*Quest Journals*
*Journal of Research in Applied Mathematics*
*Volume 7 ~ Issue 7 (2021) pp: 19-25*
*ISSN(Online) : 2394-0743 ISSN (Print): 2394-0735*
www.questjournals.org

**Research Paper**

# Bayesian Estimation of Moments and Reliability of Geometric Distribution

Randhir Singh
*Department of Statistics, Ewing Christian College, Prayagraj, India.*

***ABSTRACT***
*This paper deals with the Bayesian estimation of a function of the unknown parameter $\theta$ of the Geometric Distribution .Bayes estimates of function of the unknown parameter of the distribution have been derived and these estimates have been used further to estimate central moments up-to order four and reliability of the distribution. The estimation has been performed by taking two different forms of the prior distribution and three different types of loss functions.On the part of loss functions, the Squared Error Loss Function (SELF) and two different forms of Weighted Squared Error Loss Function (WSELF) namely, Minimum Expected Loss (MELO) Function andExponentially Weighted Minimum Expected Loss (EWMELO) Function have been considered.Estimates have also been obtained under a censored sampling scheme.*
***KEYWORDS****: Geometric Distribution, Bayes Estimator, Squared Error Loss Function (SELF), Weighted Squared Error Loss Function (WSELF).*

*Received 18 July, 2021; Revised: 01 August, 2021; Accepted 03 August, 2021 © The author(s) 2021. Published with open access at www.questjournals.org*

## I. INTRODUCTION

A discrete random variable X is said to have Geometric distribution ,if its probability mass function is given by

$$p_\theta(x) = \begin{cases} \theta(1-\theta)^x, \text{if } x = 0,1,2,\dots; 0 < \theta < 1 \\ 0, \text{Otherwise.} \end{cases} \quad (1)$$

Inthis case,

$\mu = E(X) = \theta^{-1}(1-\theta)$ (2)

$\sigma^2 = V(X) = \theta^{-2}(1-\theta)$(3)

$\mu_3 = E\{(X-\mu)\}^3 = \theta^{-2}(1-\theta) + 2\theta^{-3}(1-\theta)^2$ (4)

$\mu_4 = E\{(X-\mu)\}^4 = \theta^{-2}(1-\theta) + 9\theta^{-4}(1-\theta)^2$ (5)

$R(t,\theta) = P(X \geq t)$, is given by,

$$R(t,\theta) = (1-\theta)^t \quad (6)$$

The failure rate denoted by,$h(t,\theta)$, is given by,

$$h(t,\theta) = \frac{p_\theta(t)}{R(t,\theta)} = \theta \quad (7)$$

This is the only discrete distribution which satisfies the 'Lack of Memory' property and has constant failure rate as mentioned in Johnson and Kotz(1969). These properties are satisfied by the exponential distribution in the continuous case. As mentioned in Bhattacharya and Kumar (1988) and also in Bhattacharya and Tyagi(1990),the Geometric distribution is useful in situations where life-times of items are measured in discrete cycles. If the random variable X denotes the number of cycles completed by an item before it breaks down (breaking is similar to failure), X has the probability mass function given by (1).

Roy and Mitra (1957) studied a class of discrete distributions known as the Power Series Distribution (PSD).Gupta (1974) considered a more general class of discrete distributions known as the Modified Power Series Distribution (MPSD)and derived moments of this distribution. This class contains the class of Power Series Distribution. Both MPSD and PSD contain the Geometric distribution as particular cases.Gupta (1977), has obtained MVUE of $\phi(\theta) = \theta^r, r \geq 1$.Bhattacharya and Kumar (1988) and also Bhattacharya and Tyagi(1990) obtained Bayes estimates of the mean $\mu = \theta^{-1}(1-\theta)$ and the reliability $R(t,\theta) = (1-\theta)^t$ of the Geometric distribution. Bhattacharya and Kumar (1988) used the Beta distribution ,which is the Natural Conjugate Bayesian Density (NCBD), as the prior distribution for the unknown parameter $\theta$,while Bhattacharya and Tyagi(1990) used Generalized Beta density introduced by Holla(1968).Bayes estimates derived in these two works as mentioned were for complete sample as well as for a censored sampling scheme. On the part of the loss function, the Squared Error Loss Function (SELF) was used.This loss function suffers from the drawback of being unbounded and it gives equal weights to overestimation as well underestimation. WenboYu and JinyunXie (2016) derived Bayesian estimates of the unknown parameter of Geometric distribution .They used the symbol the symbol $1 - R$ for the parameter $\theta$ and erroneously termed R as the reliability of the distribution. They used precautionary loss function and two prior distributions, one a quasi-prior and the other the Beta distribution, the Natural Conjugate Bayesian Density. They derived Bayes estimates based on the complete sampleand also based on the record value. In recent works,Singh (2021) obtained Bayes Estimator of $\phi(\theta) = \theta^r, r \in (-\infty, \infty)$,for the MPSD and again Bayes Estimator of $\psi(\theta) = \theta^r(1-\theta)^s, r, s \in (-\infty, \infty)$, for some particular cases of the MPSD..

