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**Research Paper**

# Solution of Elastic Equilibrium Equation of Composite Laminate Plate

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***Abstract***
*In this paper, First we have derive the equations of equilibrium of composite laminated plates through energy approach. Then these equation, are reduced to single fourth order partial differential in-homogeneous equation in one of displacement components. We considered no coupling between in-plane and out of plane displacement equations.*

***Keywords:*** *composite plate , Laminates, Mechanics of composites , Elasticity coupling , Multilayer structure .*

## I. INTERODUCTION

Laminated plates are one of very commonly used structural elements. There are so many theories namely:
(i) Classical elasticity analysis applicable to non- homogeneous plates.
(ii) Moderately thick plate theories including thickness–shear and thickness-normal deformations.
(iii) Thin plate analysis.

The history of theory of elasticity shows that the development of the theory has been guided by the natural philosophy i.e., to understand the nature than in trying to make it more comfortable to human civilization. In a "Balanced" laminate such coupling does not occur. In a minimum-weight- designs of panels and plates the absence of coupling is desirable. If these have fewer number of plies than the number of available options be less. Situations are there when a variable skin thickness is desired. therefore, a single ply removed may make balanced laminate as unbalanced.

Chamis used Galerkin technique to discuss about the response of simply supported rectangular plates under uniform normal and shear loads. Chiu employed energy method for variety of boundary conditions.

We shall follow the last variety because of simplicity .The simplified form is because of symmetrical stacking of plies with respect to middle plane.

The response of laminated plates is governed by three coupled displacement equations of equilibrium. Most of research workers prefer to take three separate functions corresponding to three displacement components. we are going to extend the method of Kushwaha to composite plate theory.

## II. STRAIN-DISPLACEMENT-CURVATURE RELATIONSHIPS.

The assumptions of thin plate theory lead to the following relations.

DISPLACEMENT RELATIONS

$$\left.\begin{aligned} u &= u^{0}(x,y) - z\frac{\partial w}{\partial x} \\ v &= v^{0}(x,y) - z\frac{\partial w}{\partial y} \\ w &= w(x,y) \end{aligned}\right\} \qquad (2.1)$$

STRAIN - CURVATURE RELATIONS

$$\left.\begin{aligned} e_x &= e_x^0 + z\,k_x,\ e_y = e_y^0 + z\,k_y \\ e_{xy} &= e_{xy}^0 + z\,k_{xy} \end{aligned}\right\} \quad (2.2)$$

Where

$$e_x^0 = \frac{\partial u^0}{\partial x}, e_y^0 = \frac{\partial v^0}{\partial y}, e_{xy}^0 = \frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x}, \quad (2.3)$$

$$k_x = -\frac{\partial^2 w}{\partial x^2}, k_y = \frac{\partial^2 w^0}{\partial y^2}, k_{xy}^0 = -2\frac{\partial^2 w}{\partial x \partial y}, \quad (2.4)$$

The superscript 'o' refer to quantities of middle plane. Here we are using rectangular co-ordinate system embeded in middle plane so that xy plane is in middle plane and w is transverse displacement.

we are going to use the notation-

$$\left[N_x, N_y, N_{xy}\right] = \int_{-t/2}^{t/2} \left[\sigma_x, \sigma_y, \sigma_{xy}\right] dz, \quad (2.5)$$

$$\left[M_x, M_y, M_{xy}\right] = \int_{-t/2}^{t/2} \left[\sigma_x, \sigma_y, \sigma_{xy}\right] 2dz \quad (2.6)$$

where z-axis is perpendicular to middle plane and

t is thickness of plate.

## III. DERIVATION OF EQUATION OF EQUILIBRIUM

We take strain energy of elementary volume dx dy dz as dV along with the work done by transverse load q(x, y)

Elementary energy = dV

$$\therefore\ dV = \frac{1}{2}\left[\sigma_i e_i + \sigma_{ij} e_{ij}\right] dx\,dy\,dz + q(x, y) dx\,dy$$

$$i, j = x, y, z. \quad (3.1)$$

It is assumed

$$\frac{\partial w}{\partial z} = 0, e_{yz} = e_{xz} = 0, \quad (3.2)$$

et

$$V = \frac{1}{2}\iint\Big[N_x e_x^0 + N_y e_y^0 + N_{xy} e_{xy}^0 - M_x \frac{\partial^2 w}{\partial x^2}$$

$$-M_y \frac{d^2 w}{dy^2} - 2M_{xy}\frac{\partial^2 w}{\partial x \partial y}\Big] dx\, dy + \iint q(x, y) dx\, dy$$

$$(3.3)$$

where $N_x$, $N_y$, $N_{xy}$ are in plane stresses and $M_x$, $M_y$; $M_{xy}$ are tending and twisting moments,

The constitutive relations are

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & : & 0 \\ 0 & : & D \end{bmatrix}\begin{bmatrix} e^0 \\ k \end{bmatrix} \quad (3.4)$$

with

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \quad (3.5)$$

Where,

$$\left[A_{ij}, B_{ij}, D_{ij}\right] = \int_{-t/2}^{t/2} \bar{Q}_{ij}(\theta_k)[1, z, z^2]dz, \quad (3.6)$$

where $B_{ij} = 0$,

and $\bar{Q}_{ij}(\theta_k)$ is the reduced stiffness coefficient of $k^{th}$ layer with angle $\theta$. Using these constitutive relations we get

$$V = \frac{1}{2}\iint\Big[A_{11}e_x^{0^2} + 2A_{12}e_x^0 e_y^0 + A_{22}e_y^{02}$$

$$+A_{66}e_{xy}^{0^2} + D_{11}w_{,xx}^2 + 2D_{12}w_{,xx}w_{,yy}$$

$$+D_{22}\ w_{,yy}^2 + 4D_{66}w_{,xy}^2\Big]dx\,dy$$

$$+\iint q(x,y)dx\,dy. \qquad (3.7)$$

Now we apply the principle of virtual work then we get the following

$$A_{11}e_{x,x}^0 + A_{12}\ e_{y,x}^0 + A_{66}e_{xy,y}^0 = 0 \qquad (3.8)$$

$$A_{12}e_{x,x}^0 + A_{22}\ e_{y,y}^0 + A_{66}e_{xy,x}^0 = 0 \qquad (3.9)$$

$$D_{11}w_{,xxxx} + 2(D_{12} + 2D_{66})\,w_{,xxyy} + D_{22}\ w_{,yyyy} = q(x,y) \qquad (3.10)$$

Where the symbol (,) is for partial differentials with respect to variable written on right side of symbol.

## IV. Conclusion

we evaluated analytically, in plane stresses are singular i.e. they have square root (Cauchy Type) singularity at crack tip Components of bending moment or twisting couple are smooth.

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