Quest Journals
Journal of Research in Applied Mathematics
Volume 7 ~ Issue 6 (2021) pp: 21-28
ISSN(Online) : 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

Representations of Time Series Models Applied to the First and Second Difference Operation with Back Shift

Şenol Çelik
Bingöl University Faculty of Agriculture Biometry and Genetic Department Bingöl-Turkey
Corresponding Author: senolcelik@bingol.edu.tr

Abstract
In this study, explicit writing of time series models such as Autoregressive (AR), moving average (MA), and Autoregressive moving average (ARMA) with Back-shift operators are provided. In practice, the coefficients of the ARIMA models obtained in most studies are given and their interpretations are made, but the model is not explicitly expressed as an equation. Sometimes, ARIMA models are misspelled as equations by most researchers. In the open writing of the ARIMA model, which is provided that it is stationary by taking the difference, the difference is ignored and written as the ARMA model. In order to correct this inaccuracy, the expression of ARIMA models clearly written in this study should be taken into account.
Keywords: *ARIMA, Back-Shift, stationary*

Received 06 June, 2021; Revised: 18 June, 2021; Accepted 20 June, 2021 © The author(s) 2021. Published with open access at www.questjournals.org

I. INTRODUCTION

Time series is a general concept used for data sorted by time. Unlike other data, sequence has a significance in time series. The time series is a probabilistic process. What will be the future values of the time series and the relationships between the time series are used as an important tool in decision-making processes [1].

Time series are an important application area of statistics and sometimes econometrics, with applications in all areas of science. A time series is a sequence of measurements observed at periodic intervals of time. Monthly quantities exported product from a factory, a roadway accident numbers that occur on a weekly, hourly the height of water level in a lake, a country's annual gross domestic product and the quantities of exports and imports, investment annual revenues annual unemployment rates are examples of monthly rainfall time series in a city. Examples can be expanded with applications that can be given from economics, business, engineering, and basic sciences. Time series are used extensively in the study of geophysics, meteorology, and economic data [2].

Time series are examined under two basic headings as stationary and non-stationary series according to their deviations from the average. If the mean and variance of the studied time series show a symmetric change, or if the series is free of periodic fluctuations, such series are stationary time series. Stationarity is important in time series. Many statistical inferences are based on the assumption of the stationarity of the series. If the series is not stationary, analyzes are made after making it stationary using some techniques (such as difference taking). Many economic data are not stationary. It is also necessary to examine non-stationary time series in two parts. In practice, there are two main reasons for destabilization. These reasons are that the average of the series depends on time (the series is a deterministic trend) and that the series ' autocorrelations depend on time. Destroying the deterministic trend is simple. In the other case, the trend in the series is stochastic and some techniques such as transforming and taking a difference are used to eliminate the trend [2].

The value of a variable of time series can be mathematically generated in the form of a series X_t. In the series, the change Δ symbol between the two years

$$\Delta X = X_{t+1} - X_t = X_t - X_{t-1}$$

is written as to show the difference. At the moment when the process or change of these two-year differences over a given period is addressed, a process related to the time series is performed. There is a dynamic process and a change is expressed here [3].

In this study, it is aimed to express the ARIMA models, which are made stationary by applying the difference operation, clearly with Back-Shift operators using parameter coefficients. It is believed that with the realization of this goal, the models that the researchers will obtain in the later stages will be shown correctly.

II. METHOD

The time series consists of autoregressive (AR), moving average (MA) and autoregressive moving average (ARMA) models.
A pth-order autoregressive model AR(p) model is denoted as [4].

$$X_t = c + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + e_t$$

Closed form of AR model

$$(1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p) X_t = e_t$$

in the form [5].
AR(p) model uses a linear combination of past values of the target to make forecasts.
A qth-order moving average process, expressed MA(q), is characterized by [6].

$$X_t = -\theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q} + e_t$$

Closed form of MA model

$$X_t = (1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_p B^q) e_t$$

in the form [5].
ARMA(p,q) model composed of a pth-order autoregressive and qth-order moving average process and it is characterized by [7].

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}$$

ARIMA models that are not stationary but become stationary after the difference is written with Back shift operators as follows.

