Quest Journals
Journal of Research in Applied Mathematics
Volume 7 ~ Issue 12 (2021) pp: 04-13
ISSN(Online) : 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

Dynamic behaviors for an almost prey-predator system

Lili Wang
School of Mathematics and Statistics, Anyang Normal University, Anyang Henan 455000, China
Corresponding Author: Lili Wang

ABSTRACT: *This paper is concerned with an almost prey-predator model with water level fluctuations. By using several comparison theorems and some mathematical methods, we derive some sufficient conditions for permanence of the system. Moreover, by using the properties of almost periodic functions and constructing a suitable Lyapunov functional, sufficient conditions which guarantee the existence of uniformly asymptotically stable almost periodic solution of the system are obtained. Finally, an example together with its numerical simulations is presented to illustrate the feasibility and effectiveness of the results.*
KEYWORDS: *Permanence; Almost periodic solution; Asymptotical stability.*

R*eceived 28 Nov, 2021;* ***R****evised 10 Dec, 2021;* ***A****ccepted 12 Dec, 2021 © The author(s) 2021.*
P*ublished with open access at* ***www.questjournals.org***

I. INTRODUCTION

Dynamic behaviors of prey-predator system with water level fluctuations have been studied by many authors in the past few years; see, [1-7]. In [4], the authors studied the following system:

Let $G(t)$ and $B(t)$ be respectively the biomass of the prey and predator at time t. When a predator attacks a prey, it has access to a certain quantity of food depending on the water level. When water level is low the predator is more in contact with the prey. Let $r(t)$ be the accessibility function for the prey. It is assumed that the function $r(t)$ is almost periodic and continuous. The minimum value r_1 is reached in spring and the maximum value r_2 is attained during autumn, denoted respectively by γ_G and γ_B the maximum consumption rate of the resource by the prey and predator. Let e_B be the conversion rate of the prey in biomass and m_G, m_B be respectively the consumption rate of biomass by metabolism of the prey and predator. The predator needs a quantity $\gamma_B B(t)$ for his food, but he has access to a quantity $r(t)\dfrac{G(t)B(t)}{B(t)+D}$, where D measures the other causes of mortality outside the metabolism and predation. It gives the extent to which environment provides protection to the prey. If $\dfrac{r(t)G(t)}{B(t)+D} \geq \gamma_B$, then the predator will be satisfied with the quantity $\gamma_B B(t)$ for his food. Otherwise, i.e if $\dfrac{r(t)G(t)}{B(t)+D} \leq \gamma_B$, the predator will content himself with $r(t)\dfrac{G(t)B(t)}{B(t)+D}$. Consequently, the quantity of food received by the predator is $\min\left(\gamma_B, r(t)\dfrac{G(t)}{B(t)+D}\right)B(t)$.

Accordingly, the prey-predator model can be expressed as

$$\begin{cases} \dfrac{dG(t)}{dt} = -\min\left(\gamma_B, r(t)\dfrac{G(t)}{B(t)+D}\right)B(t) + \gamma_G G(t) - m_G G^2(t), \\ \dfrac{dB(t)}{dt} = e_B \min\left(\gamma_B, r(t)\dfrac{G(t)}{B(t)+D}\right)B(t) - m_B B(t). \end{cases} \tag{1.1}$$

The initial conditions for the system (1.1) take the form of

$$G(s) = \varphi_1(s) > 0, B(s) = \varphi_2(s) > 0, s \in (-\infty, 0], \varphi_i(0) > 0, \tag{1.2}$$

where $\varphi_i, i = 1, 2$ are bounded and continuous functions on $(-\infty, 0]$.

Let B_0, G_0 be respectively the initial density of the predator and prey with $B_0 > 0$ and $G_0 > 0$. Throughout this paper, we suppose that:

(H_1) $r_2 < \min\left(\dfrac{\gamma_B(B_0 + D)}{G_0}, \dfrac{4m_B m_G D\gamma_B}{(\gamma_G + m_B)^2}\right)$;

(H_2) $0 < D < \dfrac{(\gamma_G - r_2)m_B r_1}{m_G e_B}$.

By the theory of functional differential equations [8], it is clear that the system (1.1) has a unique positive solution which satisfies the initial condition (1.2).

