Quest Journals
Journal of Research in Applied Mathematics
Volume 7 ~ Issue 7 (2021) pp: 01-06
ISSN(Online) : 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

Non-interacting Dark Energy Model with Hybrid Expansion Law in Brans- Dicke Theory of Gravitation

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Abstract *In this analysis, the evolution of the two fluid dark energy parameter in the spatially homogeneous and isotropic Friedmann-Robertson-Walker (FRW) model filled with barotropic fluid and dark energy in Brans-Dicke theory of gravity is analyzed. A solution of the field equations of non-interacting fluidis presented using the hybrid law of expansion. The physical and kinematical aspects of the results obtained are also discussed.*

KEYWORDS: *Two fluid scenario, Dark energy. FRW model, Brans-Dicke theory.*

Received 26 June, 2021; Revised: 06 July, 2021; Accepted 08 July, 2021 © The author(s) 2021. Published with open access at www.questjournals.org

I. INTRODUCTION

The recent several astronomical observations [1-5]confirmed that the universe is undergoing a phase of accelerated expansion. The mysterious dominant component which has a large negative pressure is responsible for this cosmic acceleration known as dark energy. Several candidates like cosmological constant, some dynamical scalar fields such as quintessence, Phantom, quintom and K-essenceare proposed for dark energy [6-10]. However, quintessence models involving scalar fields give rise to time dependent equation of state parameter which is not necessarily constant. In recent years, there has been a lot of interest in modified theories of gravitation to address the problem of late time acceleration of the universe. Among the various modifications of general relativity Brans-Dicke[11], theory of gravity are noteworthy. The several authors (Sahani and Starobinsky[12];Padmanabhan[13]; Caldwell [14]; Nojiri and Odintsov[15] have studied dark energy models with different context of use.
The importance of two fluid in the discussion of modern cosmology is clearly out lined. Several authors have investigated the two fluid scenario for dark energy models in FRW universe. In particular, Liang et al. [16] have studied the cosmological evolution of a two fluid dilaton model of dark energy along with two fluid scenario for dark energy models was studied by Setare et al. [17], Chimento et al. [18] and Chimento and Pavon[19] while Saha et al. [20] studied the evolution of the dark energy parameter in FRW universe model filled with two fluid.Inspired by the above investigations and discussion, in this paper, we study the evolution of dark energy parameter in FRW universe filled with two fluids in Brans-Dicke scalar- tensor theory of gravitation.

II. METRIC AND FIELD EQUATIONS
III.

We consider the homogeneous and isotropic FRW metric in the form $ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$ (1)

where $a(t)$ is the scale factor and $k=-1,0,+1,$ respectively, for open flat and closed models of the universe. The field equations given by Brans and Dicke for the combined scalar and tensor fields are

$$R_{ij}-\frac{1}{2}g_{ij}R=-8\pi\phi^{-1}T_{ij}-\omega\phi^{-2}\left(\phi_{,i}\phi_{,j}-\frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right)-\phi^{-1}\left(\phi_{i;j}-g_{ij}\phi^{,k}_{;k}\right) \tag{2}$$

$$\phi^{,k}_{;k}=8\pi(3+2\omega)^{-1}T \tag{3}$$

where ϕ is the scalar field, ω is the dimensionless coupling constant (this should be constrained as $\geq 40{,}000$ for its consistency with solar system bounds [21, 22], T_{ij} is the two fluid energy momentum tensor consisting of barotropic fluid and dark energy, R_{ij} and R have usual meaning.

Also,

$$T^{ij}_{;j}=0 \tag{4}$$

Be the consequence of the field equations (2) and (3). Here a semicolon indicates covariant derivative and comma denotes ordinary derivative with respect to x^k.

In a commoving coordinate system Brans-Dicke field equations (2)–(4) for the metric (1), in the two fluid scenario,lead to

$$2\frac{\ddot{a}}{a}+\left(\frac{\dot{a}}{a}\right)^2+\frac{k}{a^2}+\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+\frac{\ddot{\phi}}{\phi}-2\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}=-8\pi\phi^{-1}\rho_{tot} \tag{5}$$

$$3\left[\left(\frac{\dot{a}}{a}\right)^2+\frac{k}{a^2}\right]-\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}=8\pi\phi^{-1}\rho_{tot} \tag{6}$$

$$\frac{\ddot{\phi}}{\phi}+3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}=8\pi\phi^{-1}\left(\rho_{tot}-3\rho_{tot}\right) \tag{7}$$

$$\dot{\rho}_{tot}+(\rho_{tot}+p_{tot})3\frac{\dot{a}}{a}=0 \tag{8}$$

where $\rho_{tot}=\rho_m+\rho_D$ and $p_{tot}=p_m+p_D$. Here ρ_m and p_m are energy density and pressure of barotropic fluid and ρ_D and p_D energy density and pressure of dark fluid respectively.

