Quest Journals
Journal of Research in Applied Mathematics
Volume 7 ~ Issue 12 (2021) pp: 40-43
ISSN(Online) : 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

New Classification of NG-Groups

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ABSTRACT:
We investigate and present a NG groups that consisting of consisting of non-bijective transformations cannot be subset of symmetric groups by using the notion of AntiGroups. In addition, we study some several examples and basic propertie of these groups. However, in particularly, we find all NG-groups that not subsets of S3 are AntiGroups of types [4]
KEYWORDS: *Symmetric group, NG groups. AntiGroup*

***R**eceived 20 Dec, 2021; **R**evised 28 Dec, 2021; **A**ccepted 31 Dec, 2021 © The author(s) 2021.*
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I. INTRODUCTION

The transformation group is important and good part of the group theory [1]. We recall a permutation group on a non-empty set A is a group consisting of bijections transformations from A to itself with respect to compositions mapping on anon empty set A which the one most important transformation groups for more details see [2],[3],[4]. It is well known that any permutation group on a set A with cardinality n has order not greater that n!. In [5], Y. Wu, X.Wei present the conditions of the groups generated by nonobjective transformations on a set. Author in [6], find the maximal order of these groups. In [7], A.A.A. Agboola present the notion of AntiGroups particularly of type-AG[4]. In this paper, we try to consider NG by using the conspet of AntiGroups. Particularly, we find all NG-groups that not subsets of S3 are AntiGroups of types [4].

II. PRELIMINARY

We review some basics definitions and properties of the finite group theory that we will used in our paper. For more detailed in lots of abstract algebra and finite group theory can see [8],[9], and [10] would be good supplementary sources for the theory needed here.

Definition 2.1. Let $(X \neq \phi)$. If the binary relation ~ in X satisfy the following:

1- $x \sim x$ for any $x \in X$;
2- $x \sim y$ then $y \sim x$ for any $x,y \in X$;
3- $x \sim y$ and $y \sim z$ then $x \sim z$ for any $x,y,z \in X$, then we called ~ an equivalence relation on X.

Definition 2.2. Suppose that ~ is an equivalence relation on X. For an element $x \in X$, we denoted the equivalence class of a determined by ~ and $X/\sim = \{[x]\sim \,|x \in X\}$ is called the quotient set of X relative to the equivalence relation ~.

Definition 2.3. [Classical group][10]. Suppose that X be a nonempty set and let $* : X \times X \rightarrow X$ be a binary operation on X. The $(X, *)$ is called a classical group if the following conditions hold:

(C1) closure law. i.e $\forall x, y \in X$, then $x * y \in X$.
(C2) axiom of associativity. i.e. $\forall x, y, z \in X$, then $x * (y * z) = (x * y) * z$.
(C3) axiom of existence of neutral element. i.e. $\forall x \in X$, there exists $e \in X$ such that $x * e = e * x = x$.
(C4) axiom of existence of inverse element. i.e. $\forall x \in X$, then there exists $y \in X$ such that $x * y = y * x = e$ where e is the neutral element of X.

If in addition,
(C5) $\forall x, y \in X$, $x * y = y * x$, then we say $(Z, *)$ is called an abelian group.

Definition 2.4. [AntiGroup][11] An AntiGroup G is an alternative to the classical group X that has at least one AntiLaw or at least one of :
GC1: AntiClosureLaw. i.e. $\forall\ (x, y) \in X$, then $x*y \notin X$,
GC2: AntiAssociativity. i.e. $\forall\ (x,y,z)\in$X, then $x*(y*z)\neq(x*y)*z$,
GC3: AntiNeutralElement. i.e. $\forall x \in X$, then there does not exist an element $e \in X$ such that $x * e = e * x = x$,
GC4: AntiInverseElement . i.e. $\forall x\in X$, then there does not exist $u\in X$ such that $x*u = u*x=e$ where e is an AntiNeutralElement in X.

If X is an alternative to the classical abelian group X that has at least one AntiLaw or at least one of {GC1, GC2, GC3, GC4 and
GC5: AntiCommutativity. i.e. $\forall\ (x, y)\in X$, $x*y\neq y*x$, then we called X is An AntiAbelianGroup .
Definition 2.5[11]. The NG-groups is groups which consisting of transformations on a non-empty set X and the group has no bijection as its elements.i.e. NG groups is groups that cannot be subset of Symmetric group.

