W - S(W, Z)Y\}.$$

## III. Relation between the Riemannian connection and the quarter symmetric non-metric connection

Let $\tilde{\nabla}$ be a linear connection and $\nabla$ be a Riemannian connection of a Lorentzian $\alpha$ -Sasakian manifold $M$ is given by

(3.1) $$\tilde{\nabla}_W Y = \nabla_W Y + H(W, Y),$$

where $H$ is a tensor of type (1, 2). For $\tilde{\nabla}$ to be a quarter symmetric connection in $M$, we have [6]

(3.2) $$H(W, Y) = \frac{1}{2}[T(W, Y) + T^1(W, Y) + T^1(Y, W)],$$

where

(3.3) $$g(T^1(W, Y)Z) = g(T(W, Y), Z) = g(T(Z, W), Y).$$

From (1.2) and (3.3), we get

(3.4) $$T^1(W, Y) = g(\varphi Y, W)\xi - \eta(W)\varphi Y.$$

By using (1.2) and (3.4) in (3.2), we obtain

(3.5) $$H(W, Y) = -\eta(W)\varphi Y,$$

thus a quarter symmetric connection $\tilde{\nabla}$ in a Lorentzian $\alpha$-Sasakian manifold is given by

(3.6) $$\tilde{\nabla}_W Y = \nabla_W Y - \eta(W)\varphi Y.$$

By using (3.6) in (1.1), we obtain

(3.7) $$T^1(W, Y) = \tilde{\nabla}_W Y - \tilde{\nabla}_Y W - [W, Y], = \eta(Y)\varphi W - \eta(W)\varphi Y.$$

The equation (3.7) shows that the connection $\tilde{\nabla}$ is a quarter symmetric linear connection [6], we have

(3.8) $$(\tilde{\nabla}_W g(Y, Z)) = \eta(Y)g(\varphi W, Z) - \eta(Z)g(Y, \varphi W).$$

In view of (3.7) and (3.8), we conclude that $\tilde{\nabla}$ is a quarter symmetric non-metric connection and (3.6) is the relation between the Riemannian connection and the quarter symmetric connection on a Lorentzian $\alpha$ -Sasakian manifold.

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## IV. Curvature tensor of a Lorentzian $\alpha$ -Sasakian manifold with respect to a quarter symmetric non-metric connection

The curvature tensor of a Lorentzian $\alpha$ -Sasakian manifold with respect to a quarter symmetric

non-metric connection $\tilde{\nabla}$ is given by

(4.1) $$\tilde{R}(W, Y)Z = \tilde{\nabla}_W \tilde{\nabla}_Y Z - \tilde{\nabla}_Y \tilde{\nabla}_W Z - \tilde{\nabla}_{[W,Y]} Z.$$

Using (3.6) in (4.1), we get

(4.2) $$\tilde{R}(W, Y)Z = R(W, Y)Z - \eta(Y)(\nabla_W \varphi)Z + \eta(W)(\nabla_Y \varphi)Z - (\nabla_W \eta)(Y)\varphi Z + (\nabla_Y \eta)(W)\varphi Z,$$

and in view of (2.2) and (2.3), we obtain

(4.3) $$\tilde{R}(W, Y)Z = R(W, Y)Z + \{g(Y, Z)\eta(W) - \eta(Y)g(W, Z)\}\xi + \alpha\, \eta(Z)\{\eta(W)Y - \eta(Y)W\}.$$

The equation (4.3) is the relation between the curvature tensor of $M$ with respect to a quarter symmetric non-metric connection $\tilde{\nabla}$ and the Riemannian connection $\nabla$. By using (4.3) and (2.2), we get

(4.4) $$\tilde{R}(W, \xi)Y = R(W, \xi)Y + \alpha\{\eta(Y)\eta(W) + g(W, Y)\}\xi + \alpha\, \eta(Y)\{\eta(W)\xi + W\},$$