The following standard analysis shows that the model (1.1) is biologically sound.

Lemma 1.1 Every solution of the system (1.1) with the initial conditions (1.2) exists in the interval $[0, +\infty]$ and remains positive for all $t \geq 0$.

Lemma 1.2 For all $t \geq 0, r_2 G(t) < \gamma_B(B(t) + D)$.

By Lemma 1.2, the system (1.1) is reduced to the simple form

$$\begin{cases} \dfrac{dG(t)}{dt} = G(t)(\gamma_G - m_G G(t)) - r(t)\dfrac{G(t)B(t)}{B(t)+D}, \\ \dfrac{dB(t)}{dt} = e_B r(t)\dfrac{G(t)B(t)}{B(t)+D} - m_B B(t). \end{cases} \tag{1.3}$$

II. BASIC RESULTS

In this section, we shall develop some preliminary results, which will be used to prove the main result.

Lemma 2.1 ([9])

If $a > 0, b > 0$, and $x' \geq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, then $\liminf\limits_{t\to+\infty} x(t) \geq \dfrac{b}{a}$.

If $a > 0, b > 0$, and $x' \leq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, then $\limsup\limits_{t\to+\infty} x(t) \leq \dfrac{b}{a}$.

*Corresponding Author: Lili Wang

Let D denotes R or an open subset of R. The relevant definitions and the properties of almost periodic functions, see [10].

Definition 2.1 ([10]) $f \in C(R,R)$ is an almost periodic function if and only if for any sequence $\alpha'_n \subset T$, there exists a subsequence $\alpha_n \subset \alpha'_n$ such that $f(t+\alpha_n)$ converges uniformly on R as $n \to +\infty$. Furthermore, the limit function is also an almost periodic function.

Lemma 2.2 ([10]) If $f(t), g(t)$ are almost periodic functions, then, for any $\varepsilon > 0$, $E\{\varepsilon, f\} \cap E\{\varepsilon, g\}$ is a nonempty relatively dense set in R; that is, for any given $\varepsilon > 0$, there exists a constant $l(\varepsilon) > 0$, such that in any interval of length $l(\varepsilon)$, there exists at least a $\tau \in E\{\varepsilon, f\} \cap E\{\varepsilon, g\}$ such that

$$|f(t+\tau) - f(t)| < \varepsilon, |g(t+\tau) - g(t)| < \varepsilon, \forall t \in R.$$

Consider the following almost periodic dynamic equation

$$x' = f(t,x) \tag{2.1}$$

and the associate product system of (2.1)

$$x' = f(t,x),\ y' = f(t,y). \tag{2.2}$$

Lemma 2.3 ([10]) Suppose that there exists a Lyapunov function $V(t,x,y) \in C(R^+ \times D \times D, R)$ satisfying the following conditions:

(1) $a(\|x-y\|) \le V(t,x,y) \le b(\|x-y\|)$, where $a, b \in K$, $K = \{a \in C(R^+, R^+) : a(0) = 0$ and a is increasing$\}$;

(2) $|V(t,x_1,y_1) - V(t,x_2,y_2)| \le L(\|x_1 - x_2\| + \|y_1 - y_2\|)$, where $L > 0$ is a constant;

(3) $D^+V'_{(2.2)}(t,x,y) \le -\alpha V(t,x,y)$, where $\alpha > 0$ is a constant.

Moreover, if there exists a solution $x(t)$ of (2.1) such that $x(t) \in S$, where $S \subset D$ is a compact set. Then there exists a unique uniformly asymptotically stable almost periodic solution $p(t)$ of (2.1) in S. Furthermore, if $f(t,x)$ is periodic with period ω in t, then $p(t)$ is a periodic solution of (2.1) with period ω.