Proposition2.1.([5],theorem 1) For any $f\in NG$ and the e the identity element of NG, $\sim e=\sim f$.
Proof.
For any $x\in X$, our goal is to show that $[a]_f=[a]_e$. On one hand, if $a\in[x]f$, *i.e.* $f(a)=f(x)$. Since NG is a group with identity element e, there is a transformation $f'\in NG$ such that $f'f=e=f'f$. Therefore, $e(a) = f'(f(a)) = f'(f(x))=e(x)$, Which yields that $a\in[x]_e$. On the other hand, *if* $y\in[a]e$ *i.e.* $e(a)e(y)$. Hence, $f(a)=(fe)(a)=f(e(y))=(fe)(y)=f(y)$, which implies $y\in[x]_f$. It follows that $[x]_e.= [x]_f$ for any $x\in X$, as wanted.
Remark 2.1. For Lemma 2.1, we see that $\sim_f=\sim_g$ for any element $f,g\in NG$ .The following Theorem is revised version of Theorem 2,[5].
Theorem 2.1. Let f be an element in $P(X)$ and $\hat{f}$ be the induced transformation of f on $X/\sim f$, *i.e* $\hat{f}$: $X/\sim_f \longrightarrow X/\sim_f$, $[x]_f \mapsto[f(x)]_f$.Then the following hold: There exists a groups $NG\subseteq P(X)$ containing f as the identity element iff $f^2=f$.
There is a groups $NG\subseteq P(X)$ containing f as the identity element iff $\hat{f}$ is abijective on $X/\sim_f$.

We make some corrections to the original proofs of corollaries are from [5]. Actually, we adopt the restriction of finiteness on X in the first corollary from the original one. Moreover, we used the finiteness on X in the second corollary; the original one did not use it.
Corollary 2.1. Let f be an element in $P(X)$. Then $f^2 = f$ iff the induced mapping $\hat{f}$ on $X/\sim_f$ is the identity element.
Proof.
($\Rightarrow$) we suppose that $f^2 = f$. Then for any $[x]_f\in X/\sim f$, *as* $f(x)=f(f(x))$ we see that $[x]_f = [f(x)]_f$. It follows that $\hat{f}([x]_f) = [f(x)]_f = [x]f$; which implies that $\hat{f}$ is the identity mapping on $X/\sim f$.
($\Leftarrow$) we assume that $\hat{f}$ is the identity mapping on $X/\sim f$. Then for any $[x]_f \in X/\sim f$, the condition that $\hat{f}([x]_f) = [x]_f$ will imply that $[f(x)]_f = [x]_f$ and hence $f(f(x)) = f(x)$. It follows that $f^2 = f$ as required.
Corollary 2.2. Suppose that X is a finite set and f is an element in $P(X)$. Then there is a group NG $\subseteq$P(X) containing f as an element iff $Im(f) = Im(f^2)$.
Proof.
($\Rightarrow$) we suppose that there is a group NG $\subseteq$P(X) containing f as an element. Let e be the identity element of NG. Then by Theorem 2.1, the induced mapping $\hat{f}$ is a bijection on $X/\sim f$. In particular, $\hat{f}$ is surjective and thus for any $x\in X$, there is a $[y]_f \in X/\sim f$ such that $\hat{f}([y]_f) = [x]_f = [f(y)]_f$; which yields that $f(x) = f(f(y)) = (f^2)(y)$. As a result, $Im(f) \subseteq Im(f^2)$ and thus $Im(f) = Im(f^2)$.
($\Leftarrow$) we suppose that $Im(f) = Im(f^2)$. Thus, for any $f(x)\in Im(f)$ there is a $y\in X$ such that $f(x) = f(f(y))$ and hence $\hat{f}([y]_f) = [x]_f$; which implies that $\hat{f}$ is surjective on $X/\sim f$. Note that we are assuming that X is finite and so is $X/\sim f$. We have that the induced mapping $\hat{f}$ is bijective. By Theorem 2.1, the assertion follows.
Remark 2.2. Suppose that NG$\subseteq$P(X) is a group. We have seen, in Remark 2.1, that$\sim f=\sim g$ for any elements in NG and we will denote the common equivalence relation by $\sim$. Also, by Theorem 2.1, each element $f \in$NG will induce a bijection $\hat{f}$ on $X/\sim$.

The following theorem is crucial since it turns a group NG $\subseteq$ P(X) into a permutation group.
Theorem 2.2[6]. Suppose that NG$\subseteq$P(X) is a group. Set NG= $\{\hat{f} / f \in NG\}$; then $\hat{NG}$ is a permutation group on $X/\sim$ *and* $\rho : NG \longrightarrow \hat{NG}, f\mapsto\hat{f}$,is an isomorphism.
Proof.

For any $f, g \in NG$ and any $[x] \in X/\sim$, we have $\rho(fg)([x]) = [(fg)(x)] = [f(g(x))] = \rho(f)([g(x)]) = (\rho(f)\rho(g))([x])$; $\Rightarrow \rho(fg) = \rho(f)\,\rho(g)$ and thus ρ *is* a homomorphism. By the definition of $\hat{NG}$, it is obvious that ρ is surjective.
Now suppose that $\rho(f) = \rho(g)$ for two elements $f, g \in NG$, *i.e.* $[f(x)] = [g(x)]$, $\forall x \in X$: Let e be the identity element of NG, then we have $[f(x)]_e = [g(x)]_e$; $\forall x \in X$. It follows that $e(f(x)) = e(g(x))$; $\forall x \in X$ .Hence, $f(x) = (ef)(x) = e(f(x)) = e(g(x)) = g(x)$, $\forall x \in X$, and therefore $f = g$. We conclude that ρ *is* injective. *As a consequence,* ρ is an isomorphism.
Examples2-1 Let F be a field, and V be a vector space of dimension 2 over F. And $\{v_1,v_2\}$ a basis of V. For any $a \in F^*$,we define $Ta : V \rightarrow V$, $\mu = a_1v_1 + a_2v_2 \rightarrow a\, a_1v_1$.

