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Research Paper

Comparing the Trajectory of Mathematical Performance by Gender

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Abstract
It is of interest to investigate if mathematical fluid and crystallized abilities could be gender driven under certain conditions. By and large, the general performance of students in mathematics, especially the fluid and crystallized abilities, by gender could be investigated. At least, to find out whether the fluid and crystallized abilities in Mathematics are likely to differ for male and female students. At times, the difference or otherwise could be more glaring if new Mathematical interventions are administered to the students. The two types learners' mental capacity; fluid and crystallized abilities could be measured through performance in Mathematics. In order to preserve the multivariate data structure, the Hotelling's T^2 distribution was used to test of multivariate hypothesis concerning two mean vectors of male and female scores. The Hotelling's T^2 distribution is the multivariate equivalent of the t-test used in univariate test of hypothesis. The distribution is used to infer about mean vectors rather than individual means. The objective of this study is to investigate the effect of gender on the mathematics performance of science students in Kaduna Polytechnic, Nigeria. In specifics terms, this study is to determined the mean difference between the male and female students in mathematical abilities after receiving new coaching in Mathematics. In order to compare the trajectory of students' performance by gender, the Hotelling's T^2 distribution is used. After the data analysis using the SPSS, the results have revealed that male and female students' results do not differ even after new tutorial in Mathematics. Hence, we affirm that male and female students have the same performance in new tutorial in Mathematics, Basic Mathematics, CGPA at First Semester and CGPA at Second Semester. In general, the trajectory of new tutorial in Mathematics score on the performance of male and female students is the same. Hence, gender has no effect in the scores of students in new tutorial in Mathematics, scores in Basic Mathematics, CGPA at First Semester and CGPA at Second Semester. In other words, both the fluid and crystallized abilities in Mathematics do not differ for male and female students.

Key words: *Analysis of Variance (ANOVA) test, fluid ability, crystallized ability, covariances, omnibus test, dispersion matrix, mean vectors, univariate test, multivariate test, Hotelling's T^2 distribution.*

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I. Introduction

It is of interest to investigate if the general performance of students in mathematics could be influenced by gender. Perhaps, both the fluid and crystallized abilities in Mathematics are likely to differ for male and female students or otherwise. At times, the difference or otherwise could be more glaring if new intervention in Mathematics is administered to the students. The two types learners' mental capacity; fluid and crystallized abilities could be measured through performance in Mathematics. Fluid ability has the character of a purely general ability to discriminate and perceive relations between things. It increases until adolescence and then slowly declines. On the other hand, crystallized ability consists of discriminatory habits long established in a particular field, originally through the operation of fluid ability, but no longer requiring insightful perception for their successful operation. (Ackerman, 1996). The objective of this study was to investigate the effect of gender on the mathematics performance of science students in Kaduna Polytechnic, Nigeria. In specifics terms, this study is to determined the mean difference between the male and female students in mathematical abilities after receiving new intervention in Mathematics. In order to compare the trajectory of students' performance by gender, the Hotelling's T^2 distribution is used. The Hotelling's T^2 distribution is a multivariate statistical technique used for test of hypothesis concerning mean vectors rather than individual means. It is simply a special case of the Multivariate analysis of variance with only two groups (Usman, 2016).

II. METHODOLOGY

While preserving the multivariate data structure, the Hotelling's T^2 distribution is used to test of hypothesis and inferences concerning two mean vectors rather than individual means. It is the multivariate equivalent of the t-test used in univariate test of hypothesis (Morrison, 2005). In order to demonstrate the Hotelling's T^2 distribution, start by recalling the univariate theory for the test of the null hypothesis $\mathbf{H_0 : \mu_1 = \mu_2}$ against the two-sided alternative hypothesis $\mathbf{H_1 : \mu_1 \neq \mu_2}$ using the univariate independent t-test. Where; $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ are the means from two normal populations. If $\overline{X}_1$ and $\overline{X}_2$ denote the means of independent random samples from the two respective normal population, the appropriate test statistic is as follows.