III. PERMANENCE

Assume that the coefficients of (1.3) satisfy

$$(H_3)\ \gamma_G > r_2 \ge r_1 > \frac{m_B D}{e_B m_1}.$$

Theorem 3.1 Let $(G(t), B(t))$ be any positive solution of the system (1.3) with the initial condition (1.2). If (H_3) holds, then the system (1.3) is permanent, that is, any positive solution $(G(t), B(t))$ of the system (1.3) satisfies

$$m_1 \le \liminf_{t\to+\infty} G(t) \le \limsup_{t\to+\infty} G(t) \le M_1, \tag{3.1}$$

$$m_2 \le \liminf_{t\to+\infty} B(t) \le \limsup_{t\to+\infty} B(t) \le M_2, \tag{3.2}$$

especially if $m_1 \le G_0 \le M_1, m_2 \le B_0 \le M_2$, then

$$m_1 \le G(t) \le M_1,\ m_2 \le B(t) \le M_2,\ t \in [t_0, +\infty),$$

where $M_1 = \dfrac{\gamma_G}{m_G} - \dfrac{r_1 m_1}{m_G(m_1 + D)}$, $M_2 = \dfrac{e_B r_2 M_1}{m_B} - D$, $m_1 = \dfrac{\gamma_G - r_2}{m_G}$, $m_2 = \dfrac{e_B r_1 m_1}{m_B} - D$.

Proof. Assume that $(G(t), B(t))$ be any positive solution of the system (1.3) with the initial condition (1.2). It follows from the first equation of the system (1.3) that

$$G'(t) \ge G(t)[(\gamma_G - r_2) - m_G G(t)]. \tag{3.3}$$

By Lemma 2.1, we can get

$$\liminf_{t\to+\infty} G(t) \ge \frac{\gamma_G - r_2}{m_G} := m_1.$$

Then, for arbitrarily small positive constant $\varepsilon > 0$, there exists a $T_1 > 0$ such that

$$G(t) > m_1 - \varepsilon,\ \forall t \in [T_1, +\infty].$$

From the second equation of the system (1.3), when $t \in [T_1, +\infty)$,

$$B'(t) = e_B r(t)\frac{G(t)B(t)}{B(t) + D} - m_B B(t) > e_B r_1 \frac{(m_1 - \varepsilon)B(t)}{B(t) + D} - m_B B(t).$$

Let $\varepsilon \to 0$, then

$$B'(t) \ge e_B r_1 \frac{m_1 B(t)}{B(t) + D} - m_B B(t) \ge \frac{B(t)}{B(t) + D}[e_B r_1 m_1 - m_B D - m_B B(t)]. \tag{3.4}$$

By Lemma 2.1, we can get

$$\liminf_{t\to+\infty} B(t) = \frac{e_B r_1 m_1}{m_B} - D := m_2.$$

Then, for arbitrarily small positive constant $\varepsilon > 0$, there exists a $T_2 > T_1$ such that

$$B(t) > m_2 - \varepsilon,\ \forall t \in [T_2, +\infty).$$

On the other hand, from the first equation of the system (1.3), when $t \in [T_2, +\infty)$,

$$G'(t) = G(t)(\gamma_G - m_G G(t)) - r(t)\frac{G(t)B(t)}{B(t) + D} < G(t)\Big(\gamma_G - \frac{r_1(m_1 - \varepsilon)}{(m_1 - \varepsilon) + D} - m_G G(t)\Big). \tag{3.5}$$

Let $\varepsilon \to 0$, then $G'(t) \le G(t)\left(\gamma_G - \frac{r_1 m_1}{m_1 + D} - m_G G(t)\right)$.

By Lemma 2.1, we can get

$$\limsup_{t\to+\infty} G(t) = \frac{\gamma_G}{m_G} - \frac{r_1 m_1}{m_G(m_1 + D)} := M_1.$$

Then, for arbitrarily small positive constant $\varepsilon > 0$, there exists a $T_3 > T_2$ such that

$$G(t) < M_1 + \varepsilon, \forall t \in [T_3, +\infty].$$

From the second equation of the system (1.3), when $t \in [T_3, +\infty)$,

$$B'(t) = e_B r(t)\frac{G(t)B(t)}{B(t)+D} - m_B B(t) < \frac{B(t)}{B(t)+D}[e_B r_2 (M_1 + \varepsilon) - m_B D - m_B B(t)]. \tag{3.6}$$

Let $\varepsilon \to 0$, then $B'(t) \le \frac{B(t)}{B(t)+D}[e_B r_2 M_1 - m_B D - m_B B(t)]$.

By Lemma 2.1, we can get $\limsup_{t\to+\infty} B(t) = \frac{e_B r_2 M_1}{m_B} - D := M_2$.

