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Surface Gravity and Surface Area Relation in a Schwarzschild Blackhole

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ABSTRACT: The event horizon is the boundary of a blackhole and at it we have a surface gravity (K) given by $\frac{c^4}{4GM}$. This surface gravity is quite strong, but surface gravity gets stronger as we move beyond the event horizon and go further into the blackhole. At the singularity, which is the smallest point in a black hole, the fabric of space time bends infinitely and we have an infinitely small area. This shows that the surface area and the surface gravity in a blackhole actually have an inverse relationship. In this paper we will find a mathematical model for that inverse relationship.

KEYWORDS: Stokes Theorem, Singularity, Poincare Conjecture, Schwarzschild blackholes

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I. INTRODUCTION

Through the differential equations formed and Stokes theorem, we will show that if there is a gravitational singularity, it has no mass. This is contradictory to what we know about the singularity at the present (Katz, 1979). According to existing calculations, the singularity has an infinitely high density, which means there is some mass which is concentrated in an infinitely small volume (Macdonald, 2014). According to the equations formulated in this paper, we find that the singularity has no mass and hence can't contain any matter. This gives birth to many paradoxes, as mass is required for there to be a gravitational force. So how can something which has no mass, bend the fabric of spacetime infinitely?

ANALYSIS

Following the Poincare conjecture, a manifold in 3 or more dimensions is homeomorphic to the 3-sphere (Hamilton, 1997). When it comes to Schwarzschild blackholes, we can assume the area enclosed by the event horizon to be homeomorphic to the area enclosed by the singularity (Hughston & Tod, 1990). Therefore, transforming the event horizon into the singularity, we see an increase in surface gravity.

The surface area of a schwarzschild blackhole can be given as $A = 4\pi r^2$
The radius of the event horizon can be given as $r_{eh} = \frac{2GM}{c^2}$
Since we are minimizing the area and transforming it into the singularity, we notice that surface gravity gets stronger, however this relationship isn't linear. In order to express the rate of change of surface gravity with respect to area, we need a differential equation.

The derivative of surface gravity (K) with respect to surface area (A) can be expressed as follows: $\frac{dK}{dA} = \frac{dK}{dM} \times \frac{dM}{dA}$ where M is mass

We know that surface gravity at the event horizon can be given as follows: $K_{eh} = \frac{c^4}{4GM_{eh}}$

The derivative of surface gravity with respect to mass can be found by differentiating the above expression and this gets us

$$\frac{dK}{dM} = \frac{-c^4}{4GM^2}$$

Now, that we have the surface gravity and mass relation, we must move on to the mass and area relation.

$$A_{eh} = 4\pi r_{eh}{}^2$$

$$r_{eh} = \frac{2GM}{c^2}$$

By substituting the expression above for r, we can find the Area and Mass relation $A_{eh} = 4\pi(\frac{2GM_{eh}}{c^2})^2$

$$M_{eh} = \frac{c^2\sqrt{A_{eh}}}{4G\sqrt{\pi}}$$

To find the derivative of Mass with respect to area, we have to differentiate the expression above and that gives us

$$\frac{dM}{dA} = \frac{c^2}{8G\sqrt{A\pi}}$$

Since we have found $\frac{dK}{dM}$ as well as $\frac{dM}{dA}$ we can now find out $\frac{dK}{dA}$,

$$\frac{dK}{dA} = \frac{dK}{dM} \times \frac{dM}{dA} = \frac{-c^4}{4GM^2} \times \frac{c^2}{8G\sqrt{A\pi}} = \frac{-c^6}{32G^2M^2\sqrt{A\pi}}$$

Now we have to write M in terms of A in order to find an accurate relation which is as follows:

$$M_{eh} = \frac{c^2\sqrt{A_{eh}}}{4G\sqrt{\pi}}$$

$$M^2 = \frac{c^4A}{16G^2\pi}$$

Substituting M squared in the derivative we get

$$\frac{dK}{dA} = \frac{-c^6}{32G^2\frac{c^4A}{16G^2\pi}\sqrt{A\pi}} = \frac{-c^2\sqrt{\pi}}{2A^{\frac{3}{2}}}$$