MA(q) (Moving average model)
ARIMA(0,1,1) Model

$$(1 - B) X_t = (1 - \theta B) e_t$$
$$X_t - X_{t-1} = -\theta e_{t-1} + e_t$$
$$X_t = X_{t-1} - \theta e_{t-1} + e_t$$

ARIMA(0,1,2) Model

$$(1 - B) X_t = (1 - \theta_1 B - \theta_2 B^2) e_t$$
$$X_t - X_{t-1} = -\theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$
$$X_t = X_{t-1} - \theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$

ARIMA(0,1,3) Model

$$(1 - B) X_t = (1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3) e_t$$
$$X_t - X_{t-1} = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3}$$
$$X_t = X_{t-1} - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$

ARIMA(0,2,1) Model

$$(1 - B)^2 X_t = (1 - \theta B) e_t$$
$$X_t (1 - 2B + B^2) = e_t - \theta B e_t$$
$$X_t - 2X_{t-1} + X_{t-2} = -\theta e_{t-1} + e_t$$
$$X_t = 2X_{t-1} - X_{t-2} - \theta e_{t-1} + e_t$$

ARIMA(0,2,2) Model

$$(1 - B)^2 X_t = (1 - \theta_1 B - \theta_2 B^2) e_t$$
$$(1 - 2B + B^2) X_t = -\theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$
$$X_t - 2X_{t-1} + X_{t-2} = -\theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$
$$X_t = 2X_{t-1} - X_{t-2} - \theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$

ARIMA(0,2,3) Model

$$(1 - B)^2 X_t = (1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3) e_t$$
$$(1 - 2B + B^2) X_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3}$$
$$X_t - 2X_{t-1} + X_{t-2} = -\theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$
$$X_t = 2X_{t-1} - X_{t-2} - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$

AR (p) Autoregressive models

ARIMA(1,1,0) Model

$$(1-B)(1-\phi B)X_t = e_t$$
$$(1-\phi B - B + \phi B^2)X_t = e_t$$
$$X_t(1 - B(1+\phi) + B^2\phi) = e_t$$
$$X_t - (1+\phi)X_{t-1} + \phi X_{t-2} = e_t$$
$$X_t = (1+\phi)X_{t-1} - \phi X_{t-2} + e_t$$

ARIMA(2,1,0) Model

$$(1-B)(1-\phi_1 B - \phi_2 B^2)X_t = e_t$$
$$(1-\phi_1 B - \phi_2 B^2 - B + \phi_1 B^2 + \phi_2 B^3)X_t = e_t$$
$$X_t(1 - B(1+\phi_1) + B^2(\phi_1 - \phi_2) + B^3\phi_2) = e_t$$
$$X_t - B(1+\phi_1)X_t + B^2(\phi_1 - \phi_2)X_t + B^3\phi_2 X_t = e_t$$
$$X_t = (1+\phi_1)X_{t-1} + (-\phi_1 + \phi_2)X_{t-2} - \phi_2 X_{t-3} + e_t$$

ARIMA(3,1,0) Model

$$(1-B)(1-\phi_1 B - \phi_2 B^2 - \phi_3 B^3)X_t = e_t$$
$$(1-\phi_1 B - \phi_2 B^2 - \phi_3 B^3 - B + \phi_1 B^2 + \phi_2 B^3 + \phi_3 B^4)X_t = e_t$$
$$X_t(1 - B - \phi_1 B - \phi_2 B^2 + \phi_1 B^2 - \phi_3 B^3 + \phi_2 B^3 + \phi_3 B^4) = e_t$$
$$X_t[1 - B(1+\phi_1) + B^2(\phi_1 - \phi_2) + B^3(\phi_2 - \phi_3) + B^4\phi_3] = e_t$$
$$X_t - B(1+\phi_1)X_t + B^2(\phi_1 - \phi_2)X_t + B^3(\phi_2 - \phi_3)X_t + B^4\phi_3 X_t = e_t$$
$$X_t = B(1+\phi_1)X_t - B^2(\phi_1 - \phi_2)X_t - B^3(\phi_2 - \phi_3)X_t - B^4\phi_3 X_t + e_t$$
$$X_t = (1+\phi_1)X_{t-1} + (-\phi_1 + \phi_2)X_{t-2} + (-\phi_2 + \phi_3)X_{t-3} - \phi_3 X_{t-4} + e_t$$

ARIMA(1,2,0) Model

$$(1-B)^2(1-\phi B)X_t = e_t$$
$$(1-2B+B^2)(1-\phi B)X_t = e_t$$
$$(1-\phi B - 2B + 2\phi B^2 - \phi B^3)X_t = e_t$$
$$X_t(1 - B(\phi+2) + 2\phi B^2 - \phi B^3) = e_t$$
$$X_t - B(\phi+2)X_t + 2\phi B^2 X_t - \phi B^3 X_t = e_t$$
$$X_t = (\phi+2)X_{t-1} - 2\phi X_{t-2} + \phi X_{t-3} + e_t$$

