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Research Paper

A New Approximation Method from Modified Newton's Gregory Backward and Modified Gauss's Backward Interpolation Formula

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ABSTRACT: *A number of different methods have been developed to construct useful interpolation formulas for evenly and unevenly spaced points. In this paper a new interpolation formula which is obtained using Modified Newton's Gregory Backward formula and a modified form of Gauss backward interpolation formula by taking the mean of the two methods in which we retreat the subscripts in Modified Gauss's Backward Formula by one unit and replacing s by s-1 and letting the subscript of Newton's Gregory Backward be* $n = \frac{1}{n}$ *for* $n = 1,2,3,\ldots$ *and replacing* s *by* $s - 1$*. We also made comparison of the newly developed interpolation formula with existing interpolation methods and results show that the new formula is very efficient and possess good accuracy for evaluating functional values between given data.*

KEYWORDS: *Newton's Gregory Backward Formula, Gauss's Formula, Difference Table, Interpolation.*

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I. INTRODUCTION

Polynomial Interpolation theory has a number of important uses. Its primary use is to furnish some mathematical tools that are used in developing methods in the areas of approximation theory, numerical integration and the numerical solution of differential equations (E.Atkinson, 1989). The general problem of interpolation consists, then, in representing a function, known or unknown, in a form chosen in advance, with the aid of given values which this function takes for definite values of the independent variable (Scarborough, 1966). A number of different methods have been developed to construct useful interpolation formulas for evenly and unevenly spaced data.

II. LITERATURE REVIEW

It is justified to say that "there is no single person who did so much for this field, as for so many others, as Newton", (H. H. Goldstine, 1977). His eagerness becomes clear in a letter he wrote to Oldenburg (1960), where he first describes a method by which certain functions may be expressed in series of powers. The contributions of Newton to the subject are contained in:

(1) A letter to Smith in 1675 (I. Newton, 1959);
(2) a manuscript entitled Methodus Differentialis (I. Newton, 1981), published in 1711, although earlier versions were probably written in the middle 1670s;
(3) a manuscript entitled Regula Differentiarum, written in 1676, but first discovered and published in the 20th century (D. C. Fraser, 1927);
(4) Lemma V in Book III of his celebrated Principia (I. Newton, 1960), which appeared in 1687. The latter was published first and contains two formulae. The first deals with equal-interval data, which Newton seems to have discovered independently of Gregory. The second formula deals with the more general case of arbitrary-interval data.

Around 300 BC, they were using not only linear, but also more complex forms of interpolation to predict the positions of the sun, moon, and the planets they knew of. Farmers, timing the planting of their crops, were the primary users of these predictions. Also, in Greece sometime around 150 BC, Toomer (1978) believes that Hipparchus of Rhodes used linear interpolation to construct a "chord function", which is similar to a sinusoidal function, to compute positions of celestial bodies. Farther east, Chinese evidence of interpolation dates back to around 600 AD. Liu Zhuo used the equivalent of second order Gregory-Newton interpolation to construct an "Imperial Standard Calendar" see Martzloff (1997) and Y˘an and Shírán (1987).

The general interpolation formula for equidistant data was first written down in 1670 by Gregory (1939) that can be found in a letter written to Collins. Particular cases of it, but, had been published several decades earlier by Briggs, the man who brought to fruition the work of Napier on logarithms. In the introductory chapters to his major works (H. Briggs, 1624, 1633), he described the precise rules by which he carried out his computations, including interpolations, in constructing the tables contained therein.