Arriving at a general equation for surface gravity K

$$k = \int \frac{dk}{dA} \times dA$$

$$k = \int \frac{-c^2\sqrt{\pi}}{2\sqrt{A}^3} \times dA = \frac{-c^2\sqrt{\pi}}{2}\int A^{-3/2} \times dA$$

$$k = \frac{-c^2\sqrt{\pi}}{2} \times \frac{-2}{\sqrt{A}} = \frac{c^2\sqrt{\pi}}{\sqrt{A}}$$

$$A = 4\pi r^2$$

$$\sqrt{A} = 2\sqrt{\pi}r$$

$$k = \frac{c^2}{2r}$$

$$k_{eh} = \frac{c^2\sqrt{\pi}}{\sqrt{4\pi r^2{}_{eh}}}$$

$$r_{eh} = \frac{2GM}{c^2}$$

$$k_{eh} = \frac{c^2\sqrt{\pi}}{2\sqrt{\pi}\times 2GMc^{-2}} = \frac{c^4}{4GM}$$

Since this expression for k corresponds with the earlier expression of k which was given by Schawrzchild, we can confirm that the derivative of surface gravity with respect to surface area holds true.

By using Stokes theorem, we can show the difference between surface gravity at the singularity and the surface gravity at the event horizon. This means that the boundary condition is from the event horizon to the singularity.

$$k_s - k_{eh} = \int_{eh}^{s} \frac{dk}{dA} \times dA = \frac{c^2\sqrt{\pi}}{\sqrt{A_s}} - \frac{c^2\sqrt{\pi}}{\sqrt{A_{eh}}}$$

$$= \frac{c^2}{2r_s} - \frac{c^2}{2r_{eh}}$$

$$r_{s\to 0}$$

$$k_{s\to\infty}$$

Therefore, surface gravity at the singularity approaches infinity.

By using Stokes theorem, we can show the difference between Mass at the event horizon and mass at the singularity. This means that the boundary condition is from the singularity to the event horizon.

$$M_{eh} - M_s = \int_{s}^{eh} \frac{dM}{dA} \times dA = \int_{s}^{eh} \frac{c^2}{8G\sqrt{A\pi}} \times dA = \frac{c^2}{8G\sqrt{\pi}}\int A^{-1/2}$$

$$= \frac{c^2}{8G\sqrt{\pi}} \times 2\sqrt{A} = \frac{c^2\sqrt{A_{eh}}}{4G\sqrt{\pi}} - \frac{c^2\sqrt{A_s}}{4G\sqrt{\pi}} = \frac{c^2(r_{eh}-r_s)}{2G}$$

$$r_{s\to 0}$$

$$\frac{c^2(r_{eh})}{2G} = M_{eh} - M_s$$

$$r_{eh} = \frac{2GM}{c^2}$$

$$M_{eh} - M_s = \frac{c^2 \times 2GM_{eh}}{2Gc^2} = M_{eh}$$

This proves that the mass of the singularity approaches 0. So if there is some form of a singularity that has the radius approaching 0, and the mass also approaches 0, then how is it able to bend the fabric of spacetime. Since gravity is what causes the bending of space time and mass is required to have a gravitational force, it seems paradoxical that something with no mass approaches infinite surface gravity. However, according to existing ideas, at the singularity the fabric of space-time bends infinitely (Claes, 2006). That means that some of the mass of the blackhole should be concentrated at the region of the singularity. However, according to these calculations, the singularity has no mass and yet surface gravity tends towards infinity. This paradox could perhaps show that mass may not be the sole characteristic that determines the distortion of space-time.

Now we will move onto another approach that demonstrates that there may be no singularity at all. In calculus a key concept is that in the graph of a function, at the maximum or minimum point, the derivative is equal to 0. If this is true and the singularity is the point of maximum gravity. Then $\frac{dK}{dA} = 0$ and $\frac{dk}{dM} = 0$ at the singularity. According to expressions calculated earlier, $\frac{dK}{dA} = \frac{-c^2\sqrt{\pi}}{2A^{\frac{3}{2}}}$ and $\frac{dK}{dM} = \frac{-c^4}{4GM^2}$. Since 0 isn't equal to $-c^2$ and it isn't equal to $-c^4$, we can say that perhaps there is no maximum or minimum point. Instead, the further we reduce the area, the further we increase the surface gravity, and we can keep going on like this forever, but never reach 0 area. Hence, there may be no singularity (Forshaw & Smith, 2004).

II. CONCLUSION

In conclusion, a gravitational singularity may not exist for a Schwarzschild blackhole with these specific boundary conditions. If a gravitational singularity does exist, it approaches no mass and hence the expression for its surface gravity becomes paradoxical, even though at a surface area equal to 0, the surface gravity becomes infinitely large as shown earlier. These paradoxes highlight that it isn't possible for a gravitational singularity to exist in both states of 0 area and 0 mass.

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