ARIMA(2,2,0) Model

$$(1-B)^2(1-\phi B - \phi B^2)X_t = e_t$$
$$(1-2B+B^2)(1-\phi_1 B - \phi_2 B^2)X_t = e_t$$
$$X_t(1-\phi_1 B - \phi_2 B^2 - 2B + 2\phi_1 B^2 + 2\phi_2 B^3 + B^2 - \phi_1 B^3 - \phi_2 B^4 = e_t$$
$$X_t(1 + B(-\phi_1 - 2) + B^2(2\phi_1 - \phi_2 + 1) + B^3(-\phi_1 + 2\phi_2) + B^4(-\phi_2) = e_t$$
$$X_t + B(-\phi_1 - 2)X_t + B^2(2\phi_1 - \phi_2 + 1)X_t + B^3(-\phi_1 + 2\phi_2)X_t + B^4(-\phi_2)X_t = e_t$$
$$X_t = (\phi_1 + 2)X_{t-1} + (-2\phi_1 + \phi_2 - 1)X_{t-2} + (-2\phi_2 + \phi_1)X_{t-3} + \phi_2 X_{t-4} + e_t$$

ARIMA(3,2,0) Model

$$(1-B)^2(1-\phi_1 B - \phi_2 B^2 - \phi_3 B^3)X_t = e_t$$
$$(1-2B+B^2)(1-\phi_1 B - \phi_2 B^2 - \phi_3 B^3)X_t = e_t$$

$$X_t(1-\phi_1 B - \phi_2 B^2 - \phi_3 B^3 - 2B + 2\phi_1 B^2 + 2\phi_2 B^3 + 2\phi_3 B^4 + B^2 - \phi_1 B^3 - \phi_2 B^4$$
$$-\phi_3 B^5 = e_t$$

$$X_t(1 - 2B - \phi_1 B - \phi_2 B^2 + 2\phi_1 B^2 + B^2 - \phi_3 B^3 + 2\phi_2 B^3 - \phi_1 B^3 + 2\phi_3 B^4 - \phi_2 B^4$$
$$-\phi_3 B^5 = e_t$$

$$[1 - B(\phi_1 + 2) + B^2(2\phi_1 - \phi_2 + 1) + B^3(-\phi_1 + 2\phi_2 - \phi_3) + B^4(2\phi_3 - \phi_2) - B^5\phi_3]X_t$$
$$= e_t$$

$$X_t - B(\phi_1 + 2)X_t + B^2(2\phi_1 - \phi_2 + 1)X_t + B^3(-\phi_1 + 2\phi_2 - \phi_3)X_t + B^4(2\phi_3 - \phi_2)X_t$$
$$-B^5\phi_3 X_t = e_t$$

$$X_t - (\phi_1 + 2)X_{t-1} + (2\phi_1 - \phi_2 + 1)X_{t-2} + (-\phi_1 + 2\phi_2 - \phi_3)X_{t-3} + (2\phi_3 - \phi_2)X_{t-4}$$
$$-\phi_3 X_{t-5} = e_t$$

$X_t = (\phi_1 + 2)X_{t-1} + (-2\phi_1 + \phi_2 - 1)X_{t-2} + (\phi_1 - 2\phi_2 + \phi_3)X_{t-3} + (-2\phi_3 + \phi_2)X_{t-4}$
$+\phi_3 X_{t-5} + e_t$
it can be written in the form.

ARMA(p,d,q) Autoregressive Moving Average Model
ARIMA(1,1,1) Model

$$(1 - B)(1 - \phi B)X_t = (1 - \theta B)e_t$$
$$(1 - \phi B - B + \phi B^2)X_t = e_t - \theta B e_t$$
$$(1 - B(1 + \phi) + \phi B^2)X_t = e_t - \theta e_{t-1}$$
$$X_t - (1 + \phi)X_{t-1} + \phi X_{t-2} = e_t - \theta e_{t-1}$$
$$X_t = (1 + \phi)X_{t-1} - \phi X_{t-2} - \theta e_{t-1} + e_t$$

ARIMA(1,1,2) Model

$$(1 - B)(1 - \phi B)X_t = (1 - \theta_1 B - \theta_2 B^2)e_t$$
$$X_t(1 - \phi B - B + \phi B^2) = -\theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$
$$X_t(1 - B(1 + \phi) + \phi B^2) = -\theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$
$$X_t = (1 + \phi)X_{t-1} - \phi X_{t-2} - \theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$

ARIMA(1,1,3) Model

$$(1 - B)(1 - \phi B)X_t = (1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3)e_t$$
$$X_t(1 - \phi B - B + \phi B^2) = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3}$$
$$X_t(1 - B(1 + \phi) + \phi B^2) = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3}$$
$$X_t - (1 + \phi)X_{t-1} + \phi X_{t-2} = -\theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$
$$X_t = (1 + \phi)X_{t-1} - \phi X_{t-2} - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$