By the beginning of the 20th century, the problem of interpolation by finite or divided differences had been studied by astronomers, mathematicians, statisticians, and actuaries. Many of them introduced their own system of notation and terminology, leading to confusion and researchers reformulating existing results. The point was discussed by Joffe (1917), who also made an attempt to standardize yet another system. It is, however, Sheppard's (1899) notation for central and mean differences that has survived in later publications. Most of the now well-known variants of Newton's original formulae had been worked out. This is not to say, however, that there are no more advanced developments to report on quite to the contrary. Already in 1821, Cauchy (1821) studied interpolation by means of a ratio of two polynomials and showed that the solution to this problem is unique, the Waring–Lagrange formula being the special case for the second polynomial equal to one. It was Cauchy also who, in 1840, found an expression for the error caused by truncating finite-difference interpolation series (A. Cauchy, 1841). The absolute value of this so-called Cauchy remainder term can be minimized by choosing the abscissa as the zeroes of the polynomials introduced later by Tchebychef (1874). Due to errors, it can be seen that our numerical result is an approximate value of the (sometimes unknown) exact result, except for the rare case where the exact answer is sufficiently simple rational number (Sri.Nandakumar M., 2011).

A generalization of a different nature was published in 1878 by Hermite, who studied and solved the problem of finding a polynomial of which also the first few derivatives assume pre-specified values at given points, where the order of the highest derivative may differ from point to point. Birkhoff (1906) studied the even more general problem: given any set of points, find a polynomial function that satisfies pre-specified criteria concerning its value and/or the value of any of its derivatives for each individual point. Birkhoff interpolation, also known as lacunary interpolation, initially received little attention, until Schoenberg (1966) revived interest in the subject.

III. METHODOLOGY

3.1 DERIVATION OF THE METHOD

Consider the NGB of the form;

$$P_n(x_s) = f_0 + \binom{s}{1}\Delta f_{-1} + \binom{s+1}{2}\Delta^2 f_{-2} + \binom{s+2}{3}\Delta^3 f_{-3} + \binom{s+3}{4}\Delta^4 f_{-4} + \binom{s+4}{5}\Delta^5 f_{-5} + \cdots + \binom{s+n-1}{n}\Delta^n f_{-n}$$

$$= f_0 + s\Delta f_{-1} + \frac{s(s+1)\Delta^2}{2!} f_{-2} + \frac{s(s+1)(s+2)\Delta^3}{3!} f_{-3} + \frac{s(s+1)(s+2)(s+3)\Delta^4}{4!} f_{-4} + \ldots$$

(3.1)

Let $n = \frac{1}{n}$ for $n = 1, 2, 3, \ldots$ and applying the lozenge's principle, we obtained

$$P_n(x_s) = f_0 + \binom{s}{1}\Delta f_{-1} + \binom{s+1}{2}\Delta^2 f_{-1} + \binom{s+1}{3}\Delta^3 f_{-1} + \binom{s+1}{4}\Delta^4 f_{-1} + \binom{s+1}{5}\Delta^5 f_{-1} + \cdots$$

$$= f_0 + s\Delta f_{-1} + \frac{s(s+1)\Delta^2}{2!} f_{-1} + \frac{s(s^2-1)\Delta^3}{3!} f_{-3} + \frac{s(s^2-1)(s-2)\Delta^4}{4!} f_{-4}$$

(3.2)

where equation (3.2) is called Modified Newton Gregory Backward (MNGB)

3.2 GAUSS'S BACKWARD FORMULA

$$P_n(x_s) = f_0 + \binom{s}{1}\Delta f_{-1} + \binom{s+1}{2}\Delta^2 f_{-1} + \binom{s+1}{3}\Delta^3 f_{-2} + \binom{s+1}{4}\Delta^4 f_{-2} + \cdots + \binom{s+\left[\frac{n}{2}\right]}{n}\Delta^n f_{-n}$$

$$= f_0 + s\Delta f_{-1} + \frac{s(s+1)\Delta^2}{2!} f_{-1} + \frac{s(s^2+1)(s+2)\Delta^3}{3!} f_{-3} + \frac{s(s^2+1)(s+2)\Delta^4}{4!} f_{-2} +$$

(3.3)

Advancing the subscript of (3.3) one unit and replace s by $(s-1)$ we have:

$$P_n(x_s) = f_1 + (s-1)\Delta f_0 + \frac{s(s-1)\Delta^2}{2!} f_0 + \frac{s(s-1)(s-2)\Delta^3}{3!} f_{-1} + \frac{s(s^2-1)(s-2)\Delta^4}{4!} f_{-1}$$