(4.5) $$\tilde{R}(W, \xi)Y = \alpha\, (\alpha + 1)\{\eta(Y)W - \eta(W)Y\}.$$

Taking inner product of (4.3) with respect to $U$, we have

(4.6) $$\tilde{R}(W, Y, Z, U) = R(W, Y, Z, U) + \alpha\, \{g(Y, Z)\eta(W) - \eta(Y)g(W, Z)\}\eta(U) + \alpha\, \eta(Z)\{\eta(W)g(Y, U) - \eta(Y)g(W, U)\}.$$

In view of (4.6), we can state the following:

**Proposition 4.1.** *A Lorentzian $\alpha$ -Sasakian manifold with respect to quarter symmetric non-metric connection is a quasi constant curvature if the manifold is of constant curvature with respect to the Levi-Civita connection.*

Also from (4.6), we have

(4.7) $$\tilde{R}(W, Y, Z, U) = -\tilde{R}(Y, W, Z, U).$$

But

(4.8) $$\tilde{R}(W, Y, Z, U) \neq -\tilde{R}(W, Y, Z, U).$$

From (4.3) it is obvious that

(4.9) $$\tilde{R}(W, Y)Z + \tilde{R}(Y, Z)W + \tilde{R}(Z, W)Y = 0.$$

Hence the curvature tensor with respect to quarter symmetric non-metric connection satisfies first Bianchi identity.

Contracting (4.6) with $U$, $W$ we get

(4.10) $$\tilde{S}(Y,Z) = S(Y,Z) - \alpha\, g(Y,Z) - n\,\alpha\, \eta(Y)\eta(Z).$$

Again contracting (4.10), yields

(4.11) $$\tilde{r} = r,$$

where $\tilde{S}$ and $S$, $\tilde{r}$ and $r$ are the Ricci tensor and Scalar curvature of the connections $\tilde{\nabla}$ and $\nabla$ respectively.

Hence we can state the following:

**Proposition 4.2.** *If $M$ is a Lorentzian $\alpha$-Sasakian manifold with respect to a quarter symmetric non-metric connection $\tilde{\nabla}$, then*

(1) *The curvature tensor $\tilde{R}$ is given by (4.6)*
(2) *The Ricci tensor $\tilde{S}$ is given by (4.10)*
(3) *The first Bianachi identity is given by (4.9)*
(4) $\tilde{r} = r$
(5) *The Ricci tensor $\tilde{S}$ is symmetric*
(6) *If $M$ is Einstein or $\eta$-Einstein with respect to Riemannian connection, then $M$ is $\eta$-Einstein with respect to quarter symmetric non-metric connection.*

## V. CONCIRCULAR AND CONFORMAL CURVATURE TENSOR ON A LORENTZIAN $\alpha$-SASAKIAN MANIFOLD WITH RESPECT TO THE QUARTER SYMMETRIC NON-METRIC CONNECTION

We define the Concircular curvature tensor $\tilde{C}^*$ and Conformal curvature tensor $\tilde{C}$ on a Lorentzian $\alpha$-Sasakian manifold with respect to a quarter symmetric non-metric connection $\tilde{\nabla}$ by

(5.1) $$\tilde{C}^*(U,Y)Z = \tilde{R}(U,Y)Z - \frac{\tilde{r}}{n(n-1)}\{g(Y,Z)U - g(U,Z)Y\},$$

(5.2) $$\tilde{C}(U,Y)Z = \tilde{R}(U,Y)Z - \frac{1}{n-2}\{\tilde{S}(Y,Z)U - \tilde{S}(U,Z)Y + g(Y,Z)\tilde{Q}U - g(U,Z)\tilde{Q}Y\} + \frac{\tilde{r}}{(n-1)(n-2)}\{g(Y,Z)U - g(U,Z)Y\},$$

for any $U, Y, Z \in T_pM$, where $\tilde{Q}$ is the symmetric endomorphism of the tangent space at each point corresponds to $\tilde{S}$ and $\tilde{r}$ are the Ricci tensor and the scalar curvature with respect to quarter symmetric non-metric connection.