ARIMA(1,2,1) Model

$$(1 - B)^2(1 - \phi B)X_t = (1 - \theta B)e_t$$
$$(1 - 2B + B^2)(1 - \phi B)X_t = (1 - \theta B)e_t$$
$$X_t(1 - \phi B - 2B + 2\phi B^2 - \phi B^3) = e_t - \theta B e_t$$
$$X_t(1 - B(\phi + 2) + 2\phi B^2 + B^2 - \phi B^3) = e_t - \theta e_{t-1}$$
$$X_t - B(\phi + 2)X_t + B^2(2\phi + 1)X_t - B^3\phi X_t = e_t - \theta e_{t-1}$$
$$X_t = (\phi + 2)X_{t-1} + (-2\phi - 1)X_{t-2} + \phi X_{t-3} + e_t - \theta e_{t-1}$$

or

$$X_t = (\phi + 2)X_{t-1} - (2\phi + 1)X_{t-2} + \phi X_{t-3} - \theta e_{t-1} + e_t$$

it be obtained.

ARIMA(1,2,2) Model

$$(1 - B)^2(1 - \phi B)X_t = (1 - \theta_1 B - \theta_2 B^2)e_t$$
$$(1 - 2B + B^2)(1 - \phi B)X_t = -\theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$
$$(1 - \phi B - 2B + 2\phi B^2 - \phi B^3)X_t = -\theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$
$$(1 - B(\phi + 2) + 2\phi B^2 + B^2 - \phi B^3)X_t = -\theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$
$$X_t - B(\phi + 2)X_t + B^2(2\phi + 1)X_t - B^3\phi X_t = -\theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$
$$X_t = (\phi + 2)X_{t-1} - (2\phi + 1)X_{t-2} + \phi X_{t-3} - \theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$

ARIMA(1,2,3) Model

$$(1 - B)^2(1 - \phi B)X_t = (1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3)e_t$$
$$(1 - 2B + B^2)(1 - \phi B)X_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3}$$
$$(1 - \phi B - 2B + 2\phi B^2 - \phi B^3)X_t = -\theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$
$$(1 - B(\phi + 2) + 2\phi B^2 + B^2 - \phi B^3)X_t = -\theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$
$$X_t - B(\phi + 2)X_t + B^2(2\phi + 1)X_t - B^3\phi X_t = -\theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$
$$X_t = (\phi + 2)X_{t-1} + (1 - 2\phi)X_{t-2} + \phi X_{t-3} - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$

ARIMA(2,1,1) Model

$$(1 - B)(1 - \phi_1 B - \phi_2 B^2)X_t = (1 - \theta B)e_t$$
$$(1 - \phi_1 B - \phi_2 B^2 - B + \phi_1 B^2 + \phi_2 B^3)X_t = e_t - \theta e_{t-1}$$
$$(1 - B(1 + \phi_1) + B^2(\phi_1 - \phi_2) + B^3\phi_2)X_t = -\theta e_{t-1} + e_t$$
$$X_t - (1 + \phi_1)X_{t-1} + (\phi_1 - \phi_2)X_{t-2} + \phi_2 X_{t-3} = -\theta e_{t-1} + e_t$$
$$X_t = (1 + \phi_1)X_{t-1} + (-\phi_1 + \phi_2)X_{t-2} - \phi_2 X_{t-3} - \theta e_{t-1} + e_t$$

ARIMA(2,1,2) Model

$$(1-B)(1-\phi_1 B-\phi_2 B^2)X_t=(1-\theta_1 B-\theta_2 B^2)e_t$$
$$X_t(1-\phi_1 B-\phi_2 B^2-B+\phi_1 B^2+\phi_2 B^3)=-\theta_1 e_{t-1}-\theta_2 e_{t-2}+e_t$$
$$X_t(1-B(1+\phi_1)+B^2(\phi_1-\phi_2)+B^3\phi_2)=-\theta_1 e_{t-1}-\theta_2 e_{t-2}+e_t$$
$$X_t-(1+\phi_1)X_{t-1}+(\phi_1-\phi_2)X_{t-2}+\phi_2 X_{t-3}=-\theta_1 e_{t-1}-\theta_2 e_{t-2}+e_t$$
$$X_t=(1+\phi_1)X_{t-1}+(-\phi_1+\phi_2)X_{t-2}-\phi_2 X_{t-3}-\theta_1 e_{t-1}-\theta_2 e_{t-2}+e_t$$