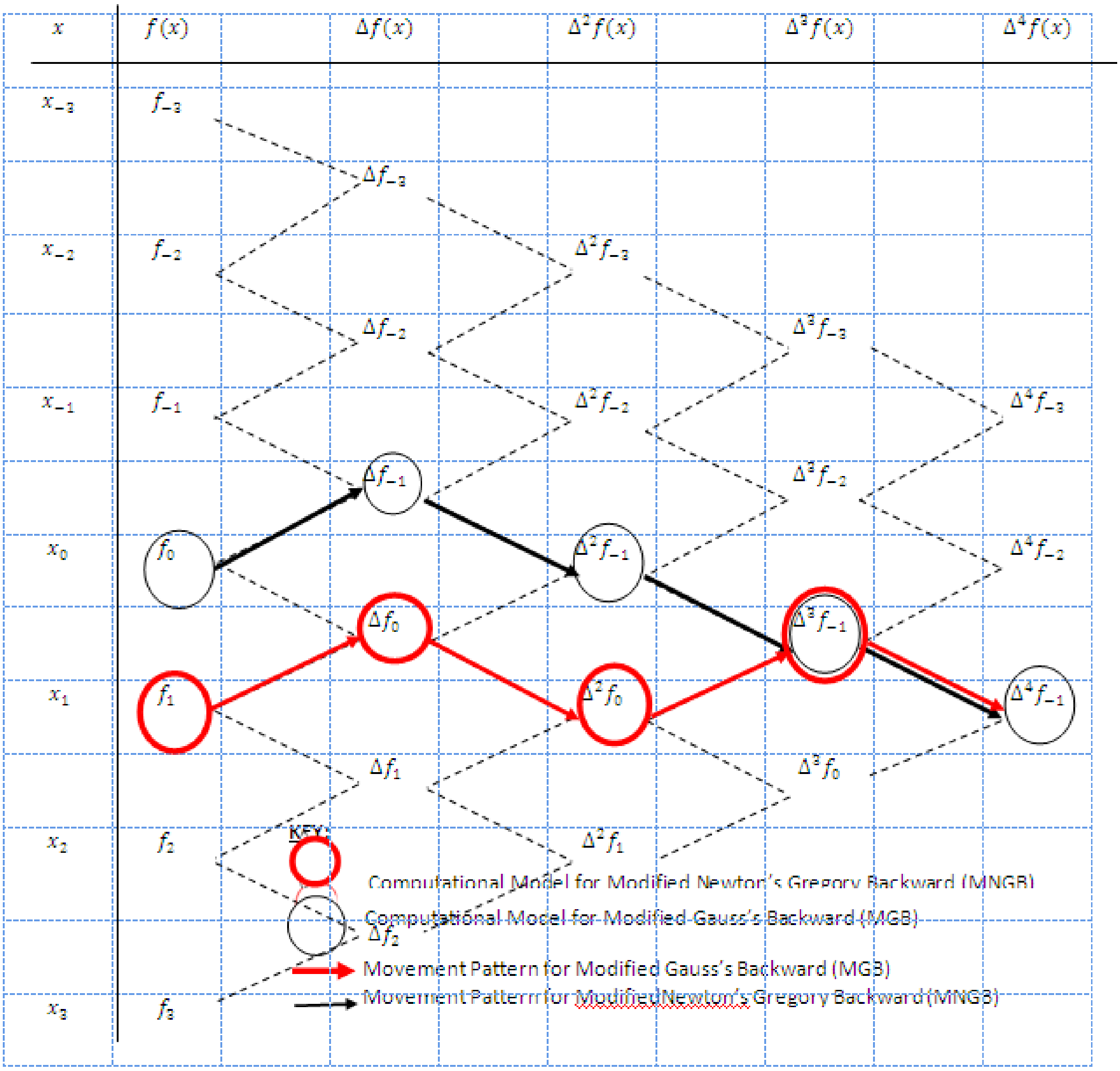
Note that equation (3.4) can be referred to as Modified Gauss's Backward Formula (MGBF)