Using (2.13) and (4.2) in (5.1), yields

(5.3) $$\tilde{C}^*(U,Y)Z = C^*(U,Y)Z + \alpha\,\{g(Y,Z)\eta(U) - g(U,Z)\eta(Y)\}\xi + \alpha\,\eta(Z)\{\eta(U)Y - \eta(Y)U\}.$$

If we consider $\tilde{C}^* = C^*$, then (5.3) reduces to

(5.4) $$g(U,W) = -n\eta(U)\eta(W).$$

In view of (5.4) in (4.10), we get

(5.5) $$\tilde{S}(Y,Z) = S(Y,Z).$$

Hence we can state the following:

**Theorem 5.3.** *If in a Lorentzian $\alpha$ -Sasakian manifold the concircular curvature tensor is invariant under quarter symmetric non-metric connection, then the Ricci tensors are equal with respect to Levi-Civita and quarter symmetric non-metric connections.*

Let us consider a Lorentzian $\alpha$ -Sasakian manifold with respect to quarter symmetric non-metric connection satisfying the condition $\tilde{C}^*(\xi, U)\cdot\tilde{S} = 0$, then we get

(5.6) $$\tilde{S}(\tilde{C}^*(\xi,U)Y,Z) + \tilde{S}(Y,\tilde{C}^*(\xi,U))Z = 0.$$

By virtue of (5.3), we get

(5.7)
$$\tilde{S}(C^*(\xi,U)Y,Z) - \alpha g(U,Y)\tilde{S}(\xi,Z) - 2\alpha\eta(U)\eta(Y)\tilde{S}(\xi,Z) - \alpha\eta(Y)\tilde{S}(U,Z) + \tilde{S}(Y,C^*(\xi,U)Z) - \alpha g(U,Z)\tilde{S}(Y,\xi) - 2\alpha\eta(U)\eta(Z)\tilde{S}(Y,\xi) - \alpha\eta(Z)\tilde{S}(Y,U) = 0.$$

In view of (2.8) and (2.13), (5.7) gives

(5.8) $$\tilde{S}(U,Y) = A\,g(U,Y) + B\eta(U)\eta(Y)$$

By virtue of (5.5) in (5.8) yields

(5.9) $$S(U,Y) = A\,g(U,Y) + B\eta(U)\eta(Y)$$

Where

$$A = \frac{\alpha[n\alpha(n-1)(1+\alpha)-r] - n\alpha(n-1)(1+\alpha)[n\alpha(n-1)(1-\alpha)+r]}{n\alpha(n-1)(1+\alpha)-r}$$

and

$$B = \frac{n\alpha[n\alpha(n-1)(1+\alpha)-r] - 2n(n-1)\alpha^2[n\alpha(n-1)(1-\alpha)+r]}{n\alpha(n-1)(1+\alpha)-r}$$

Hence

**Theorem 5.4.** A Lorentzian $\alpha$- Sasakian manifold with respect to quarter symmetric non-metric connection satisfying the condition $\tilde{C}^*(\xi,U)\cdot\tilde{S} = 0$ is a $\eta$- Einstein manifold.

Also use of (4.3) and (4.10) in (5.2), reduces to

**(5.10)**
$$\begin{aligned}\tilde{C}(U,Y,Z,W) = {} & R(U,Y,Z,W) + \alpha\{g(Y,Z)\eta(U)\eta(W) - \eta(Y)\eta(W)g(U,Z)\} + \alpha\{\eta(Y)\eta(Z)[g(Y,W) \\ & - g(U,W) - \frac{1}{n-2}\{S(Y,Z)g(U,W) - \alpha g(Y,Z)g(U,W) - n\alpha\eta(Y)\eta(Z)g(U,W) \\ & - S(U,Z)g(Y,W) + \alpha g(U,Z)g(Y,W) + n\alpha\,\eta(U)\eta(Z)g(Y,W) + g(Y,Z)S(U,W) \\ & - \alpha\,g(Y,Z)g(U,W) - n\alpha\,g(Y,Z)\eta(U)\eta(W) - g(U,Z)S(Y,W) + \alpha\,g(U,Z)g(Y,W) \\ & + n\alpha g(U,Z)\eta(Y)\eta(W)\} + \frac{\tilde{r}}{(n-1)(n-2)}\{\tilde{g}(Y,Z)g(U,W) - \tilde{g}(U,Z)g(Y,W)\}.\end{aligned}$$