Then, for arbitrarily small positive constant $\varepsilon > 0$, there exists a $T_4 > T_3$ such that

$$B(t) < M_2 - \varepsilon, \forall t \in [T_4, +\infty].$$

In special case, if $m_1 \le G_0 \le M_1, m_2 \le B_0 \le M_2$, by Lemma 2.1, it follows from (3.3)-(3.4), (3.5)-(3.6) that

$$m_1 \le G(t) \le M_1, m_2 \le B(t) \le M_2, t \in [t_0, +\infty),$$

This completes the proof.

IV. ALMOST PERIODIC SOLUTIONS

In this section, by constructing a suitable Lyapunov functional, we shall study the existence of a unique almost periodic solution of (1.1), which is uniformly asymptotically stable. By the relationship between (1.1) and (1.3), we only need to study the system (1.3) exists a unique almost periodic solution, which is uniformly asymptotically stable.

Let S be the set of all solutions $(G(t), B(t))$ of the system (1.3) satisfying $m_1 \le G(t) \le M_1, m_2 \le B(t) \le M_2$ for all $t \in R^+$.

Lemma 4.1 $S \ne \varnothing$.

Proof. By Theorem 3.1, we see that for any $t_0 \in R^+$ with $m_1 \le G_0 \le M_1$, $m_2 \le B_0 \le M_2$, the system (1.3) has a solution $(G(t), B(t))$ satisfying $m_1 \le G(t) \le M_1, m_2 \le B(t) \le M_2, t \in [t_0, +\infty)$.

Since $r(t)$ is almost periodic, it follows from Lemma 2.5 that there exists a sequence $t_0 \in R^+$, $\{t_n\}$ $t_n \to +\infty$ as $n \to +\infty$ such that $t_0 \in R^+$ as $r(t+t_n) \to r(t)$ $n \to +\infty$ uniformly on R^+.

Now, we claim that $\{G(t+t_n)\}$ and $\{B(t+t_n)\}$ are uniformly bounded and equi-continuous on any bounded interval in R^+. In fact, for any bounded interval $[\alpha, \beta] \subset R^+$, when n is large enough, $\alpha + t_n > t_0$, then $t + t_n > t_0, \forall t \in [\alpha, \beta]$. So, $m_1 \le G(t+t_n) \le M_1$, $m_2 \le B(t+t_n) \le M_2$ for any $t \in [\alpha, \beta]$, that is, $\{G(t+t_n)\}$ and $\{B(t+t_n)\}$ are uniformly bounded. On the other hand, $\forall t_1, t_2 \in [\alpha, \beta]$, from the mean value theorem of differential calculus on time scales, we have

$$|G(t_1+t_n) - G(t_2+t_n)| \le \left(M_1(\gamma_G + m_G M_1) + r_2 \frac{M_1 M_2}{m_2 + D}\right)|t_1 - t_2|, \tag{4.1}$$

$$|B(t_1+t_n) - B(t_2+t_n)| \le \left(e_B r_2 \frac{M_1 M_2}{m_2 + D} + m_B M_2\right)|t_1 - t_2|. \tag{4.2}$$

The inequalities (4.1) and (4.2) show that $\{G(t+t_n)\}$ and $\{B(t+t_n)\}$ are equi-continuous on $[\alpha, \beta]$. By the arbitrary of $[\alpha, \beta]$, the conclusion is valid.

By Ascoli-Arzela theorem, there exists a subsequence of $\{t_n\}$, we still denote it as $\{t_n\}$, such that

$$G(t+t_n) \to p(t), B(t+t_n) \to q(t),$$

as $n \to +\infty$ uniformly in t on any bounded interval in R^+.

Furthermore,

$$\begin{cases} \dfrac{dG(t+t_n)}{dt} = G(t+t_n)(\gamma_G - m_G G(t+t_n)) - r(t+t_n)\dfrac{G(t+t_n)B(t+t_n)}{B(t+t_n)+D}, \\ \dfrac{dB(t+t_n)}{dt} = e_B r(t+t_n)\dfrac{G(t+t_n)B(t+t_n)}{B(t+t_n)+D} - m_B B(t+t_n). \end{cases}$$

Let $n \to +\infty$, then

$$\begin{cases} \dfrac{dp(t)}{dt} = p(t)(\gamma_G - m_G p(t)) - r(t)\dfrac{p(t)q(t)}{q(t)+D}, \\ \dfrac{dq(t)}{dt} = e_B r(t)\dfrac{p(t)q(t)}{q(t)+D} - m_B q(t). \end{cases}$$

It is clear that $(p(t), q(t))$ is a solution of the system (1.3). Moreover,

$$m_1 \le p(t) \le M_1, m_2 \le q(t) \le M_2, \forall t \in R^+.$$

This completes the proof.