The Equation of state parameters of barotropic fluid and dark fluid respectively given by

$$\omega_m=\frac{p_m}{\rho_m} \tag{9}$$

and

$$\omega_D=\frac{p_D}{\rho_D} \tag{10}$$

In the following section, we consider the two cases: non-interacting two fluid model and interacting fluid model. We determine, in both the cases, $a(t)$, ρ_m, p_m, ρ_D, p_D, ω_m, ω_D by solving the Brans-Dicke field equations. We also study their physical behavior.

First we consider that two fluid does not interact with each other. Hence the general form of conservation equation (8) leads us to the following separate conservation equations for the barotropic fluid and dark fluid respectively:

$$\dot{\rho}_m+(\rho_m+p_m)3\frac{\dot{a}}{a}=0 \tag{11}$$

$$\dot{p}_D+(\rho_D+p_D)3\frac{\dot{a}}{a}=0 \tag{12}$$

It may be observed that there is a structural difference between Eqs. (11) and (12). Since equation of state parameter of barotropic fluid ω_m is constant while ω_D is allowed to be a function of cosmic time, integration of Eq. (11) gives us

$$\rho_m=\rho_0 a^{-3(1+\omega_m)} \tag{13}$$

where ρ_0 is constant of integration.

Now using (13) in (5) and (6) we, first, obtain ρ_D and p_D in terms scale factor $a(t)$ as

$$8\pi\phi^{-1}\rho_D = 3\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right] - \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + 3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} - 8\pi\phi^{-1}\rho_0 a^{-3(1+\omega_m)} \tag{14}$$

$$-8\pi\phi^{-1}p_D = 2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\ddot{\phi}}{\phi} - 2\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + 8\pi\phi^{-1}\rho_0\omega_m a^{-3(1+\omega_m)} \tag{15}$$

To determine the scale factor $a(t)$ and the Brans-Dicke scalar field ϕ, we usethe condition for trace free energy momentum tensor of the two fluid

$$\rho_{tot} - 3\rho_{tot} = 0 \tag{16}$$

and the cosmological scale factor as a hybrid expansion law as

$$a(t) = te^t \tag{17}$$

Integrating Eq. (7) using Eqs. (16) and (17) we obtain

$$\phi = \frac{\phi_0}{t^2 e^{2t}} + \phi_1 \tag{18}$$

where ϕ_0 and ϕ_1 are constants of integration. We take $\phi_0 = 1$ and $\phi_1 = 0$ without loss of any generality. By using the scalefactor given by Eq. (17) and the scalar field given by Eq. (18)in (14) and (15), we obtain,
Pressure of the model as,

$$p_D = \frac{1}{-8\pi\phi^{-1}}\left\{2\left(1+\frac{2}{t}\right) + \left(1+\frac{1}{t}\right)^2 + \frac{k}{t^2e^{2t}} + \frac{\omega}{2}4\left(1+\frac{1}{t}\right)^2 + \left[4+\frac{8}{t}+\frac{6}{t^2}\right] + 4\left(1+\frac{1}{t}\right)^2 + 8\pi\rho_0\omega_m\left(te^t\right)^{(-1-3\omega_m)}\right\}. \tag{19}$$

Energy density of the model is,

$$\rho_D = \frac{1}{8\pi\phi^{-1}}\left\{3\left[\left(1+\frac{1}{t}\right)^2 + \frac{k}{t^2e^{2t}}\right] - [2\omega-6]\left(1+\frac{1}{t}\right)^2 - 8\pi\rho_0\left(te^t\right)^{(-1-3\omega_m)}\right\}. \tag{20}$$

