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Research Paper

Some Results On e^* - Closed sets

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ABSTRACT :
In this paper ,weintroduce and discuss about the strongly e^* - closed sets and strongly e^* - open sets. Some characterizations and several properties ofstrongly e^* - closed sets and strongly e^* - open sets are obtained.
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KEY WORDS: *strongly e^* - closed sets, strongly e^* - open sets, generalized e-closed set, e – Normal.*

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I. STRONGLY E^* - CLOSED SET

Definion:1.1
λ is called strongly e^* - closed set(Se^* - closed) if cl(int(λ))⊆ V, whenever λ⊆ V and V is e- open set inX. The complement of Se*- closed set is Se*- open set.

Definion:1.2
A function $h: Y \to Z$is said to be Strongly e^* - closed if for each closed set β of Y, $h(\beta)$ is a Strongly e^* - closed set in Z.

Theorem;1.1
$h(\lambda)$ is called Strongly e^* - closed, if a function $h: Y \to Z$is continuous and strongly e^* - closed and λ is e- closed of Y.

Proof:
Consider $h(\lambda) \subseteq$ W, where W is an open set of Z. Since $h$is Continuous, h^{-1} (W) is an open set in Y. Therefore, $\lambda \subseteq h^{-1}$ (W). Since λ is e – open, λ⊆ clint$_\delta$ (λ) ∪ intcl$_\delta$(λ)
And clint$_\delta$(λ) ∪ intcl$_\delta$(λ) $\subseteq h^{-1}$ (W).
h(clint$_\delta$(λ) ∪ intcl$_\delta$(λ)) ⊆ W
Since, $h$is Strongly e*- open set.h (clint$_\delta$(λ) ∪ intcl$_\delta$(λ)) is also Strongly e*- open set.
clint(h (clint$_\delta$(λ) ∪ intcl$_\delta$(λ))) ⊆ W
and h (λ) ⊆h (clint$_\delta$(λ) ∪ intcl$_\delta$(λ))
clint$h$ (λ) ⊆ clint$h$(clint$_\delta$(λ) ∪ intcl$_\delta$(λ)) ⊆ W
clint$h$ (λ) ⊆ W
Hence, h (λ) is Strongly e^* - closed set.

Example:1.1
Let X={a,b,c,d} and let $\sigma = \{\emptyset, x, \{a\}, \{c\}, \{a,b\}\{a,c\}, \{a,b,c\}, \{a,c,d\}\}$ is topological Space. Here {a}⊆{a,b,d}, where {a,b,d} is an e-open set ,Then the set $\{a\}$is Strongly e*- closed set in (X, σ)

Definion:1.3
U iscalled generalized e - closed set if cl(U) ⊆ V, whenever U ⊆ V and V is e- open set in X.

Theorem:1.2
If M&N are two subsets of $Y$and if M is Strongly e^* - closed, N is generalized e-closed set, then M∩N is generalized e-closed set.

Proof:
Since int(Y) = Y.Then M becomes generalized e-closed set & Since Intersection of two generalized e-closed set is generalized e-closed set. Hence M∩N is generalized e-closed set.

Definion:1.3
cl$_{Se*}$(λ) = ∩{ λ⊆ V /where λ is Se*- closed sets }
int$_{Se*}$(λ) = ∪{ λ⊇ V /where λ is Se*- openset}

Definion:1.4

If for each Strongly e^* - open set V containing v, λ∩(V-{v})≠ ∅ then that point v of a space Y is said to be Se*- limit point of λ of Y.

Theorem:1.3

The arbitrary union of Se*- neighbourhood of a point is also Se*- neighbourhood of that point.

Proof:

Obvious

Theorem:1.4

If a subset R of Q contained in another subset S of Q then Se*- derived set of R contained in Se*- derived set of S.

Proof:

Consider y is in Se*- derived set of Q.Since, definition of Se*- limit point of R, y ∈Se*- limit point of S.

Hence y∈ Se*- derived set of R ⊆ Se*- derived set of Q.

Se*- derived set of R is contained in Se*- derived set of S.

Theorem:1.5

The union of Strongly e*- derived set of two subsets of Q is equal to Strongly e*- derived set of union of two subsets of Q.

Proof:

Consider R & S are two subsets of Q. Since R⊆ R∪S & S⊆ R∪S. From the theorem(1.2), Se*- derived set of R ⊆ Se*- derived set of Q.

& Se*- derived set of R ⊆ Se*- derived set of (R∪S).

Se*- derived set of S ⊆ Se*- derived set of (R∪S).

ν_x of x such that y ∈ ν_x ⊆ λ. By the definition of Se* neighbourhood of x,thereexists a Se* open set $\mu_x$suchthat y ∈ ν_x ⊆μ_x⊆ λ. set of (R∪S).

Se*- derived set of R∪ Se*- derived set of S ⊆ Se*- derived set of (R∪S). ------ (1)

On the otherhand,

We take x ∉(Se*- derived set of R) ∪(Se*- derived set of S).

x ∉ Se*- derived set of (R∪S).

By the definition of Se*- limit point of a subset,

Se*- derived set of (R∪S)⊆(Se*- derived set of R) ∪ (Se*- derived set of S) ------(2)

By (1) & (2),

Se*- derived set of (R∪S)= (Se*- derived set of R)∪ (Se*- derived set of S).