Remark: From the proofs of Theorem 3.1 and Lemma 4.1, we know that if the conditions of Theorem 3.1 hold, S is a positive invariant set of the system (1.3).

Theorem 4.1 Suppose the condition (H_3) holds, assume further that

$$(H_4)\quad \lambda < 0\text{, where } \lambda = \max\left\{-m_G m_1 + \frac{e_B r_2[(1+D)M_1 + M_2]}{(m_2+D)^2}, \frac{D r_2 M_2 - e_B r_1(m_1+m_2)}{(m_2+D)^2}\right\};$$

then there exists a unique uniformly asymptotically stable almost periodic solution $(G(t), B(t))$ of the system (1.1) which is bounded by S^* for all $t \in R^+$.

Proof. Let $x(t) = \ln(G(t))$, $y(t) = \ln(B(t))$, then the system (1.3) can be transformed into

$$\begin{cases} x'(t) = \gamma_G - m_G \exp(x(t)) - r(t)\dfrac{\exp(y(t))}{\exp(y(t))+D}, \\ y'(t) = e_B r(t)\dfrac{\exp(x(t))}{\exp(y(t))+D} - m_B. \end{cases} \tag{4.3}$$

From Lemma 4.1, the system (4.3) has a bounded solution $(x(t), y(t))$ satisfying $\ln m_1 < x(t) < \ln M_1$, $\ln m_2 < y(t) < \ln M_2, \forall t \in R^+$, then $x(t)| < M_1, |y(t)| < M_2$, where

$$M_1 = \max\{|\ln m_1|, |\ln M_1|\},\ M_2 = \max\{|\ln m_2|, |\ln M_2|\}.$$

For $(x, y) \in R^2$, we define the norm $\|(x, y)\| = |x| + |y|$. Consider the product system of system (4.3)

$$\begin{cases} x'(t) = \gamma_G - m_G \exp(x(t)) - r(t)\dfrac{\exp(y(t))}{\exp(y(t))+D}, \\ y'(t) = e_B r(t)\dfrac{\exp(x(t))}{\exp(y(t))+D} - m_B, \\ u'(t) = \gamma_G - m_G \exp(u(t)) - r(t)\dfrac{\exp(v(t))}{\exp(v(t))+D}, \\ v'(t) = e_B r(t)\dfrac{\exp(u(t))}{\exp(v(t))+D} - m_B. \end{cases} \tag{4.4}$$

Suppose $X = (x(t), y(t))$, $Y = (u(t), v(t))$ be any two solutions of system (4.3), then $\|X\| \le A, \|Y\| \le A$, where $A = M_1 + M_2$. Set

$$S^* = \{(x(t), y(t)) \mid \ln m_1 < x(t) < \ln M_1, \ln m_2 < y(t) < \ln M_2, \forall t \in R^+\}.$$

Consider a Lyapunov functional defined on $R^+ \times S^* \times S^*$ as follows

$$V(t, X, Y) = |x(t) - u(t)| + |y(t) - v(t)|. \tag{4.5}$$

Since $\| X - Y\| = |x(t) - u(t)| + |y(t) - v(t)|$, we have

$$\frac{1}{2}\| X - Y\| \le V(t, X, Y) \le 2\| X - Y\|.$$

Let $a, b \in C(R^+, R^+)$, $a(x) = \frac{1}{2}x, b(x) = 2x$, thus the condition (1) in Lemma 2.3 is satisfied.