In the present work the Bayesian Estimation has been performed for central moments up-to order four and reliability of the Geometric distribution, by taking the Beta distribution, the Natural Conjugate Bayesian Density as well as the Generalized Beta density introduced by Holla(1968).Bayesian Estimation under a Censored Sampling Scheme as given in Bhattacharya and Kumar (1988) andalso in Bhattacharya and Tyagi (1990) have also been considered. Although expressions for central moments of any order can be evaluated, yet, we have restricted up-to order four as central moments of order four are sufficient to describe all characteristics of a distribution.

On the part of loss function, we have considered the Following:

(i) Squared Error Loss Function(SELF): In this case,

$$w(\theta, \delta) = (\theta - \delta)^2 \qquad (8)$$

This loss function is symmetric but is unbounded.

(ii) Minimum Expected Loss (MELO)Function: In this case

$$w(\theta, \delta) = \theta^{-2}(\theta - \delta)^2 \qquad (9)$$

This loss function is asymmetric but is bounded.This form of loss function was used by Tummala and Sathe (1978) and by Zellner (1979)

(iii) Exponentially Weighted Minimum Expected Loss (EWMELO) Function. In this case

$$w(\theta, \delta) = \theta^{-2}e^{-a\theta}(\theta - \delta)^2, a > 0 \qquad (10)$$

This form of loss function is also asymmetric and bounded. This form of loss function was used for the first time by Singh, the author (1997),in his work for D.Phil.SELFand two forms of WSELF were used by Singh,the author,(1999) in the study of reliability of a multicomponent systemand (2010) in Bayesian Estimation of the mean and distribution function of Maxwell's distribution.Recently, the author again used these loss functions in Bayesian estimation for the MPSD (2021) and for estimating Loss and Risk Functions of a continuous distribution (2021).

## II. NOTATIONS AND RESULTS USED:

Let $X_1, X_2, X_3, \ldots X_N$ be a random sample of size N from the p.m.f given by (1).
Then,

$$T_N = \sum_{i=1}^{N} X_i \qquad (11)$$

We shall use the following result as given by Abramowitz and Stegun (1964):

$$\Gamma(x) = \int_0^\infty u^{x-1}e^{-u}du \qquad (12)$$

$$\Gamma(x)b^{-x} = \int_0^\infty u^{x-1}e^{-bu}du \qquad (13)$$

$$\frac{\Gamma(b-a)\Gamma(a)M(a,b,z)}{\Gamma(b)} = \int_0^1 u^{a-1}(1-u)^{b-a-1}e^{-zu}du \qquad (14)$$

Where, $M(a, b, z)$ is the Confluent Hypergeometric Function and has a series representation given by,

$$M(a,b,z) = \sum_{n=0}^{\infty} \frac{(a)_n z^n}{(b)_n n!} \qquad (15)$$

Where,$(a)_0 = 1$ amd

$$(a)_n = \prod_{i=1}^{n}(a + i - 1) \qquad (16)$$

For observed value $t_N = \sum_{i=1}^{N} x_i$ of the statistic $T_N = \sum_{i=1}^{N} X_i$, the likelihood function, denoted by $L(\theta)$, is given by,

$$L(\theta) = k(1-\theta)^{t_N}\theta^N \qquad (17)$$

Where, k is function of $x_1, x_2, x_3, \ldots x_N$and does not contain $\theta$.