Remark: 1.1

Se*- derived set of (R∪S)= (Se*- derived set of R)∩(Se*- derived set of S).

Theorem:1.6

If $h: Y \to Z$is a Strongly e^* - closed function, then Se^* - closure of λ contained in h(cl(λ)), for every λ of Y.

Proof:

Consider λ⊆Y. Since h is Se^* - closed function,then$h$(cl(λ)) is Strongly e^* - closed containing h(λ), hence Se^* - closure of h(λ) contained in h(cl(λ)).Therefore Se^* - closure of λ contained in h(cl(λ)).

Theorem:1.7

λ is Se*- open iff λ contains a Se*- open neighbourhood of each of its points where Y be a Topological Space and λ be a subset of X.

Proof:

Take λ is a Se*- open sets. Consider s∈ λ. λ is a Se* neighbourhood of Y. λ contains a Se* neighbourhood of each of its points.

On the Otherhand, Consider s∈λ , there exists a neighbourhoodν_x of x such that y ∈ ν_x ⊆ λ. By the definition of Se* neighbourhood of x,thereexists a Se* open set $\mu_x$suchthat y ∈ ν_x ⊆μ_x⊆ λ. Since,x∈ λ, therefore Se*- open set suchthat x∈ μ_x, x∈∪ {μ_x : x∈ λ }

λ ⊆∪ {μ_x : x∈ λ }----(1)

z ∈ μ_y for some, x∈ λ this implies z ∈λ, λ ⊇∪ {μ_x : x∈ λ } -----(2)

From (1) &(2), λ = ∪ {μ_x : x∈ λ }.

Since the arbitrary union of Se*- open sets is also Se*- open sets. λ is a Se*- open sets.

Theorem:1.8

If λ is Se^* - closed subset of Y and y ∈Y – λ then thereexists a Se* neighbourhood μ of Y suchthat μ∩λ = ∅.

Proof:

Consider λis a Se^* - closed. Y – λ is Se*- open set. Therefore μ be a neighbourhood of y suchthatY – λ ⊇ μ.∴ μ∩λ = ∅.

Definion:1.5

If for each pair of non empty disjoint e- closedsets λ and μ thereexists a disjoint e-open sets S & T, suchthatλ ⊆ S & μ ⊆ T, S∩T = ∅, then the topological Space X satisfies this condition is called e – Normal.

Definion:1.6

If for each pair of non empty disjoint e- closed sets λ and μ thereexists a disjoint Se^*-open sets S & T, suchthat λ ⊆ S & μ ⊆ T, S∩T = ∅, then the topological Space X satisfies this condition is called Se^*- Normal.

Theorem:1.9

If h: Y→Z is continuous, Se^* - closed function from a Normal Space Y onto a Space Z, then Z is e - Normal.

Proof:

Consider M and W be disjoint closed sets of a Space Y. h^{-1}(λ) and h^{-1}(μ) are disjoint closed sets of Y,Sinceh is continuous. As Y is Normal , there exists disjoint opensets S and T of Y suchthat h^{-1}(λ) ⊆ S and h^{-1}(μ) ⊆T,Since h is Se^* - closed then there exists Se*- open sets λ and μ in Z suchthat M ⊆ λ, W ⊆ μ, h^{-1}(λ) ⊆ S and h^{-1}(μ) ⊆T,Since S & T are disjoint, then λ and μ are also disjoint μ∩λ = ∅, ∴ Z is e – Normal.

Theorem:1.10

Every Se^* - Normal is e - Normal.

Proof:

ConsiderY is Strongly e^* - Normal Space.Let λ and μ be two disjoint e- closed sts.There exists disjoint e open sets S and T such that λ ⊆ S & μ ⊆ T. Since Y is Strongly e^* - Normal and e- closed set is closed,thereforeλ and μ are closed sets. Hence Y is e- Normal.

Definion:1.7

If for each Strongly e^* - closed set γ and a point y not belongs to γ there exists disjointopen sets S and T suchthat y belongs to γ and γ contained in T then the topological space Y satisfies this condition is called Strongly e^* - regular.

Theorem:1.11

A function g:Y→Z is continuous and Y is Strongly e^* - regular, then g is Se^*- continuous.

Proof:

Consider y ∈ Y, since Z is Strongly e^* - regular vthere exists a Se^*closed set λ.
Hence y∉λ there exists an open set S in Z contaning g(y) and T in Z containing g(y).
λ⊆ T and y∈ S in Z. Hence cl((int(λ))⊆T.Since g is continuous, g^{-1} (λ) is Se^*closed set in Y.Hence g is Se^*- continuous.

Theorem:1.12

Let bijective f:Y→Z where g is any bijective function and if g^{-1} is Strongly e^* - continuous function then g is e^* - open.

Proof:

Consider M be an open set in Y, then Y/M is closed in Y & $g^{-1}$is Se^*- continuous,
$(g^{-1})^{-1}$(Y/M) is Stronglye*-closed inZ. Hence g(Y/M) = Y/g(M) is Strongly e^*-closed in Z.Hence g is Strongly e^*- open.

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