Using (2.14) in (5.10), it follows that

(5.11)
$$\tilde{C}(U,Y,Z,W) = C(U,Y,Z,W) + \frac{2}{n-2}\{g(Y,Z)g(U,W) - g(U,Z)g(Y,W)\} + (\alpha + \frac{n\alpha}{n-2})\{g(Y,Z)\eta(U)\eta(W) - g(U,Z)\eta(Y)\eta(W)\} + (\alpha - \frac{n\alpha}{n-2})\{g(Y,W)\eta(U)\eta(Z) - g(U,W)\eta(Y)\eta(Z)\}.$$

This is the relation between conformal curvature tensor $C$ and $\tilde{C}$ with respect to the Riemannian connection and quarter symmetric non-metric connection respectively.

Let us consider the Lorentzian $\alpha$ -Sasakian manifold is to be conformally flat with respect to quarter symmetric non-metric connection i.e, $\tilde{C}(U,Y,Z,W) = 0$. Now by virtue of (5.2), we obtain

(5.12)
$$\tilde{R}(U,Y,Z,W) = \frac{1}{n-2}\{\tilde{S}(Y,Z)g(U,W) - \tilde{S}(U,Z)g(Y,W) + \tilde{S}(U,W)g(Y,Z) - \tilde{S}(Y,W)g(U,Z)\} + \frac{r}{(n-1)(n-2)}\{g(Y,Z)g(U,W) - g(U,Z)g(Y,W)\}.$$

On plugging $Y = W = \xi$ in (5.12), we get

(5.13)
$$\check{S}(U,Z) = \left\{\frac{\alpha(\alpha+1)(n-1)^2(n-2)-r}{(n-1)(n-2)}\right\}g(U,Z) + \left\{\frac{2\alpha(\alpha+1)(n-1)^2-r}{(n-1)(n-2)}\right\}\eta(U)\eta(Z).$$

Substituting (5.13) in (5.12), obtain

(5.14)
$$\tilde{R}(U,Y,Z,W) = A[g(Y,Z)g(U,W) - g(U,Z)g(U,Z)g(Y,W)] + B\{g(U,W)\eta(Y)\eta(Z) - g(Y,W)\eta(U)\eta(Z) + g(Y,Z)\eta(U)\eta(W) - g(U,Z)\eta(Y)\eta(W)\}.$$

Where
$$A = \frac{2\alpha(\alpha+1)(n-1)^2(n-2)-r-r(n-3)}{(n-1)(n-2)(n-3)} \text{ and } B = \frac{2\alpha(\alpha+1)(n+1)^2(n-2)-r}{(n-1)(n-2)(n-3)}$$

Hence we can state the following:

**Theorem 5.5.** *If a Lorentzian $\alpha$ -Sasakian manifold is conformally flat with respect to quarter symmetric non-metric connection, then the manifold is of quasi constant curvature with respect to Levi-Civita connection.*

## VI. PROJECTIVE CURVATURE TENSOR ON A LORENTZIAN *A*-SASAKIAN MANIFOLD WITH RESPECT TO QUARTER SYMMETRIC NON-METRIC CONNECTION

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The projective curvature tensor $\tilde{P}$ on a Lorentzian $\alpha$-Sasakian manifold with respect to the quarter symmetric non-metric connection is given by

(6.1) $$\tilde{P}(U,Y)Z = \tilde{R}(U,Y)Z - \frac{1}{n-1}\{\tilde{S}(Y,Z)U - \tilde{S}(U,Z)Y\},$$

for any $U, Y, Z \in T_pM$, where $\tilde{S}$ is the Ricci tensor of the manifold with respect to quarter symmetric non-metric connection.

