Quest Journals
Journal of Research in Applied Mathematics
Volume 7 ~ Issue 10 (2021) pp: 72-74
ISSN(Online) : 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

Proof of Collatz Conjecture

Paras Gupta

ABSTRACT
Collatz conjecture states that : start with any positive integer n. If the integer is odd, multiply it by 3 and add 1. If the number is even, keep on dividing it by 2 until an odd integer is obtained. After repeating this sequence again and again, one will be obtained as the final result. It is also known as the 3n+1 problem or the Ulam conjecture. This paper presents the proof of the collatz conjecture using the basic concepts of number theory.
KEYWORDS: *Divisible, odd, even*

Received 17 October, 2021; Revised: 30 October, 2021; Accepted 01 November, 2021

PROOF

f(w)={w/2; if w is even

3w+1; if w is odd}

Let z be an odd number

3z+1=even, since 3*odd=odd

And odd+1=even

therefore after applying f to an odd number, we get an even number_(1)

Now, Let x be an even number. After applying f to an even number, it will either keep on dividing by 2 and become 1 or will become odd at either x/2 or x/4 or x/8……., or in general x/2n, where n =2^s, where s is a whole number.

Let it become odd at x/2n

=> x is divisible by 2n

Now f(x/2n)=3x+2n/2n

=> 3x+2n/2n is even, since after applying f to an odd number, we get an even number(from 1)

3x+2n/2n is even if and only if 3x+2n/2n is divisible by 2 or 3x+2n is divisible by 4n(or 8n or 16n or 32n…)

Now 3x+2n/2n will either keep on dividing by 2 and become 1 or it will become odd at either 3x+2n/2n divided by 2 or 3x+2n/2n divided by 4 or 3x+2n/2n divided by 8…….

Or 3x+2n/4n or 3x+2n/8n or 3x+2n/16n….., or in general 3x+2n/2n.2k or 3x+2n/4nk, where k=2^t(t is a whole number) _(2)

Let it become odd at 3x+2n/4nk
Now f(3x+2n/4nk)=9x+6n+4nk/4nk
Now 9x+6n+4nk/4nk is even_(from 1)

=> 9x+6n+4nk/4nk is even if and only if 9x+6n+4nk/4nk is divisible by 2 or 4 or 8… or 9x+6n+4nk is divisible by 8nk(or 16nk or 32nk or 64nk…..)_ (similar reason as 2)

Now 9x+6n+4nk/4nk will either keep on dividing by 2 and become 1 or it will become odd at either 9x+6n+4nk/4nk divided by 2 or 9x+6n+4nk/4nk divided by 4 or 9x+6n+4nk/4nk divided by 8

Or 9x+6n+4nk/8nk or 9x+6n+4nk/16nk or 9x+6n+4nk/32nk…., or in general 9x+6n+4nk/8nka, where a=2^v(v is a whole number)_(3)

Let it become odd at 9x+6n+4nk/8nka

Now f(9x+6n+4nk/8nka)=27x+18n+12nk+8nka/8nka

Now 27x+18n+12nk+8nka/8nka is even_(from 1)

Now 27x+18n+12nk+8nka/8nka will either keep on dividing by 2 and become 1 or it will become odd at either 27x+18n+12nk+8nka/8nka divided by 2 or 27x+18n+12nk+8nka/8nka divided by 4 or 27x+18n+12nk+8nka/8nka divided by 8…………

Or 27x+18n+12nk+8nka/16nka or 27x+18n+12nk+8nka/32nka or 27x+18n+12nk+8nka/64nka….., or in general 27x+18n+12nk+8nka/16nkap, where p=2^g(g is a whole number)_(4)

Let it become odd at 27x+18n+12nk+8nka/16nkap

Now f(27x+18n+12nk+8nka/16nkap)=81x+54n+36nk+24nka
+16nkap/16nkap

Now 81x+54n+36nk+24nka+16nkap/16nkap is even_(from 1)

Now 81x+54n+36nk+24nka+16nkap/16nkap will either keep on dividing by 2 and become 1 or it will become odd at either 81x+54n+36nk+24nka+16nkap/16nkap divided by 2 or 81x+54n+36nk+24nka+16nkap/16nkap divided by 4 or 81x+54n+36nk+24nka+16nkap/16nkap divided by 8……..

Or 81x+54n+36nk+24nka+16nkap/32nkap
or 81x+54n+36nk+24nka+16nkap/64nkap or 81x+54n+36nk+24nka+16nkap/128…..

Or81x+54n+36nk+24nka+16nkap/32nkapr……, where r=2^h(h is a whole number)_(5)

From (2),(3),(4),(5)

It is clear that every time when f is applied to odd numbers the minimum value by which the numerator of our result has to be divisible becomes 2 for the first time, 4 for the second time, 8 for the third time, 16 for

the fourth time, 32 for the fifth time, 64 for the sixth time, 128 for the seventh time, 256 for the eighth time, 512 for the ninth time and so on.

All these values give 1 on applying f, since each divides with 2 continuously to give 1.

Hence Proved IMPOSSIBLE POSSIBILITY
The possibility that (2),(3),(4),(5),.... will keep on extending till infinity is impossible. If (2),(3),(4),(5),...keep on extending till infinity, then the numerator will tends towards infinity and the denomination will be some power of 2. Such a number will not exist because
Ltx→infinity(infinity)/2^d= infinity,where d is any natural number. This means the above sequence could repeat infinite times only for infinity. But infinity does not exist, so the above sequence will not repeat infinite times.
(The numerator is taken as infinity because the numerator increases every time the above sequence is repeated

and after repeating the above sequence infinite times, the numerator will tend towards infinity)

REFERENCE

[1]. Wikipedia