ARIMA(2,1,3) Model

$$(1-B)(1-\phi_1 B-\phi_2 B^2)X_t=(1-\theta_1 B-\theta_2 B^2-\theta_3 B^3)e_t$$
$$X_t(1-\phi_1 B-\phi_2 B^2-B+\phi_1 B^2+\phi_2 B^3)=-\theta_1 e_{t-1}-\theta_2 e_{t-2}-\theta_3 e_{t-3}+e_t$$
$$X_t(1-B(1+\phi_1)+B^2(\phi_1-\phi_2)+B^3\phi_2)=-\theta_1 e_{t-1}-\theta_2 e_{t-2}-\theta_3 e_{t-3}+e_t$$
$$X_t-(1+\phi_1)X_{t-1}+(\phi_1-\phi_2)X_{t-2}+\phi_2 X_{t-3}=-\theta_1 e_{t-1}-\theta_2 e_{t-2}-\theta_3 e_{t-3}+e_t$$
$$X_t=(1+\phi_1)X_{t-1}+(-\phi_1+\phi_2)X_{t-2}-\phi_2 X_{t-3}-\theta_1 e_{t-1}-\theta_2 e_{t-2}-\theta_3 e_{t-3}+e_t$$

ARIMA(2,2,1) Model

$$(1-B)^2(1-\phi_1 B-\phi_2 B^2)X_t=(1-\theta B)e_t$$
$$X_t(1-2B+B^2)(1-\phi_1 B-\phi_2 B^2)=e_t-\theta e_{t-1}$$
$$X_t(1-\phi_1 B-\phi_2 B^2-2B+2\phi_1 B^2+2\phi_2 B^3+B^2-\phi_1 B^3-\phi_2 B^4=-\theta e_{t-1}+e_t$$
$$X_t(1+B(-\phi_1-2)+B^2(2\phi_1-\phi_2+1)+B^3(-\phi_1+2\phi_2)+B^4(-\phi_2)=-\theta e_{t-1}+e_t$$
$$X_t+B(-\phi_1-2)X_t+B^2(2\phi_1-\phi_2+1)X_t+B^3(-\phi_1+2\phi_2)X_t+B^4(-\phi_2)X_t=-\theta e_{t-1}+e_t$$
$$X_t=(\phi_1+2)X_{t-1}+(-2\phi_1+\phi_2-1)X_{t-2}+(-2\phi_2+\phi_1)X_{t-3}+\phi_2 X_{t-4}-\theta e_{t-1}+e_t$$

ARIMA(2,2,2) Model

$$(1-B)^2(1-\phi_1 B-\phi_2 B^2)X_t=(1-\theta_1 B-\theta_2 B^2)e_t$$
$$X_t(1-2B+B^2)(1-\phi_1 B-\phi_2 B^2)=-\theta_1 e_{t-1}-\theta_2 e_{t-2}+e_t$$
$$X_t(1-\phi_1 B-\phi_2 B^2-2B+2\phi_1 B^2+2\phi_2 B^3+B^2-\phi_1 B^3-\phi_2 B^4=-\theta_1 e_{t-1}-\theta_2 e_{t-2}+e_t$$
$$X_t(1+B(-\phi_1-2)+B^2(2\phi_1-\phi_2+1)+B^3(-\phi_1+2\phi_2)+B^4(-\phi_2)=-\theta_1 e_{t-1}-\theta_2 e_{t-2}+e_t$$
$$X_t+B(-\phi_1-2)X_t+B^2(2\phi_1-\phi_2+1)X_t+B^3(-\phi_1+2\phi_2)X_t+B^4(-\phi_2)X_t=-\theta_1 e_{t-1}-\theta_2 e_{t-2}+e_t$$

$$X_t=(\phi_1+2)X_{t-1}+(-2\phi_1+\phi_2-1)X_{t-2}+(-2\phi_2+\phi_1)X_{t-3}$$
$$+\phi_2 X_{t-4}-\theta_1 e_{t-1}-\theta_2 e_{t-2}+e_t$$

ARIMA(2,2,3) Model

$$(1-B)^2(1-\phi B-\phi B^2)X_t=(1-\theta_1 B-\theta_2 B^2-\theta_3 B^3)e_t$$
$$(1-2B+B^2)(1-\phi_1 B-\phi_2 B^2)X_t=(1-\theta_1 B-\theta_2 B^2-\theta_3 B^3)e_t$$
$$X_t(1-\phi_1 B-\phi_2 B^2-2B+2\phi_1 B^2+2\phi_2 B^3+B^2-\phi_1 B^3-\phi_2 B^4$$
$$=-\theta_1 e_{t-1}-\theta_2 e_{t-2}-\theta_3 e_{t-3}+e_t$$

$$X_t(1+B(-\phi_1-2)+B^2(2\phi_1-\phi_2+1)+B^3(-\phi_1+2\phi_2)+B^4(-\phi_2)$$
$$=-\theta_1 e_{t-1}-\theta_2 e_{t-2}-\theta_3 e_{t-3}+e_t$$

$$X_t+B(-\phi_1-2)X_t+B^2(2\phi_1-\phi_2+1)X_t+B^3(-\phi_1+2\phi_2)X_t+B^4(-\phi_2)X_t$$
$$=-\theta_1 e_{t-1}-\theta_2 e_{t-2}-\theta_3 e_{t-3}+e_t$$

$$X_t=(\phi_1+2)X_{t-1}+(-2\phi_1+\phi_2-1)X_{t-2}+(-2\phi_2+\phi_1)X_{t-3}$$
$$+\phi_2 X_{t-4}-\theta_1 e_{t-1}-\theta_2 e_{t-2}-\theta_3 e_{t-3}+e_t$$