In addition,

$$\begin{aligned}
&|V(t, X, Y) - V(t, \tilde{X}, \tilde{Y})| \\
&= \big|\, |x(t) - u(t)| + |y(t) - v(t)| - |\tilde{x}(t) - \tilde{u}(t)| - |\tilde{y}(t) - \tilde{v}(t)| \,\big| \\
&\le \big| (x(t) - u(t)) - (\tilde{x}(t) - \tilde{u}(t)) \big| + \big| (y(t) - v(t)) - (\tilde{y}(t) - \tilde{v}(t)) \big| \\
&\le |x(t) - \tilde{x}(t)| + |u(t) - \tilde{u}(t)| + |y(t) - \tilde{y}(t)| + |v(t) - \tilde{v}(t)| \\
&= \| X - \tilde{X}\| + \| Y - \tilde{Y}\|.
\end{aligned}$$

Let $L = 1$, then the condition (2) of Lemma 2.3 is satisfied.

Finally, calculate the $V'(t, X, Y)$ along the solutions of (4.3), we can obtain

$$\begin{aligned}
&V'(t, X, Y) \\
&= \operatorname{sgn}(x(t) - u(t))(x(t) - u(t))' + \operatorname{sgn}(y(t) - v(t))(y(t) - v(t))' \\
&= \operatorname{sgn}(x(t) - u(t))\Big[m_G(\exp(u(t)) - \exp(x(t))) + r(t)\Big(\frac{\exp(v(t))}{\exp(v(t)) + D} - \frac{\exp(y(t))}{\exp(y(t)) + D}\Big)\Big] \\
&+ \operatorname{sgn}(y(t) - v(t)) e_B r(t)\Big[\frac{\exp(x(t))}{\exp(y(t)) + D} - \frac{\exp(u(t))}{\exp(v(t)) + D}\Big] \\
&= \operatorname{sgn}(x(t) - u(t))\Big[m_G(\exp(u(t)) - \exp(x(t))) + D r(t)\frac{\exp(v(t)) - \exp(y(t))}{(\exp(v(t)) + D)(\exp(y(t)) + D)}\Big] \qquad (4.6) \\
&+ \operatorname{sgn}(y(t) - v(t)) e_B r(t)\frac{\exp(x(t) + v(t)) - \exp(y(t) + u(t))}{(\exp(y(t)) + D)(\exp(v(t)) + D)} \\
&+ \operatorname{sgn}(y(t) - v(t)) D e_B r(t)\frac{\exp(x(t)) - \exp(u(t))}{(\exp(y(t)) + D)(\exp(v(t)) + D)}.
\end{aligned}$$

By using the mean value theorem, we have

$$\begin{aligned}
&\exp\{x(t)\} - \exp\{u(t)\} = \xi_1(t)(x(t) - u(t)), \\
&\exp\{y(t)\} - \exp\{v(t)\} = \xi_2(t)(y(t) - v(t)), \\
&\exp\{x(t) + v(t)\} - \exp\{y(t) + u(t)\} = \xi_3(t)[(x(t) - u(t)) + (v(t) - y(t))], \qquad (4.7)
\end{aligned}$$

where $\xi_1(t)$ lies between $\exp\{x(t)\}$ and $\exp\{u(t)\}$;

$\xi_2(t)$ lies between $\exp\{y(t)\}$ and $\exp\{v(t)\}$;

$\xi_3(t)$ lies between $\exp\{x(t)+v(t)\}$ and $\exp\{y(t)+u(t)\}$.

From (4.6) and (4.7), we have

$V'(t, X, Y)$

$$= \operatorname{sgn}(x(t)-u(t))\big[m_G(\exp(u(t))-\exp(x(t))) + Dr(t)\frac{\exp(v(t))-\exp(y(t))}{(\exp(v(t))+D)(\exp(y(t))+D)}\big]$$

$$+\operatorname{sgn}(y(t)-v(t))e_B r(t)\frac{\exp(x(t)+v(t))-\exp(y(t)+u(t))}{(\exp(y(t))+D)(\exp(v(t))+D)}$$

$$+\operatorname{sgn}(y(t)-v(t))e_B r(t)\frac{\exp(x(t)+v(t))-\exp(y(t)+u(t))}{(\exp(y(t))+D)(\exp(v(t))+D)}$$

$$+\operatorname{sgn}(y(t)-v(t))De_B r(t)\frac{\exp(x(t))-\exp(u(t))}{(\exp(y(t))+D)(\exp(v(t))+D)}$$

$$= \big[-m_G\xi_1(t) + \frac{e_B r(t)(\xi_3(t)+D\xi_1(t))}{(\exp(y(t))+D)(\exp(v(t))+D)}\big]|x(t)-u(t)|$$