3.3 NEWLY DERIVED METHOD

Taking the mean of equation (3.2) and (3.4) we obtain:

$$P_n(x_s) = \frac{1}{2}(f_0 + f_1) + \frac{s(\Delta f_{-1}\Delta f_0) - \Delta f_0}{2} + \frac{s^2(\Delta^2 f_{-1} + \Delta^2 f_0) + s(\Delta^2 f_{-1} - \Delta^2 f_0)}{4} + \frac{s(s-1)(2s-1)\Delta^3 f_{-1}}{16} + \frac{s(s^2-1)(s-2)\Delta^4 f_{-1}}{24}$$

Table 1: Combination of MNGB and MGB



IV. IMPLEMENTATION OF THE METHODS

We seek solution to problems 4.1 – 4.5 using the following Methods:

i. NGB Formula
ii. Gauss backward
iii. Stirling's interpolation formula
iv. Bessel's interpolation formula
v. Laplace –Everett's formula
vi. The New method

Problem 4.1: Given the following table values of e^{-x}, find the value of $e^{-1.9}$

x	1.0	1.25	1.50	1.75	2.00	2.25	2.50
f(x)	0.3679	0.2865	0.2231	0.1738	0.153	0.1054	0.0821

NEWTON'S GREGORY BACKWARD (NGB)

$$P_n(x_s) = f_0 + s\Delta f_{-1} + \frac{s(s+1)\Delta^2}{2!}f_{-2} + \frac{s(s+1)(s+2)\Delta^3}{3!}f_{-3} + \frac{s(s+1)(s+2)(s+3)\Delta^4}{4!}f_{-4} + \cdots$$

where, $s = \frac{x-x_n}{h} = \frac{1.9-2.50}{0.25} = -2.4, x = 1.9$

$P_4(1.9) =$

$0.0821 + (-2.4)(-0.0233) + \frac{(-2.4)(-1.4)(0.0066)}{2!} + \frac{(-2.4)(-1.4)(-0.4)(-0.0020)}{3!} + \frac{(-2.4)(-1.4)(-0.4)(0.6)(0.0002)}{4!} =$
0.1495672

GAUSS'S BACKWARD (GB)

$P_n(x_s) = f_0 + s\Delta f_{-1} + \frac{s(s+1)\Delta^2}{2!}f_{-1} + \frac{s(s^2-1)(s+2)\Delta^3}{3!}f_{-3} + \frac{s(s^2-1)(s+2)\Delta^4}{4!}f_{-2} + \ldots$

$$P_4(1.9) = 0.2231 + (1.6)(0.0634) + \frac{1.6(2.6)(0.0141)}{2!} + \frac{1.6(1.56)(-0.0039)}{3!} + \frac{1.6(1.56)(3.6)(0.0006)}{4!}$$
$$= 0.14959024$$

STIRLING'S INTERPOLATION FORMULA

$$P_n(x_s) = f_0 + (\Delta f_{-1} + \Delta f_0) + \frac{S^2\Delta^2 f_{-1}}{2!} + \frac{S(S^2-1)}{3!}\frac{(\Delta^3 f_{-2} + \Delta^3 f_{-1})}{2} + \frac{S^2(S^2-1)\Delta^4 f_{-2}}{4!} + \cdots$$

$$P_4(1.9) = 0.2231 + \frac{(1.6)(-0.01127)}{2} + \frac{2.56(0.0141)}{2!} + \frac{1.6(1.56)}{3!}\frac{(-0.0072)}{2} + \frac{2.56(1.56)(0.0006)}{4!}$$
$$= 0.14959024$$