ARIMA(2,1,3) Model

$$(1-B)(1-\phi_1 B-\phi_2 B^2)X_t=(1-\theta_1 B-\theta_2 B^2-\theta_3 B^3)e_t$$

$$X_t(1-\phi_1 B-\phi_2 B^2-B+\phi_1 B^2+\phi_2 B^3)=-\theta_1 e_{t-1}-\theta_2 e_{t-2}-\theta_3 e_{t-3}+e_t$$
$$X_t(1-B(1+\phi_1)+B^2(\phi_1-\phi_2)+B^3\phi_2)=-\theta_1 e_{t-1}-\theta_2 e_{t-2}-\theta_3 e_{t-3}+e_t$$
$$X_t-(1+\phi_1)X_{t-1}+(\phi_1-\phi_2)X_{t-2}+\phi_2 X_{t-3}=-\theta_1 e_{t-1}-\theta_2 e_{t-2}-\theta_3 e_{t-3}+e_t$$
$$X_t=(1+\phi_1)X_{t-1}+(-\phi_1+\phi_2)X_{t-2}-\phi_2 X_{t-3}-\theta_1 e_{t-1}-\theta_2 e_{t-2}-\theta_3 e_{t-3}+e_t$$

ARIMA(3,1,3) Model

$$(1-B)(1-\phi_1 B-\phi_2 B^2-\phi_3 B^3)X_t=(1-\theta_1 B-\theta_2 B^2-\theta_3 B^3)e_t$$

$$(1-\phi_1 B-\phi_2 B^2-\phi_3 B^3-B+\phi_1 B^2+\phi_2 B^3+\phi_3 B^4)X_t$$

$$= -\theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$

$$X_t(1 - B - \phi_1 B - \phi_2 B^2 + \phi_1 B^2 - \phi_3 B^3 + \phi_2 B^3 + \phi_3 B^4)$$
$$= -\theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$

$$X_t[1 - B(1 + \phi_1) + B^2(\phi_1 - \phi_2) + B^3(\phi_2 - \phi_3) + B^4\phi_3]$$

$$= -\theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$

$$X_t - B(1 + \phi_1)X_t + B^2(\phi_1 - \phi_2)X_t + B^3(\phi_2 - \phi_3)X_t + B^4\phi_3 X_t$$
$$= -\theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$

$$X_t = B(1 + \phi_1)X_t - B^2(\phi_1 - \phi_2)X_t - B^3(\phi_2 - \phi_3)X_t - B^4\phi_3 X_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}$$
$$-\theta_3 e_{t-3} + e_t$$

$$X_t = (1 + \phi_1)X_{t-1} + (-\phi_1 + \phi_2)X_{t-2} + (-\phi_2 + \phi_3)X_{t-3} - \phi_3 X_{t-4} - \theta_1 e_{t-1}$$
$$-\theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$

ARIMA(3,1,2) Model

$$(1 - B)(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3)X_t = (1 - \theta_1 B - \theta_2 B^2)e_t$$

$$(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3 - B + \phi_1 B^2 + \phi_2 B^3 + \phi_3 B^4)X_t = -\theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$

$$X_t(1 - B - \phi_1 B - \phi_2 B^2 + \phi_1 B^2 - \phi_3 B^3 + \phi_2 B^3 + \phi_3 B^4) = -\theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$

$$X_t[1 - B(1 + \phi_1) + B^2(\phi_1 - \phi_2) + B^3(\phi_2 - \phi_3) + B^4\phi_3] = -\theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$

$$X_t - B(1 + \phi_1)X_t + B^2(\phi_1 - \phi_2)X_t + B^3(\phi_2 - \phi_3)X_t + B^4\phi_3 X_t = -\theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$

$$X_t = B(1 + \phi_1)X_t - B^2(\phi_1 - \phi_2)X_t - B^3(\phi_2 - \phi_3)X_t - B^4\phi_3 X_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$

$$X_t = (1 + \phi_1)X_{t-1} + (-\phi_1 + \phi_2)X_{t-2} + (-\phi_2 + \phi_3)X_{t-3} - \phi_3 X_{t-4} - \theta_1 e_{t-1} \ -\theta_2 e_{t-2} {}_+ e_t$$