Let $\pi(\theta)$ be the prior probability density function of $\theta$,then the posterior posterior probability density function of $\theta$ ,denoted by $\pi(\theta / t_N)$,is given by,

$$\pi(\theta / t_N) = \frac{L(\theta)\pi(\theta)}{\int_0^1 L(\theta)\pi(\theta)d\theta} \qquad (18)$$

Under the Squared Error Loss Function (SELF), $L(\psi(\theta), d) = (\psi(\theta) - d)^2$,the Bayes Estimate of $\psi(\theta)$,denoted by $\hat{\psi}_B$ is given by,

$$\widehat{\psi}_B = \int_0^1 \psi(\theta)\pi(\theta / t_N)d\theta \qquad (19)$$

Similarly,under the Weighted Squared Error Loss Function (WSELF), $L(\psi(\theta), d) = W(\theta)(\psi(\theta) - d)^2$, where, $W(\theta)$is a function of $\theta$,the Bayes Estimate of $\psi(\theta)$,denoted by $\widehat{\psi}_W$ is given by,

$$\widehat{\psi}_W = \frac{\int_0^1 W(\theta)\psi(\theta)\pi(\theta / t_N)d\theta}{\int_0^1 W(\theta)\pi(\theta / t_N)d\theta} \qquad (20)$$

We have taken two different forms of $W(\theta)$,as given below:

(i). $W(\theta) = \theta^{-2}$.The Bayes Estimate of $\psi(\theta)$,denoted by $\widehat{\psi}_M$, is known as the Minimum Expected Loss (MELO) Estimate and is given by,

$$\widehat{\psi}_M = \frac{\int_0^1 \theta^{-2}\psi(\theta)\pi(\theta / t_N)d\theta}{\int_0^1 \theta^{-2}\pi(\theta / t_N)d\theta} \qquad (21)$$

This loss function was used by Tummala and Sathe (1978) for estimating reliability of certain life time distributions and by Zellner (1979) for estimating functions of parameters in econometric models.

(ii). $W(\theta) = \theta^{-2}e^{-a\theta}$.The Bayes Estimate of $\psi(\theta)$,denoted by $\widehat{\psi}_E$, is known as the Exponentially Minimum Expected Loss EW(MELO) Estimate and is given by,

$$\widehat{\psi}_E = \frac{\int_0^1 \theta^{-2}e^{-a\theta}\psi(\theta)\pi(\theta / t_N)d\theta}{\int_0^1 \theta^{-2}e^{-a\theta}\pi(\theta / t_N)d\theta} \qquad (22).$$

Now, we shall obtain Bayes Estimate of $\psi(\theta) = \theta^r(1-\theta)^s$, $r, s \in (-\infty, \infty)$

## III. BAYESIAN ESTIMATION

Since, in this case,$0 < \theta < 1$,we have taken two different prior distributions, namely,$\pi_1(\theta)$ and $\pi_2(\theta)$ as given below:

$$\pi_1(\theta) = \begin{cases} \frac{\theta^{p-1}(1-\theta)^{q-1}}{B(p,q)}, \text{if } p > 0, q > 0, 0 < \theta < 1 \\ 0, \text{Otherwise.} \end{cases} \qquad (23)$$

And,

$$\pi_2(\theta) = \begin{cases} \frac{e^{-b\theta}\theta^{p-1}(1-\theta)^{q-1}}{B(p,q)M(p,p+q,-b)}, \text{if } p > 0, q > 0, 0 < \theta < 1, b \geq 0 \\ 0, \text{Otherwise.} \end{cases} \qquad (24)$$

Where,

$$B(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \qquad (25)$$

$\pi_2(\theta)$ is known the Generalized Beta Density and was introduced by Holla (1968). This prior probability density function was also used by Bhattacharya (1968).