$$t = \frac{\overline{X}_1 - \overline{X}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Where;

$$\overline{X}_j = \frac{1}{n}\sum_{j=1}^{n} X_{ij} \quad for \quad i = 1, 2, \cdots, p \quad j = 1, 2, \cdots, n$$

$$s_{ij}^2 = \frac{1}{n-1}\sum_{j=1}^{n} \left(X_{ij} - \overline{X}_i \right)^2 \quad for \quad i = 1, 2, \cdots, p \quad j = 1, 2, \cdots, n$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 1}$$

This test statistic follows the t-distribution with $n_1 + n_2 - 2$ degrees of freedom. The null hypothesis is rejected if the observed $|t|$ exceeds the critical value of t-distribution with $n_1 + n_2 - 2$ degrees of freedom. Rejecting H_0 when $|t|$ exceeds $t_{\alpha/2, n_1+n_2-2}$ is tantamount to rejecting the null hypothesis when:

$$t^2 = \frac{\left(\overline{X}_1 - \overline{X}_2 \right)^2}{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)} = \frac{n_1 n_2}{n_1 + n_2}\left(\overline{X}_1 - \overline{X}_2 \right)\left(s_p^2 \right)^{-1}\left(\overline{X}_1 - \overline{X}_2 \right) > t^2_{\alpha/2, n_1+n_2-2}$$

The variable t^2 above is the squared distance from the sample means $\overline{X}_1$ and $\overline{X}_2$. The units of distance are expressed in terms of the pooled standard deviation of $\overline{X}_1$ and $\overline{X}_2$. (Marcoulides, & Hershberger, 2012). The moment $\overline{X}_1$, s_1^2 and $\overline{X}_2$, s_2^2 are observed, then the null hypothesis is rejected, at α/2 level of significance, if:

$$\frac{n_1 n_2}{n_1 + n_2}\left(\overline{X}_1 - \overline{X}_2 \right)\left(s_p^2 \right)^{-1}\left(\overline{X}_1 - \overline{X}_2 \right) > t^2_{\alpha/2, n_1+n_2-2}$$

Where; $t_{\alpha/2, n_1+n_2-2}$ denotes the upper 100α of the *t*-distribution with $n_1 + n_2 - 2$ degrees of freedom. If the null hypothesis is not rejected, it is then concluded that the values of the normal population means are equal. (Pituch, & Stevens, 2016). A natural generalization of the squared distance in its multivariate analogue is as follows:

$$T^2 = \frac{n_1 n_2}{n_1 + n_2}\left(\overline{\mathbf{X}}_1 - \overline{\mathbf{X}}_2 \right)^{\mathbf{T}} \mathbf{s}_{\mathbf{p}}^{\mathbf{-1}} \left(\overline{\mathbf{X}}_1 - \overline{\mathbf{X}}_2 \right)$$

Where;

$$\underset{(p\times 1)}{\bar{\mathbf{X}}_{\mathbf{j}}} = \frac{1}{n}\sum_{j=1}^{n} X_{ij} \quad for \quad i = 1, 2, \cdots, p \quad j = 1, 2, \cdots, n$$

$$\underset{(\mathbf{p}\times\mathbf{p})}{\mathbf{S}} = \frac{1}{n-1}\sum_{j=1}^{n}\left(\mathbf{X}_{\mathbf{ij}} - \bar{\mathbf{X}}_{\mathbf{i}}\right)\left(\mathbf{X}_{\mathbf{ij}} - \bar{\mathbf{X}}_{\mathbf{i}}\right)^{\mathbf{T}} \quad for \quad i = 1, 2, \cdots, p \quad j = 1, 2, \cdots, n$$

In this case, the two multivariate normal populations are the average scores from male and female; each having four variables, average scores in Mathematics tutorials, scores in Basic Mathematics, Cumulative Grade Point Average (CGPA) at First Semester and CGPA at Second Semester. In order to test the null hypothesis concerning two multivariate normal populations where the difference is zero; we proceed as follows (Everitt, & Hothorn, 2011).