ARIMA(3,1,1) Model

$$(1 - B)(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3)X_t = (1 - \theta B)e_t$$

$$(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3 - B + \phi_1 B^2 + \phi_2 B^3 + \phi_3 B^4)X_t = -\theta e_{t-1} + e_t$$

$$X_t(1 - B - \phi_1 B - \phi_2 B^2 + \phi_1 B^2 - \phi_3 B^3 + \phi_2 B^3 + \phi_3 B^4) = -\theta e_{t-1} + e_t$$

$$X_t[1 - B(1 + \phi_1) + B^2(\phi_1 - \phi_2) + B^3(\phi_2 - \phi_3) + B^4\phi_3] = -\theta e_{t-1} + e_t$$

$$X_t - B(1 + \phi_1)X_t + B^2(\phi_1 - \phi_2)X_t + B^3(\phi_2 - \phi_3)X_t + B^4\phi_3 X_t = -\theta e_{t-1} + e_t$$

$$X_t = B(1 + \phi_1)X_t - B^2(\phi_1 - \phi_2)X_t - B^3(\phi_2 - \phi_3)X_t - B^4\phi_3 X_t - \theta e_{t-1} + e_t$$

$$X_t = (1 + \phi_1)X_{t-1} + (-\phi_1 + \phi_2)X_{t-2} + (-\phi_2 + \phi_3)X_{t-3} - \phi_3 X_{t-4} - \theta e_{t-1} + e_t$$

ARIMA(3,2,3) Model

$$(1 - B)^2(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3)X_t = (1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3)e_t$$
$$(1 - 2B + B^2)(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3)X_t = (1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3)e_t$$

$$X_t(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3 - 2B + 2\phi_1 B^2 + 2\phi_2 B^3 + 2\phi_3 B^4 + B^2 - \phi_1 B^3 - \phi_2 B^4$$
$$-\phi_3 B^5 = -\theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$

$$X_t(1 - 2B - \phi_1 B - \phi_2 B^2 + 2\phi_1 B^2 + B^2 - \phi_3 B^3 + 2\phi_2 B^3 - \phi_1 B^3 + 2\phi_3 B^4 - \phi_2 B^4$$
$$-\phi_3 B^5 = -\theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$

$$[1 - B(\phi_1 + 2) + B^2(2\phi_1 - \phi_2 + 1) + B^3(-\phi_1 + 2\phi_2 - \phi_3) + B^4(2\phi_3 - \phi_2) - B^5\phi_3]X_t = -\theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$

$$X_t - B(\phi_1 + 2)X_t + B^2(2\phi_1 - \phi_2 + 1)X_t + B^3(-\phi_1 + 2\phi_2 - \phi_3)X_t + B^4(2\phi_3 - \phi_2)X_t - B^5\phi_3 X_t = -\theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$

$$X_t - (\phi_1 + 2)X_{t-1} + (2\phi_1 - \phi_2 + 1)X_{t-2} + (-\phi_1 + 2\phi_2 - \phi_3)X_{t-3} + (2\phi_3 - \phi_2)X_{t-4} - \phi_3 X_{t-5} = -\theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$

$$X_t = (\phi_1 + 2)X_{t-1} + (-2\phi_1 + \phi_2 - 1)X_{t-2} + (\phi_1 - 2\phi_2 + \phi_3)X_{t-3} + (-2\phi_3 + \phi_2)X_{t-4} + \phi_3 X_{t-5} - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} + e_t$$

it can be written in the form.

ARIMA(3,2,2) Model

$$(1 - B)^2(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3)X_t = (1 - \theta_1 B - \theta_2 B^2)e_t$$

$$(1 - 2B + B^2)(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3)X_t = (1 - \theta_1 B - \theta_2 B^2)e_t$$

$$X_t(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3 - 2B + 2\phi_1 B^2 + 2\phi_2 B^3 + 2\phi_3 B^4 + B^2 - \phi_1 B^3 - \phi_2 B^4 - \phi_3 B^5 = -\theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$

$$X_t(1 - 2B - \phi_1 B - \phi_2 B^2 + 2\phi_1 B^2 + B^2 - \phi_3 B^3 + 2\phi_2 B^3 - \phi_1 B^3 + 2\phi_3 B^4 - \phi_2 B^4 - \phi_3 B^5 = -\theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$

$$[1 - B(\phi_1 + 2) + B^2(2\phi_1 - \phi_2 + 1) + B^3(-\phi_1 + 2\phi_2 - \phi_3) + B^4(2\phi_3 - \phi_2) - B^5\phi_3]X_t = -\theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$