The posterior p. d .f. of $\theta$ , corresponding to the prior $\pi_1(\theta)$, denoted by $\pi_1(\theta / t_N)$,is given by,

$$\pi_1(\theta / t_N) = \begin{cases} \frac{(1-\theta)^{t_N+q-1}\theta^{N+p-1}}{B(N+p,t_N+q)}, \text{if } p > 0, q > 0, 0 < \theta < 1 \\ 0, \text{Otherwise.} \end{cases} \qquad (26)$$

Similarly, posterior p. d .f. of $\theta$ , corresponding to the prior $\pi_2(\theta)$, denoted by $\pi_2(\theta / t_N)$,is given by

$$\pi_2(\theta / t_N) = \begin{cases} \frac{e^{-b\theta}(1-\theta)^{t_N+q-1}\theta^{N+p-1}}{K}, \text{if } p > 0, q > 0, 0 < \theta < 1, b \geq 0 \\ 0, \text{Otherwise.} \end{cases} \qquad (27)$$

Where,

$$K = B(N+p, t_N+q)M(p+N, t_N+N+p+q, -b) \qquad (28)$$

Under the SELF and corresponding to the posterior distribution given by (26),Bayes Estimate of $\psi(\theta) = \theta^r(1-\theta)^s$,denoted by $\widehat{\psi}_{1B}$ is given by,

$$\widehat{\psi}_{1B} = \frac{B(N+p+r, t_N+q+s)}{B(N+p, t_N+q)} \qquad (29)$$

For $r = -1$ and $s = 1$,$\widehat{\psi}_{1B}$ reduces to the estimate of $\theta^{-1}(1-\theta)$,the mean of the distribution as given by Bhattacharya and Kumar (1988)

Similarly, under the WSELF, when $W(\theta) = \theta^{-2}$ and corresponding to the posterior distribution given by (26),the MELO Estimate of $\psi(\theta) = \theta^r(1-\theta)^s$,denoted by $\widehat{\psi}_{1M}$ ,is given by,

$$\widehat{\psi}_{1M} = \frac{B(N+p+r-2, t_N+q+s)}{B(N+p, t_N+q)} \qquad (30)$$

Under the WSELF, when $W(\theta) = \theta^{-2}e^{-a\theta}$ and corresponding to the posterior distribution given by (26),the EWMELO Estimate of $\psi(\theta) = \theta^r(1-\theta)^s$,denoted by $\widehat{\psi}_{1E}$ is given by,

$$\widehat{\psi}_{1E} = \frac{B(N+p+r-2, t_N+q+s)M_2}{B(N+p, t_N+q)M_1} \qquad (31)$$

Where,
$M_1 = M(N + p - 2, p + q + t_N + N - 2, -a)$ (32)
$M_2 = M(p + N + r - 2, p + q + t_N + N + r + s - 2, -a)$ (33)
On the other hand,under the SELF and corresponding to the posterior distribution given by (27),Bayes Estimate of $\psi(\theta) = \theta^r(1-\theta)^s$,denoted by $\widehat{\psi}_{2B}$ ,is given by,

$$\widehat{\psi}_{2B} = \frac{B(N+p+r, t_N+p+q+s)M_4}{B(N+p, t_N+q)M_3} \quad (34)$$

Where,
$M_3 = M(N + p , p + q + t_N + N - b)$ (35)
$M_4 = M(p + N + r, p + q + t_N + N + r + s, -b)$ (36)
For $r = -1$ and $s = 1$,$\widehat{\psi}_{2B}$ reduces to the estimate of $\theta^{-1}(1-\theta)$,the mean of the distribution as given by Bhattacharya and Tyagi (1990) .Moreover, for $r = -1$ ,$s = 1$ and $b = 0$, $\widehat{\psi}_{2B}$ reduces to the estimate of $\theta^{-1}(1-\theta)$,as given by Bhattacharya and Kumar (1988)
Similarly, under the WSELF, when $W(\theta) = \theta^{-2}$ and corresponding to the posterior distribution given by (27),the MELO Estimate of $\psi(\theta) = \theta^r(1-\theta)^s$,denoted by $\widehat{\psi}_{2M}$ is given by,

$$\widehat{\psi}_{2M} = \frac{B(p+N+r-2, t_N++q+s)M_6}{B(N+p, t_N+q)M_5} \quad (37)$$

Where,
$M_5 = M(p + N - 2, p + q + t_N + N - 2, -b)$ (38)
$M_6 = M(p + N + r - 2, p + q + t_N + N + r + s - 2, -b)$ (39)
Finally,under the WSELF, when $W(\theta) = \theta^{-2}e^{-a\theta}$ and corresponding to the posterior distribution given by (27),theEWMELO Estimate of $\psi(\theta) = \theta^r(1-\theta)^s$,denoted by $\widehat{\psi}_{2E}$ is given by,

$$\widehat{\psi}_{2E} = \frac{B(p+N+r-2, t_N++q+s)M_8}{B(N+p, t_N+qM_7} \quad (40)$$