In other words, the linear transformation Ta has matrix $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ with respect to the ordered basis v_1,v_2
Set NG ={*Ta:* $a \in F^*$ }, then NG is group. But, NG is not a subset of symmetric group.
Examples2-2 Suppose X={1,2,3}. We know, S3 ={(1,2,3), (2,3,1), (3,1,2), (1,3,2), (3,2,1), (2,1,3)} is a group but it is not an abelian group. We introduce the elements of Trans(X) as: {(1,1,1), (1,1,2), (1,1,3), (1,2,1), (1,2,2), (1,2,3), (1,3,1), (1,3,2), (1,3,3), (2,1,1), (2,1,2), (2,1,3), (2,2,1), (2,2,2), (2,2,3), (2,3,1), (2,3,2), (2,3,3), (3,1,1), (3,1,2), (3,1,3), (3,2,1), (3,2,2), (3,2,3), (3,3,1), (3,3,2), (3,3,3)}. There exits some groups that are subsets of Trans(*X*), but not subsets of the S_n. We make composition of all transformations we get:
If we apply same way, we get The groups of order 2 are : NG1={(1,1,3),(3,3,1)}, NG2={(1,2,1),(2,1,2)},NG3={(1,2,2),(2,1,1)},NG4={(1,3,3),(3,1,1)},NG5={(2,2,3),(3,3,2)}, and NG6={(2,3,2),(3,2,3)}.
Proposition 2-2: Let NG1 and NG2 be two NG-groups, then NG1 ∪ NG2 and NG1 ∩ NG2 are not necessary to be NG-groups.
Proof: Let X = {1, 2, 3}, we know, NG1 and NG2 are NG-groups. We assume NG1 ∪ NG2 is NG- groups. But, it contradiction with example 2-2. Since order of NG1 ∪ NG2 is four. In addition, NG1 ∩ NG2= ϕ, and ϕ ⊂ S3. Therefore, NG1 ∩ NG2 is not NG-group.

III. OUR RESULTS

In this section, we introduce the concepts in NG groups by using the notation of AntiGroups. Particularly we consider NG groups that not subsets of S_3.

Proposition3-1. Every NG groups of S_3 is Antigroups of type [4].
Proof: straight from the definition2.4

Theorem 3.1[6]. Suppose that *X* is a set with cardinality n with n≥ 3. Suppose NG is a group consisting of non-bijective transformations on A, where the binary operation on NG is the composition of transformations. Then the order of NG is not greater than (n-1)! and there are such groups having order (n-1)!:

Now, from previous theorem 3.1 we can extend proposition 3.1 as follows:
Proposition3-2. Every maximal order of NG groups is Antigroups of type [4].
Proof: straight from the definition2.4 and definition2.2.
Definition 3.1. Suppose that NG1 and NG2 NG groups that not subsets of S3 The direct product of NG1 and NG2 denoted by NG1×NG2 is defined by NG1×NG2= {(f, g) : f ∈ NG1, g ∈NG2}.
Examples 3-1 let consider example 2-2. Suppose X={1,2,3}. We have NG groups :
NG1, NG2, NG3, NG4, NG5, and G6. We have only NG from Gi⊗Gj where(i≠J), I and j =1,2,3,4,5,6.is just:NG1⊗NG7==NG1= NG5⊗NG1,NG6⊗NG2=NG2= NG2⊗NG3, NG3⊗NG2=NG3= NG4⊗NG3, NG3⊗NG4=NG4= NG4⊗NG1,NG1⊗NG5=NG5= NG5⊗NG6, and NG2⊗NG6=NG6= NG6⊗NG5.
So, we have only same NG groups that not subset of S3.
Proposition 3.3. Suppose that NG1 and NG2 are NG groups that not subsets of S3. let ⊗ be a binary operation on NG1×NG2 defined by (f⊗y) = (f ∘ g) ∀ (f∈NG1 and g∈NG2. If The (NG1 ×NG2, ⊗) is NG, the it is an AntiGroup of type [4].
Proof. The proof follows from the definition of AntiGroups of type-AG[4] and example3.1 .
Proposition 3-4: Let NG1 and NG2 be two NG-groups not subset of S3, then NG1 ∪ NG2 is antiGroup of type[4].
Proof: from proposition 2.2 NG1 ∪ NG2 is NG group.
From Proposition3-2. NG1 ∪ NG2 is maximal , then is Antigroups of type [4].

IV. CONCLUSION

We show that the NG group is AntiGroups. In particularly, we find all NG-groups that not subsets of S3 are AntiGroups of types [4]. However, any Ng groups nit subsets of S3 if NGi×NGj , (i≠j)≤6 are NG groups, then (NGi×NGj)is AntiGroup of type [4].
Moreover, the union of any NG that not subset of S3 ids antiGroup of type[4].

ACKNOWLEDGMENTS I would like to thank staff of Higher institute of Suluq for help on this paper.

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