BESSEL INTERPOLATION FORMULA

$$P_n(x_s) = \frac{1}{2}(f_0 + f_1) + \left(S - \frac{1}{2}\right)\Delta f_0 + \frac{S(S-1)}{2!}\frac{(\Delta^2 f_{-1} + \Delta^2 f_0)}{2} + \frac{S(S-\frac{1}{2})\Delta^3 f_{-1}}{3!} + \frac{S(S-1)(S-2)}{4!}\frac{(\Delta^4 f_{-2} + \Delta^4 f_{-1})}{2}$$

$$P_4(1.9) = \frac{1}{2}(0.2231 + 0.1738) + 1.1(-0.0493) + \frac{1.6(0.6)}{2!}\frac{0.0141 + 0.0108)}{2} + \frac{1.6(0.6) - 0.0033)(1.1)}{3!} + \frac{1.6(1.56)(-0.4)}{4!} + \frac{(0.0006 + 0.0011)}{2} = 0.14957984$$

LAPLACE-EVERETT'S FORMULA

$$P_n(x_s) = \left[vf_0 + \frac{(v^2-1)\Delta^2 f_{-1}}{3!} + \frac{v(v^2-1)(v^2-2^2)\Delta^4 f_{-2}}{5!}\right] + \left[sf_1 + \frac{s(s^2-1)\Delta^2 f_0}{3!} + \frac{s(s^2-1)(s^2-2^2)\Delta^4 f_{-1}}{5!}\right]$$

$$P_4(1.9) = [(-0.6)(0.2231) + \frac{(-0.04)(-0.6)(0.0141)}{3!} + \frac{1.6(1.56)(-1.44)(0.0011)}{5!} = 0.149575264$$

THE NEW METHOD

$$P_n(x_s) = \frac{1}{2}(f_0 + f_1) + \frac{s(\Delta f_{-1} + \Delta f_0) - \Delta f_0}{2} + \frac{s^2(\Delta^2 f_{-1} + \Delta^2 f_0) + s(\Delta^2 f_{-1} - \Delta^2 f_0)}{4} + \frac{s(s-1)(2s-1)\Delta^3 f_{-1}}{3!} + \frac{s(s^2-1)(s-2)\Delta^4 f_{-1}}{24!}$$

$P_4(1.9) =$
$\frac{1}{2}(0.2231 + 0.1738) + \frac{1.6[(-0.0634)(-0.0493)]-(-0.0493)}{2} + \frac{2.56(0.0141+0.0108)+1.6(0.0141-0.0108)}{4} +$
$\frac{1.6(0.6)(2.2)(-0.0033)}{12} + \frac{1.6(1.56)(-0.4)(-0.0011)}{24} = 0.14956944$

Problem 4.2: For the following table of values, estimate $f(7.5)$

x	1	2	3	4	5	6	7	8
$f(x)$	1	8	27	64	125	216	343	512

Table 2: Difference table for Problem 4.2

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1	1				
		7			
2	8		12		
		19		6	
3	27		18		0
		37		6	
4	64		24		0
		61		6	
5	125		30		0
		91		6	
6	216		36		0
		127		6	
7	343		42		
		169			
8	512				

NEWTON GREGORY BACKWARD (NGB)

$$P_n(x_s) = f_0 + s\Delta f_{-1} + \frac{s(s+1)\Delta^2}{2!}f_{-2} + \frac{s(s+1)(s+2)\Delta^3}{3!}f_{-3} + \frac{s(s+1)(s+2)(s+3)\Delta^4}{4!}f_{-4} +$$

where, $s = \frac{x-x_n}{h} = \frac{7.5-8}{1} = 0.5$

$$P_n(7.5) = 512 + (-0.5)(169) + \frac{(-0.5)(0.5)(42)}{2!} + \frac{(-0.5)(0.5)(1.5)(6)}{3!} + \frac{(-0.5)(0.5)(1.5)(2.5)(0)}{4!}$$
$$= 421.875$$

GAUSS'S BACKWARD (GB)

$P_n(x) = f_0 + s\Delta f_{-1} + \frac{s(s+1)\Delta^2}{2!}f_{-1} + \frac{s(s^2-1)(s+2)\Delta^3}{3!}f_{-3} + \frac{s(s^2-1)(s+2)\Delta^4}{4!}f_{-2} +$

$$P_4(7.5) = 6 + 3.5(37) + \frac{3.5(4.5)(24)}{2!} + \frac{3.5(11.25)(6)}{3!} + \frac{3.5(11.25)(1.5)(0)}{4!} = 421.875$$