$$\mathbf{H}_{\mathbf{0}} : \boldsymbol{\mu}_{\mathbf{1}} = \boldsymbol{\mu}_{\mathbf{2}}$$

$$\mathbf{H}_{\mathbf{1}} : \boldsymbol{\mu}_{\mathbf{1}} \neq \boldsymbol{\mu}_{\mathbf{2}}$$

In this case; $\boldsymbol{\mu}_{\mathbf{1}}$ and $\boldsymbol{\mu}_{\mathbf{2}}$ are the mean vectors of treatment group and control group respectively. Based on random samples of sizes 75 and 49 male and female students examined respectively; the test statistic is as follows:

$$T^2 = \frac{n_1 n_2}{n_1 + n_2}\left(\bar{\mathbf{X}}_{\mathbf{1}} - \bar{\mathbf{X}}_{\mathbf{2}}\right)^{\mathbf{T}} \mathbf{s}_{\mathbf{p}}^{\mathbf{-1}}\left(\bar{\mathbf{X}}_{\mathbf{1}} - \bar{\mathbf{X}}_{\mathbf{2}}\right)$$

The formula above is the two-sample Hotelling's T^2 distribution; if the observed T^2 exceeds the critical value or the *p-value* less than the level of significance, the null hypothesis is rejected, (Timm, 2002).

Where; $\bar{\mathbf{X}}_{\mathbf{1}}$ and $\bar{\mathbf{X}}_{\mathbf{2}}$ denote the sample mean vectors from treatment and control groups respectively are defined as follows.

$$\bar{\mathbf{X}}_{\mathbf{1}} = \begin{pmatrix} \bar{X}_{1(1)} \\ \bar{X}_{2(1)} \\ \bar{X}_{3(1)} \\ \bar{X}_{4(1)} \end{pmatrix} \text{ and } \bar{\mathbf{X}}_{\mathbf{2}} = \begin{pmatrix} \bar{X}_{1(2)} \\ \bar{X}_{2(2)} \\ \bar{X}_{3(2)} \\ \bar{X}_{4(2)} \end{pmatrix}$$

The four variables in each mean vector are scores in Mathematics tutorial, scores in Basic Mathematics, cumulative grade point average (CGPA) at First Semester and CGPA at Second Semester respectively. Furthermore, $\mathbf{S}_{\mathbf{p}}$ is the pooled sample dispersion matrix defined as follows.

$$\mathbf{s}_{\mathbf{p}} = \frac{(n_1 - 1)\mathbf{s}_{\mathbf{1}} + (n_2 - 1)\mathbf{s}_{\mathbf{2}}}{n_1 + n_2 - 1}$$

The sample dispersion matrix from the treatment and control groups respectively are defined as follows.

$$\mathbf{S}_{\mathbf{1}} = \begin{pmatrix} S_{11(1)} & S_{12(1)} & S_{13(1)} & S_{14(1)} \\ S_{21(1)} & S_{22(1)} & S_{23(1)} & S_{24(1)} \\ S_{31(1)} & S_{32(1)} & S_{33(1)} & S_{34(1)} \\ S_{41(1)} & S_{42(1)} & S_{43(1)} & S_{44(1)} \end{pmatrix} \text{ and } \mathbf{S}_{\mathbf{2}} = \begin{pmatrix} S_{11(2)} & S_{12(2)} & S_{13(2)} & S_{14(2)} \\ S_{21(2)} & S_{22(2)} & S_{23(2)} & S_{24(2)} \\ S_{31(2)} & S_{32(2)} & S_{33(2)} & S_{34(2)} \\ S_{41(2)} & S_{42(2)} & S_{43(2)} & S_{44(2)} \end{pmatrix}$$

The elements on the main diagonal of the dispersion matrices above are the variances of scores in Mathematics tutorial, scores in Basic Mathematics, CGPA at First Semester and CGPA at Second Semester respectively while the off-diagonal elements are the covariances. The data would be analyzed using the SPSS.