Where,
$M_7 = M(p + N - 2, p + q + t_N + N - 2, -(a + b))$ (41)
$M_8 = M(p + N + r - 2, p + q + t_N + N + r + s - 2 , -(a + b))$ (42)
<u>Remark (1):</u>For $s = 0$ ,we get Bayes estimator of $\phi(\theta) = \theta^r, r \in (-\infty, \infty)$ while ,for r= 0 ,we get Bayes estimator of $(1-\theta)^s, s \in (-\infty, \infty)$
Bayes estimators of central moments up-to order four and reliability of the distribution, under three loss functions and two prior and corresponding posterior distributions are shown in the following tables:

**TABLE;3.1**

| Characteristic | Loss Function→ Prior↓ | SELF |
|---|---|---|
| $\mu$ | $\pi_1(\theta)$ | $\frac{B(N+p-1, t_N+q+1)}{B(N+p, t_N+q)}$ |
| $\sigma^2$ | $\pi_1(\theta)$ | $\frac{B(N+p-2, t_N+q+1)}{B(N+p, t_N+q)}$ |
| $\mu_3$ | $\pi_1(\theta)$ | $\frac{B(N+p-2, t_N+q+1)}{B(N+p, t_N+q)} + \frac{2B(N+p-3, t_N+q+2)}{B(N+p, t_N+q)}$ |
| $\mu_4$ | $\pi_1(\theta)$ | $\frac{B(N+p-2, t_N+q+1)}{B(N+p, t_N+q)} + \frac{9B(N+p-4, t_N+q+2)}{B(N+p, t_N+q)}$ |
| $R(t,\theta)$ | $\pi_1(\theta)$ | $\frac{B(N+p, t_N+q+t)}{B(N+p, t_N+q)}$ |

**TABLE;3.2**

| Characteristic | Loss Function→ Prior↓ | MELO |
|---|---|---|
| $\mu$ | $\pi_1(\theta)$ | $\frac{B(N+p-3, t_N+q+1)}{B(N+p, t_N+q)}$ |
| $\sigma^2$ | $\pi_1(\theta)$ | $\frac{B(N+p-4, t_N+q+1)}{B(N+p, t_N+q)}$ |
| $\mu_3$ | $\pi_1(\theta)$ | $\frac{B(N+p-4, t_N+q+1)}{B(N+p, t_N+q)} + \frac{2B(nN+p-5, t_N+q+2)}{B(N+p, t_N+q)}$ |
| $\mu_4$ | $\pi_1(\theta)$ | $\frac{B(N+p-4, t_N+q+1)}{B(N+p, t_N+q)} + \frac{9B(N+p-6, t_N+q+2)}{B(N+p, t_N+q)}$ |
| $R(t,\theta)$ | $\pi_1(\theta)$ | $\frac{B(N+p-2, t_N+q+t)}{B(N+p, t_N+q)}$ |

**TABLE;3.3**

| Characteristic | Loss Function→ Prior↓ | EWMELO |
|---|---|---|
| $\mu$ | $\pi_1(\theta)$ | $\frac{B(N+p-3, t_N+q+1)M(p+N-3, p+q+t_N+N-2, -a)}{B(N+p, t_N+q)M(p+N-2, p+q+t_N+N-2, -a}$ |
| $\sigma^2$ | $\pi_1(\theta)$ | $\frac{B(N+p-4, t_N+q+1)M(p+N-4, p+q+t_N+N-3, -a)}{B(N+p, t_N+q)M(p+N-2, p+q+t_N+N-2, -a)}$ |
| $\mu_3$ | $\pi_1(\theta)$ | $\frac{B(N+p-4, t_N+q+1)M(p+N-4, p+q+t_N+N-3, -a)}{B(N+p, t_N+q)M(p+N-2, p+q+t_N+N-2, -a)} + \frac{2B(N+p-5, t_N+q+2)M(p+N-5, p+q+t_N+N-3, -a)}{B(N+p, t_N+q)M(p+N-2, p+q+t_N+N-2, -a)}$ |
| $\mu_4$ | $\pi_1(\theta)$ | $\frac{B(N+p-4, t_N+q+1)M(p+N-4, p+q+t_N+N-3, -a)}{B(N+p, t_N+q)M(p+N-2, p+q+t_N+N-2, -a)} + \frac{9B(N+p-6, t_N+q+2)M(p+N-6, p+q+t_N+N-4, -a)}{B(N+p, t_N+q)M(p+N-2, p+q+t_N+N-2, -a)}$ |
| $R(t, \theta)$ | $\pi_1(\theta)$ | $\frac{B(N+p-2, t_N+q+t)M(p+N-2, p+q+t_N+N+t-2, -a)}{B(N+p, t_N+q)M(p+N-2, p+q+t_N+N-2, -a)}$ |