$$X_t - B(\phi_1 + 2)X_t + B^2(2\phi_1 - \phi_2 + 1)X_t + B^3(-\phi_1 + 2\phi_2 - \phi_3)X_t + B^4(2\phi_3 - \phi_2)X_t - B^5\phi_3 X_t = -\theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$

$$X_t - (\phi_1 + 2)X_{t-1} + (2\phi_1 - \phi_2 + 1)X_{t-2} + (-\phi_1 + 2\phi_2 - \phi_3)X_{t-3} + (2\phi_3 - \phi_2)X_{t-4} - \phi_3 X_{t-5} = -\theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$

$$X_t = (\phi_1 + 2)X_{t-1} + (-2\phi_1 + \phi_2 - 1)X_{t-2} + (\phi_1 - 2\phi_2 + \phi_3)X_{t-3} + (-2\phi_3 + \phi_2)X_{t-4} + \phi_3 X_{t-5} - \theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t$$

it can be written in the form.

ARIMA(3,2,1) Model

$$(1 - B)^2(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3)X_t = (1 - \theta B)e_t$$

$$(1 - 2B + B^2)(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3)X_t = (1 - \theta B)e_t$$

$$X_t(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3 - 2B + 2\phi_1 B^2 + 2\phi_2 B^3 + 2\phi_3 B^4 + B^2 - \phi_1 B^3 - \phi_2 B^4 - \phi_3 B^5 = -\theta e_{t-1} + e_t$$

$$X_t(1 - 2B - \phi_1 B - \phi_2 B^2 + 2\phi_1 B^2 + B^2 - \phi_3 B^3 + 2\phi_2 B^3 - \phi_1 B^3 + 2\phi_3 B^4 - \phi_2 B^4 - \phi_3 B^5 = -\theta e_{t-1} + e_t$$

$$[1 - B(\phi_1 + 2) + B^2(2\phi_1 - \phi_2 + 1) + B^3(-\phi_1 + 2\phi_2 - \phi_3) + B^4(2\phi_3 - \phi_2) - B^5\phi_3]X_t = -\theta e_{t-1} + e_t$$

$$X_t - B(\phi_1 + 2)X_t + B^2(2\phi_1 - \phi_2 + 1)X_t + B^3(-\phi_1 + 2\phi_2 - \phi_3)X_t + B^4(2\phi_3 - \phi_2)X_t - B^5\phi_3 X_t = -\theta e_{t-1} + e_t$$

$$X_t - (\phi_1 + 2)X_{t-1} + (2\phi_1 - \phi_2 + 1)X_{t-2} + (-\phi_1 + 2\phi_2 - \phi_3)X_{t-3} + (2\phi_3 - \phi_2)X_{t-4} - \phi_3 X_{t-5} = -\theta e_{t-1} + e_t$$

$$X_t = (\phi_1 + 2)X_{t-1} + (-2\phi_1 + \phi_2 - 1)X_{t-2} + (\phi_1 - 2\phi_2 + \phi_3)X_{t-3} + (-2\phi_3 + \phi_2)X_{t-4} + \phi_3 X_{t-5} - \theta e_{t-1} + e_t$$

it can be written in the form.

III. CONCLUSION

In the study, explicit spelling of time series was obtained using Back-Shift operators. The greatest ease of this study for researchers is that the ARIMA models they obtained in their study express them in the form of equations. In practice, in most studies, stationary time series are not obtained directly, but ARIMA models that become stationary after the difference operation is taken. Here, the AR and MA values of ARIMA models for the first and second difference series are clearly written with Back-Shift operators in all combinations in the range 0-3. In short, the resulting models are given collectively in the form of equations.

REFERENCES

[1]. Eğrioğlu, E., Baş, E. 2020. Zaman Serileri Analizi ve Öngörü Yöntemleri. Nobel Akademik Yayıncılık, Ankara.
[2]. Akdi, Y. 2010. Zaman Serileri Analizi (Birim Kökler ve Kointegrasyon). Gazi Kitabevi, Ankara.
[3]. Kutlar, A. 2019. Stata ile Uygulamalı Zaman Serileri. Nobel Akademik Yayıncılık Eğitim Danışmanlık Tic. Ltd. Şti., Ankara.
[4]. Cooray, T. M. J. A. 2008. Applied Time Series. Analysis and Forecasting. Narosa Publishing House Pvt. Ltd., New Delhi.
[5]. Kadılar, C., Çekim, H. Ö. 2020. SPSS ve R Uygulamalı Zaman Serileri Analizine Giriş. Seçkin Yayıncılık San. ve Tic. A. Ş., Ankara.
[6]. Cryer, J. D. 1986. Time Series Analysis. PWS Publishing, USA.
[7]. Hamilton, J. D. 1994. Time Series Analysis. Princeton University Press Princeton, New Jersey.