III. RESULTS

Based on random samples of sizes 75 and 49 male and female students examined respectively; by using the following Hotelling's T^2 test statistic:

$$T^2 = \frac{n_1 n_2}{n_1 + n_2}\left(\bar{\mathbf{X}}_{\mathbf{1}} - \bar{\mathbf{X}}_{\mathbf{2}}\right)^{\mathbf{T}} \mathbf{s}_{\mathbf{p}}^{\mathbf{-1}}\left(\bar{\mathbf{X}}_{\mathbf{1}} - \bar{\mathbf{X}}_{\mathbf{2}}\right)$$

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Where; $\overline{\mathbf{X}}_1$ and $\overline{\mathbf{X}}_2$ denote the sample mean vectors from male and female students respectively are defined as follows.

$$\overline{\mathbf{X}}_1 = \begin{pmatrix} \overline{X}_{1(1)} \\ \overline{X}_{2(1)} \\ \overline{X}_{3(1)} \\ \overline{X}_{4(1)} \end{pmatrix} = \begin{pmatrix} 67.99 \\ 60.22 \\ 2.91 \\ 2.93 \end{pmatrix}$$

$$\overline{\mathbf{X}}_2 = \begin{pmatrix} \overline{X}_{1(2)} \\ \overline{X}_{2(2)} \\ \overline{X}_{3(2)} \\ \overline{X}_{4(2)} \end{pmatrix} = \begin{pmatrix} 63.94 \\ 61.99 \\ 2.94 \\ 2.94 \end{pmatrix}$$

The four variables in each mean vector are scores in tutorial in Mathematics, scores in Basic Mathematics, CGPA at First Semester and CGPA at Second Semester respectively. Furthermore, $\mathbf{S_p}$ is the pooled sample dispersion matrix defined as follows.

$$\mathbf{s_p} = \frac{(n_1 - 1)\mathbf{s_1} + (n_2 - 1)\mathbf{s_2}}{n_1 + n_2 - 1}$$

The sample dispersion matrix from the treatment and control groups respectively are defined as follows.

$$\mathbf{S_1} = \begin{pmatrix} S_{11(1)} & S_{12(1)} & S_{13(1)} & S_{14(1)} \\ S_{21(1)} & S_{22(1)} & S_{23(1)} & S_{24(1)} \\ S_{31(1)} & S_{32(1)} & S_{33(1)} & S_{34(1)} \\ S_{41(1)} & S_{42(1)} & S_{43(1)} & S_{44(1)} \end{pmatrix} = \begin{pmatrix} 152.36 & 116.17 & 2.71 & 2.71 \\ 116.17 & 189.99 & 3.96 & 3.25 \\ 2.71 & 3.96 & 0.18 & 0.16 \\ 2.71 & 3.25 & 0.16 & 0.18 \end{pmatrix}$$

$$\mathbf{S_2} = \begin{pmatrix} S_{11(2)} & S_{12(2)} & S_{13(2)} & S_{14(2)} \\ S_{21(2)} & S_{22(2)} & S_{23(2)} & S_{24(2)} \\ S_{31(2)} & S_{32(2)} & S_{33(2)} & S_{34(2)} \\ S_{41(2)} & S_{42(2)} & S_{43(2)} & S_{44(2)} \end{pmatrix} = \begin{pmatrix} 250.27 & 175.92 & 5.42 & 5.56 \\ 175.92 & 223.45 & 6.29 & 6.16 \\ 5.42 & 6.29 & 0.24 & 0.23 \\ 5.56 & 6.16 & 0.23 & 0.25 \end{pmatrix}$$

The elements on the main diagonal of the dispersion matrices above are the variances of average scores in new tutorial in Mathematics, scores in Basic Mathematics, CGPA at First Semester and CGPA at Second Semester respectively while the off-diagonal elements are the covariances. By using the Hotelling's T^2 test statistic above, the null hypothesis must be rejected if the observed T^2 exceeds the critical value. Alternatively, the null hypothesis must be rejected if the *p-value* is less than the level of significance. From the data using the SPSS, the analysis proceeds as follows:

Hypothesis
H_0: The trajectory of students' performance in Mathematics is the same for male and female students.
H_1: The trajectory of students' performance in Mathematics differ for male and female students.