$$+\frac{Dr(t)\xi_2(t)-e_B r(t)\xi_3(t)}{(\exp(y(t))+D)(\exp(v(t))+D)}|y(t)-v(t)|$$

$$\le \big[-m_G m_1 + \frac{e_B r_2(M_1+M_2+DM_1)}{(m_2+D)^2}\big]|x(t)-u(t)|$$

$$+\frac{Dr_2M_2-e_Br_1(m_1+m_2)}{(m_2+D)^2}|y(t)-v(t)|$$

$$\le \lambda V(t, X, Y), \qquad (4.8)$$

where $\lambda = \max\Big\{-m_G m_1 + \frac{e_B r_2[(1+D)M_1+M_2]}{(m_2+D)^2}, \frac{Dr_2M_2-e_Br_1(m_1+m_2)}{(m_2+D)^2}\Big\}$. From the condition (H_5), $\lambda<0$ and $\lambda \in \mathrm{R}^+$, the condition (3) of Lemma 2.3 is satisfied.

To sum up, from Lemma 2.3, there exists a unique uniformly asymptotically stable almost periodic solution $(x(t), y(t))$ of the system (4.3) which is bounded by S^* for all $t\in R^+$, which means that there exists a uniqueness uniformly asymptotically stable almost periodic solution $(G(t), B(t))$ of (1.3) which is bounded by S for all $t\in R^+$; that is, there exists a uniqueness uniformly asymptotically stable almost periodic solution $(G(t), B(t))$ of (1.1) which is bounded by S for all $t\in R^+$. This completes the proof.

Corollary Assume that $(H_1)-(H_4)$ hold. Suppose that the nonnegative coefficient $r(t)$ is periodic of period ω; then the system (1.1) has a unique uniformly asymptotically stable periodic solution of period ω.

Remark Suppose the conditions $(H_1)-(H_4)$ hold, then the system (1.1) has a unique uniformly asymptotically stable almost periodic solution.

Remark Assume that $(H_1)-(H_4)$ hold. Suppose that the nonnegative coefficient $r(t)$ is periodic of period ω; then the system (1.1) has a unique uniformly asymptotically stable periodic solution of period ω.

REFERENCES

[1]. M.A. Menouer, A. Moussaoui, ``Effects of consecutive water level fluctuations and harvesting on predator-prey interactions," Chaos Soliton. Fract., Vol. 91, pp.434-442, 2016.
[2]. A. Moussaoui, ``A reaction-diffusion equations modelling the effect of fluctuating water levels on prey-predator interactions," Appl. Math. Comput., Vol. 268, pp.1110-1121, 2015.
[3]. A. Moussaoui, S. Bassaid, E.H. Ait Dads, ``The impact of water level fluctuations on a delayed prey-predator model," Nonlinear Anal. RWA, Vol. 21, pp.170-184, 2015.
[4]. N. Chiboub Fellah, S.M. Bouguima, A. Moussaoui, ``The effect of water level in a prey–predator interaction:A nonlinear analysis study," Chaos Soliton. Fract., Vol. 45, pp.205-212, 2012.
[5]. U. Kahl, S. Hlsmann, R.J. Radke, J. Benndorf, ``The impact of water level fluctuations on the year class strength of roach: Implications for fish stock management," Limnologica, Vol. 38, pp.258-268, 2008.
[6]. H. Coops, M. Beklioglu, T.L. Crisman, ``The role of water-level fluctuations in shallow lake ecosystems workshop conclusions," Hydrobiologia, Vol. 506, pp.23-27, 2003.
[7]. H. Coops, S.H. Hosper, ``Water-level management as a tool for the restoration of shallow lakes in the Netherlands," Lake Reserv. Manage, Vol. 18, pp.293-298, 2002.
[8]. J. Hale, Theory of Functional Differential Equations, Springer-Verlag, Heidelberg, 1977.
[9]. F. Chen, ``On a nonlinear nonautonomous predator–prey model with diffusion and distributed delay," J. Comput. Appl. Math., 180, pp.33-49, 2005.
[10]. C. He, Almost Periodic Differential Equations, Higher Eduction Publishing House, Beijing, 1992.