**TABLE;3.4**

| Characteristic | Loss Function→ Prior↓ | SELF |
|---|---|---|
| $\mu$ | $\pi_2(\theta)$ | $\frac{B(N+p-1, t_N+q+1)M(p+N-1, p+q+t_N+N, -b)}{B(N+p, t_N+q)M(p+N, p+q+t_N+N, -b}$ |
| $\sigma^2$ | $\pi_2(\theta)$ | $\frac{B(N+p-2, t_N+q+1)M(p+N-2, p+q+t_N+N-1, -b)}{B(N+p, t_N+q)M(p+N, p+q+t_N+N, -b)}$ |
| $\mu_3$ | $\pi_2(\theta)$ | $\frac{B(N+p-2, t_N+q+1)M(p+N-2, p+q+t_N+N-1, -b)}{B(N+p, t_N+q)M(p+N-2, p+q+t_N+N, -b)} + \frac{2B(N+p-3, t_N+q+2)M(p+N-3, p+q+t_N+N-1, -b)}{B(N+p, t_N+q)M(p+N, p+q+t_N+N, -b)}$ |
| $\mu_4$ | $\pi_2(\theta)$ | $\frac{B(N+p-2, t_N+q+1)M(p+N-2, p+q+t_N+N-1, -b)}{B(N+p, t_N+q)M(p+N, p+q+t_N+N, -b)} + \frac{9B(N+p-4, t_N+q+2)M(p+N-4, p+q+t_N+N-2, -b)}{B(N+p, t_N+q)M(p+N, p+q+t_N+N, -b)}$ |
| $R(t, \theta)$ | $\pi_2(\theta)$ | $\frac{B(N+p, t_N+q+t)M(p+N, p+q+t_N+N+t, -b)}{B(N+p, t_N+q)M(p+N, p+q+t_N+N, -b)}$ |

**TABLE;3.5**

| Characteristic | Loss Function→ Prior↓ | MELO |
|---|---|---|
| $\mu$ | $\pi_2(\theta)$ | $\frac{B(N+p-3, t_N+q+1)M(p+N-3, p+q+t_N+N-2, -b)}{B(N+p, t_N+q)M(p+N-2, p+q+t_N+N-2, -b)}$ |
| $\sigma^2$ | $\pi_2(\theta)$ | $\frac{B(N+p-4, t_N+q+1)M(p+N-4, p+q+t_N+N-3, -b)}{B(N+p, t_N+q)M(p+N-2, p+q+t_N+N-2, -b)}$ |
| $\mu_3$ | $\pi_2(\theta)$ | $\frac{B(N+p-4, t_N+q+1)M(p+N-4, p+q+t_N+N-3, -b)}{B(N+p, t_N+q)M(p+N-2, p+q+t_N+N-2, -b)} + \frac{2B(N+p-5, t_N+q+2)M(p+N-5, p+q+t_N+N-3, -b)}{B(N+p, t_N+q)M(p+N-2, p+q+t_N+N-2, -b)}$ |
| $\mu_4$ | $\pi_2(\theta)$ | $\frac{B(N+p-4, t_N+q+1)M(p+N-4, p+q+t_N+N-3, -b)}{B(N+p, t_N+q)M(p+N-2, p+q+t_N+N-2, -b)} + \frac{9B(N+p-6, t_N+q+2)M(p+N-6, p+q+t_N+N-4, -b)}{B(N+p, t_N+q)M(p+N-2, p+q+t_N+N-2, -b)}$ |
| $R(t, \theta)$ | $\pi_2(\theta)$ | $\frac{B(N+p-2, t_N+q+t)M(p+N-2, p+q+t_N+N+t-2, -b)}{B(N+p, t_N+q)M(p+N-2, p+q+t_N+N-2, -b)}$ |