Level of Significance
$\alpha=0.05$

Test Statistics

$$T^2 = \frac{n_1 n_2}{n_1 + n_2}\left(\overline{\mathbf{X}}_1 - \overline{\mathbf{X}}_2\right)^{\mathbf{T}} \mathbf{s_p^{-1}} \left(\overline{\mathbf{X}}_1 - \overline{\mathbf{X}}_2\right)$$

Decision Criterion
Reject the null hypothesis if $p<0.05$

Computations
As carried out by the SPSS as given in the following tables:

Table 1: Multivariate Descriptive Statistics

Variables	Sex	N	Mean	Std. Deviation
Average scores in Mathematics tutorial	Male	75	67.99	12.344
	Female	49	63.94	15.820
	Total	124	66.39	13.903
Average scores in Basic Mathematics	Male	75	60.22	13.7839
	Female	49	61.99	14.9483
	Total	124	60.92	14.2219
CGPA at First Semester	Male	75	2.91	0.42558
	Female	49	2.94	0.48666
	Total	124	2.93	0.44902
CGPA at Second Semester	Male	75	2.93	0.41956
	Female	49	2.94	0.50466
	Total	124	2.93	0.45317

The descriptive statistics above show the means and standard deviations for Average scores in new tutorial in Mathematics, scores in Basic Mathematics, CGPA at First Semester and CGPA at Second Semester for both treatment group and control group.

Table 2: Multivariate Tests[a]

Effect		Value	F	Hypothesis df	Error df	Sig.
Group	Pillai's Trace	0.071	2.273	4	119	0.065
	Wilks' Lambda	0.929	2.273	4	119	0.065
	Hotelling's Trace	0.076	2.273	4	119	0.065
	Roy's Largest Root	0.076	2.273	4	119	0.065

a. Design: Intercept + Sex

Table 3: Tests of Between-Subjects Effects

Source	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.
Group	Tutorial in Mathematics	485.616	1	485.616	2.544	0.113
	Basic Mathematics Scores	92.829	1	92.829	0.457	0.500
	CGPA at First Semester	0.029	1	0.029	0.143	0.706
	CGPA at Second Semester	0.009	1	0.009	0.044	0.835
Error	Tutorial in Mathematics	23287.803	122	190.884		
	Basic Mathematics Scores	24785.365	122	203.159		
	CGPA at First Semester	24.771	122	0.203		
	CGPA at Second Semester	25.251	122	0.207		
Total	Tutorial in Mathematics	23773.419	123			
	Basic Mathematics Scores	24878.194	123			
	CGPA at First Semester	24.800	123			
	CGPA at Second Semester	25.260	123			

IV. CONCLUSION

From Table 2, there is agreement in all the four test statistics produced by the SPSS. Since the $p=0.065>0.05$ for all the tests Pillai's Trace, Wilks' Lambda, Hotelling's Trace and Roy's Largest Root, the null hypothesis must not be rejected and we conclude that male and female students have the same results in new tutorial in Mathematics, scores in Basic Mathematics, CGPA at First Semester and CGPA at Second Semester. Moreover, this is confirmed by the individual Analysis of Variance (ANOVA) test for each of the variables on Table 3. From table 3, we have $p=0.113>0.05$ for new tutorial in Mathematics, $p=0.500>0.05$ for average scores in Basic Mathematics, $p=0.706>0.05$ for CGPA at First Semester and $p=0.835>0.05$ for CGPA at Second Semester. Hence, we affirm that male and female students have the same level of performance in new tutorial in Mathematics, average scores in Mathematics, CGPA at First Semester and CGPA at Second Semester. In general, the trajectory of new tutorial in Mathematics score on the performance of male and female students is the same. Hence, gender has no effect in the scores of students in new tutorial in Mathematics, average scores in Basic Mathematics, CGPA at First Semester and CGPA at Second Semester. By extension, we conclude that both the fluid and crystallized abilities in Mathematics do not differ for male and female students.

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