Let Lorentzian $\alpha$-Sasakian manifold with respect to the quarter symmetric non-metric connection satisfies the condition $\tilde{P}(\xi, U)\cdot\tilde{S} = 0$. Then we get

(6.2) $$\tilde{S}(\tilde{P}(\xi,U)Y,Z) + \tilde{S}(Y,\tilde{P}(\xi,U)Z) = 0.$$

By virtue of (6.1), (6.2) yields

(6.3) $$\tilde{S}(\tilde{R}(\xi,U)Y,Z) + \tilde{S}(Y,\tilde{R}(\xi,U)Z) - \frac{1}{n-1}\{\tilde{S}(U,Y)\tilde{S}(\xi,Z)$$
$$-\tilde{S}(\xi,Y)\tilde{S}(U,Z) + \tilde{S}(U,Z)\tilde{S}(Y,\xi) - \tilde{S}(\xi,Z)\tilde{S}(Y,U)\} = 0.$$

On plugging $Z = \xi$ in (6.3), we get

(6.4) $$(\alpha^2+\alpha)\tilde{S}(U,Y) = \{2\alpha\,\eta(U)\eta(Y) - (\alpha^2-\alpha)g(U,Y)\}\tilde{S}(\xi,\xi)$$
$$+(\alpha^2+\alpha)\eta(Y)\tilde{S}(U,\xi) - (\alpha^2+\alpha)\eta(U)\tilde{S}(Y,\xi).$$

By virtue of (4.10), we have

(6.5) $$S(U,Y) = [\alpha^2(n-1)^2 - (n-2)\alpha]g(U,Y) + (2-n)(\alpha^2+2\alpha)\eta(U)\eta(Y).$$

Hence we can state the following:

**Theorem 6.6.** *If a Lorentzian $\alpha$-Sasakian manifold with respect to quarter symmetric non-metric connection satisfies the condition $\tilde{P}(\xi, U)\cdot\tilde{S} = 0$, then it is $\eta$-Einstein manifold.*

## 7. Second-order parallel tensor on a Lorentzian $\alpha$-Sasakian manifold with respect to quarter symmetric non-metric connection

**Deftnition 7.1.** *A covariant $\sigma$ of second order is said to be second-order parallel tensor if $\nabla\sigma = 0$, where $\nabla$ denotes the operator of covariant differentiation with respect to the metric tensor $g$.*

Let us consider a second order parallel tensor with respect to quarter symmetric non-metric connection on a Lorentzian $\alpha$ -Sasakian manifold, such that $\nabla\sigma = 0$, then it follows that

$$\sigma(\tilde{R}(W, U)Y, Z) + \sigma(Y, \tilde{R}(W, U\sigma)Z) = 0, \tag{7.1}$$

for any $U, Y, Z, W \in T_pM$. Substituting $Y = Z = W = \xi$ in (7.1), we get

$$\sigma(\tilde{R}(\xi, U)\xi, \xi) = 0. \tag{7.2}$$

In view of (4.3), we have

$$\sigma(U, \xi) = -g(U, \xi)\sigma(\xi, \xi). \tag{7.3}$$

Differentiating (7.3) along the arbitrary vector field $Y$, we get

$$\sigma(\nabla_Y U, \xi) + \sigma(U, \nabla_Y \xi) = -g(\nabla_Y U, \xi)\sigma(\xi, \xi) - g(U, \nabla_Y \xi)\sigma(\xi, \xi) - 2g(U, \xi)\sigma(\nabla_Y \xi, \xi). \tag{7.4}$$

Now replace $U$ by $\nabla_Y U$ in (7.3), we get

$$\sigma(\nabla_Y U, \xi) = -g(\nabla_Y U, \xi)\sigma(\xi, \xi). \tag{7.5}$$

By using (7.5) in (7.4), we have

$$\sigma(U, \nabla_Y \xi) = -g(U, \nabla_Y \xi)\sigma(\xi, \xi) - 2g(U, \xi)\sigma(\nabla_Y \xi, \xi). \tag{7.6}$$

Use of (2.3) in (7.4), gives

$$\sigma(U, \varphi Y) = -g(U, \varphi Y)\sigma(\xi, \xi) - 2g(U, \xi)\sigma(\varphi Y, \xi). \tag{7.7}$$

Replacing $Y$ by $\varphi Y$ in (7.7) and then using (7.3), we get

$$\sigma(U, Y) = -g(U, Y)\sigma(\xi, \xi). \tag{7.8}$$

Hence we can state the following:

**Theorem 7.7.** *If a Lorentzian $\alpha$-Sasakian manifold, $M$ admits a second order symmetric parallel tensor, then the second order parallel tensor is a constant multiple of associated metric tensor.*

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