**ABLE;3.6**

| Characteristic | Loss Function→ Prior↓ | EWMELO |
|---|---|---|
| $\mu$ | $\pi_2(\theta)$ | $\frac{B(N+p-3,t_N+q+1)M(p+N-3,p+q+t_N+N-2,-(a+b))}{B(N+p,t_N+q)M(p+N-2,p+q+t_N+N-2,-(a+b))}$ |
| $\sigma^2$ | $\pi_2(\theta)$ | $\frac{B(N+p-4,t_N+q+1)M(p+N-4,p+q+t_N+N-3,-(a+b))}{B(N+p,t_N+q)M(p+N-2,p+q+t_N+N-2,-(a+b))}$ |
| $\mu_3$ | $\pi_2(\theta)$ | $\frac{B(N+p-4,t_N+q+1)M(p+N-4,p+q+t_N+N-3,-(a+b))}{B(N+p,t_N+q)M(p+N-2,p+q+t_N+N-2,-(a+b))} + \frac{2B(N+p-5,t_N+q+2)M(p+N-5,p+q+t_N+N-3,-(a+b))}{B(N+p,t_N+q)M(p+N-2,p+q+t_N+N-2,-(a+b))}$ |
| $\mu_4$ | $\pi_2(\theta)$ | $\frac{B(N+p-4,t_N+q+1)M(p+N-4,p+q+t_N+N-3,-(a+b))}{B(N+p,t_N+q)M(p+N-2,p+q+t_N+N-2,-(a+b))} + \frac{9B(N+p-6,t_N+q+2)M(p+N-6,p+q+t_N+N-4,-(a+b))}{B(N+p,t_N+q)M(p+N-2,p+q+t_N+N-2,-(a+b))}$ |
| $R(t,\theta)$ | $\pi_2(\theta)$ | $\frac{B(N+p-2,t_N+q+t)M(p+N-2,p+q+t_N+N+t-2,-(a+b))}{B(N+p,t_N+q)M(p+N-2,p+q+t_N+N-2,-(a+b))}$ |

## IV. BAYESIAN ESTIMATION UNDER A CENSORED SAMPLING SCHEME

The likelihood function corresponding to**c**ensored sampling scheme,as mentioned inBhattacharya and Kumar (1988) and also in Bhattacharya andTyagi (1990),denoted by$L_c(\theta)$, is given by,

$$L_c(\theta), = \theta^m(1-\theta)^{t_*} \qquad (43)$$

Where,$T_* = \sum_{j=1}^{m} X_j + (N-m)R$ and $t_*$ is an observed value of $T_*$.N is the number of items tested, R is a preassigned number of cycles. m is the number of items having life-times less than or equal to R.$X_j$ is the recorded life-time of$j^{th}$ item (j=1,2… m) ,say a spring. For rest of $(N-m)$,items, observations are greater than R but their exact values are unknown.

Taking the prior probability density functions in (23) and (24) the corresponding posterior probability density functions are given as follows

$$\pi_{1c}(\theta/t_*) = \begin{cases} \frac{(1-\theta)^{t_*+q-1}\theta^{m+p-1}}{B(m+p,t_*+q)}, \text{if } p>0, q>0, 0<\theta<1 \\ 0, \text{Otherwise.} \end{cases} \qquad (44)$$

Similarly, posterior p. d .f. of θ , corresponding to the prior $\pi_2(\theta)$, denoted by $\pi_{2c}(\theta/t_*)$,is given by

$$\pi_2(\theta/t_*) = \begin{cases} \frac{e^{-b\theta}(1-\theta)^{t_*+q-1}\theta^{m+p-1}}{K}, \text{if } p>0, q>0, 0<\theta<1, b\ \geq 0 \\ 0, \text{Otherwise.} \end{cases} \qquad (45)$$

Where,

$$K = B(m+p,t_*+q)M(p+m,t_*+m+p+q,-b) \qquad (46)$$

Under the censored sampling scheme Bayes estimators of central moments up-to order four and reliability of the distribution, under three loss functions and two prior and corresponding posterior distributions can be obtained in expression derive earlier, by replacing N by m and $t_N$by $t_*$ respectively.