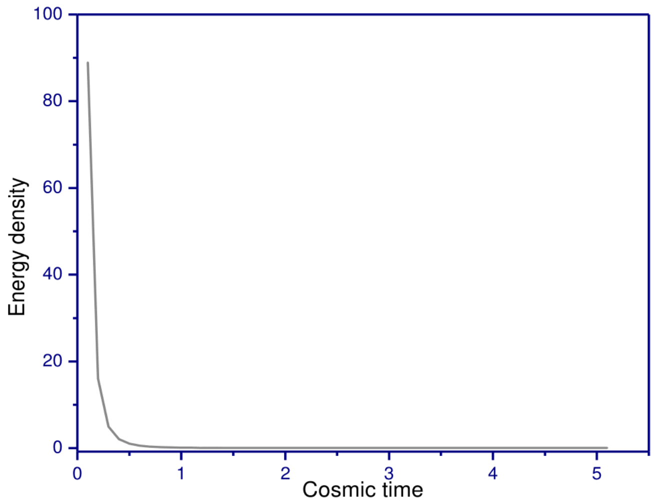

Figure (1): behavior of energy density parameter of the model verses time t with the appropriate choice of constants.

Above equation (20) gives the expression of energy density of the model. Its graphical performance is shown in the figure (1). From figure (1), it is observed that the energy density be the positive decreasing function of time, at an initial stage of the universe it is very high but with the expansion it is decreases and approaches to zero at infinite expansion.

Now using (19) and (20) in (10) we getequation of state parameter as

$$\omega_D = -\left[\frac{2\left(1+\frac{2}{t}\right)+\left(1+\frac{1}{t}\right)^2+\frac{k}{t^2e^{2t}}+\frac{\omega}{2}4\left(1+\frac{1}{t}\right)^2+\left[4+\frac{8}{t}+\frac{6}{t^2}\right]+4\left(1+\frac{1}{t}\right)^2+8\pi\rho_0\omega_m\left(te^t\right)^{(-1-3\omega_m)}}{3\left[\left(1+\frac{1}{t}\right)^2+\frac{k}{t^2e^{2t}}\right]-\left[2\omega-6\right]\left(1+\frac{1}{t}\right)^2-8\pi\rho_0\left(te^t\right)^{(-1-3\omega_m)}}\right] \qquad (21)$$

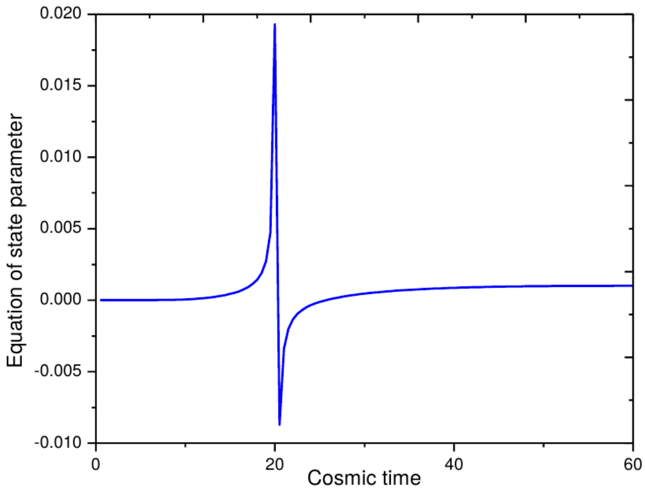

Figure (2): behavior of equation of state parameter of the model verses time t with the appropriate choice of constants.

From equation (21), we observed that the equation of state parameter is time dependent. The graphical behavior of equation of state parameter verses time t is shown in figure (2). At the initial stage when the universe started to expand for slight interval of time, the equation of state of the universe having value greater than zero i.e. the model behave as like matter dominated once at early stages while at late times it becomes less than zero. With the expansion the equation of state parameter extending from quintessence region and present in same region, this is a situation in the universe where the quintessence field dominated, while for late interval of time it shows dusty universe.

IV. CONCLUSIONS

In the analysis of evolution of the dark energy parameter for the spatially homogeneous and isotropic Friedmann-Robertson-Walker (FRW) model filled with barotropic fluid and dark energy in the scalar-tensor theory of gravitation proposed by Brans and Dicke, we find the determinate solution corresponds to the special law of variation for Hubble's parameter called hybrid law of expansion which yields a power and de-Sitter solution and using the trace of free energy momentum tensor of the non-interacting two fluid. It is observed that the energy density be the positive decreasing function of time, at an initial stage of the universe it is very high but with the expansion it is decreases and approaches to zero at infinite expansion while At the initial stage when the universe started to expand for slight interval of time, the equation of state of the universe having value greater than zero i.e. the model behave as like matter dominated once at early stages while at late times it becomes less than zero. With the expansion the Equation of state parameter extending from quintessence region and present in same region, this is a situation in the universe where the quintessence field dominated, while for late interval of time it shows dusty universe.

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