## REFERENCES

[1]. Abramowitz,M.and Stegun,I., (1964): Handbook of Mathematical Functions;U.S.Bereau of Standards,Washington D.C.
[2]. Bhattacharya,S.K. (1968); “Bayes approach to compound distributions arising from truncated mixing densities”,Ann.Inst.Stat.Math.,20.375-381
[3]. Johnson, N. L .and Kotz, S. (1969);Distributions in Statistics: Discrete Distributions.Houghton Mifflin Company,Boston.
[4]. Bhattacharya, S.K.and Kumar, S. (1988);”Discrete life testing” ,IAPQR TRANSACTIONS,Vol.13, No.2 ,pp.71-76
[5]. Bhattacharya, S.K.and Tyagi, R. (1990);”Bayesian approach to discrete reliability analysis” ,Allahabad Mathematical Society ,Second Biennial Conference (April 7-9,1990),Symposium on Statistics pp.f-23 – f-28
[6]. Gupta,R.C.,(1974); “Modified Power Series Distribution and Some of its Applications”, SankhyāB 35,288-298.
[7]. Gupta,R.C.,(1977);” Minimum Variance Unbiased Estimation in a Modified Power Series Distribution and Some of its Applications”, COMMUN.STATIST.-THEOR.,METH., A6(10)pp.977-991
[8]. Holla,M.S. (1968); “Discrete distributions with prior information” , Ann.Inst.Stat.Math.,20.151-157
[9]. Roy ,J.and Mitra,S.K., (1957); “Unbiased Minimum Variance Estimation in a Class of Discrete Distributions”, , Sankhyā, Vol.18,pp.371-378.
[10]. Singh, Randhir .(1997): D . Phil Thesis (Unpublished), Department of Mathematics and Statistics, University of Allahabad, Allahabad.(INDIA).
[11]. Singh, Randhir,(1999).”Bayesian Analysis of a Multicomponent System” ,Proceedings of NSBA-TA, 16-18 Jan.1999,pp.252-261,Editor Dr.Rajesh Singh.The conference wasorganized by the Department of Statistics,AmrawatiUniversity,Amravati-444602,Maharashtra,India.

[12]. Singh, Randhir.,(2010)."Simulation Aided Bayesian Estimation for Maxwell's Distribution", Proceedings of National Seminar on Impact of Physics on Biological Sciences (August,26,2010),held by the Department of Physics,EwingChristian College, Prayagraj,India,pp.203-210; ISBN No.: 978-81-905712-9-6

[13]. Singh, Randhir.(2021):"On Bayesian Estimation of Function of Unknown Parameter of Modified Power Series Distribution". International Journal of Innovative Science and Research Technology ISSN No:2456-2165, Volume 6,Issue 6, June-2021,pp.861-864 www.ijirst.com

[14]. [14] Randhir Singh.On Bayesian Estimation of Loss and Risk Functions. Science Journal ofApplied Mathematics and Statistics,Vol.9, No.3,2021, pp.73-77.doi:10.11648/j.sjams.20210903.11

[15]. [15]Singh, Randhir.(2021):" Bayesian Estimation of Function of Unknown Parameters of Some Particular Cases of Modified Power Series Distribution". Journal of Emerging Technologies and Innovative Research (JETIR), ISSN No:2349-5162, Volume 8,Issue 7, pp.d673-d681www.jetir.org

[16]. Tummala, V.M. and Sathe,P.T.(1978):"Minimum Expected Loss Estimators of Reliability and Parameters of Certain Life Time Distributions". IEEE Transactions on Reliability, Vol. R-27, No.4, pp.283-285

[17]. Wenbo Yu and JinyunXie (2016):"Bayesian Estimation of Reliability of Geometric Distribution under Different Loss Functions", AMSE JOURNALS-2016-Series:Advances A; Vol.59;N° 1;pp 172-185.

[18]. Zellner, A .and Park, S.B.(1979):"Minimum Expected Loss Estimators (MELO) of Functions of Parameters and Structural Coefficients of Econometric Models". JASA 74, pp.185-193.