STIRLING'S INTERPOLATION FORMULA

$$P_n(x) = f_0 + (\Delta f_{-1} + \Delta f_0) + \frac{s^2\Delta^2 f_{-1}}{2!} + \frac{s(s^2-1)}{3!}\frac{(\Delta^3 f_{-2} + \Delta^3 f_{-1})}{2} + \frac{s^2(s^2-1)\Delta^4 f_{-2}}{4!} +$$

$$P_4(7.5) = 64 + \frac{3.5(37+61)}{2} + \frac{12.25(24)}{2!} + \frac{3.5(11.25)}{3!}\frac{(6+6)}{2} + \frac{3.5(11.25)(0)}{4!} = 421.875$$

BESSEL INTERPOLATION FORMULA

$$P_n(x_s) = \frac{1}{2}(f_0 + f_1) + \left(s - \frac{1}{2}\right)\Delta f_0 + \frac{S(S-1)}{2!}\frac{(\Delta^2 f_{-1} + \Delta^2 f_0)}{2} + \frac{S(S-\frac{1}{2})\Delta^3 f_{-1}}{3!}$$
$$+ \frac{S(S-1)(S-2)}{4!}\frac{(\Delta^4 f_{-2} + \Delta^4 f_{-1})}{2} +$$

$$P_4(7.5) = \frac{(64+125)}{2} + 3(61) + \frac{3.5(2.5)}{2!}\frac{(24+30)}{2} + \frac{3.5(3)(2.5)(6)}{3!} = 421.815$$

LAPLACE-EVERETT'S FORMULA

$$P_n(x_s) = \left[vf_0 + \frac{(v^2-1)\Delta^2 f_{-1}}{3!} + \frac{v(v^2-1)(v^2-2^2)\Delta^4 f_{-2}}{5!}\right] + \left[sf_1 + \frac{s(s^2-1)\Delta^2 f_0}{3!} + \frac{s(s^2-1)(s^2-2^2)\Delta^4 f_{-1}}{5!}\right]$$

$$P_4(7.5) = \left[(-2.5)(64) + \frac{(5.25)(-2.5)(24)}{3!} + \frac{(-2.5)(5.25)(5.25)(0)}{4!}\right] + \left[3.5(125) + \frac{11.25(3.5)(30)}{3!} + \frac{3.5(11.25)(8.25)(0)}{5!}\right]$$

$$= 421.875$$

THE NEW METHOD

$$P_n(x_s) = \frac{1}{2}(f_0 + f_1) + \frac{s(\Delta f_{-1}+\Delta f_0)-\Delta f_0}{2} + \frac{S^2(\Delta^2 f_{-1}+\Delta^2 f_0)+s(\Delta^2 f_{-1}-\Delta^2 f_0)}{4} + \frac{S(S-1)(2S-1)\Delta^3 f_{-1}}{3!} + \frac{S(S^2-1)(S-2)\Delta^4 f_{-1}}{24!}$$

$$P_4(7.5) = \frac{1}{2}(64+125) + \frac{3,5(37+61)-61}{2} + \frac{12.25(24+30)+3.5(24-30)}{4} + \frac{3.52.5(6)(6)}{12} + \frac{3.5(11.25)(1.5)(0)}{24} = 421.875$$

Problem 4.3: In the given table, estimate the value of $y = log_{10}337.5$

X	310	320	330	3340	350	360
$f(x)$	2.4913617	2.5051500	2.585139	2.5314789	2.544068	2.5563025

Table 3: Difference table for Problem 4.3

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
310	2.4913617				
		0.0137883			
320	2.5051500		-0.0004244		
		0.0133639		0.0000255	
330	2.585139		-0.0003989		-0.0000025
		0.012965		0.000023	
3340	2.5314739		-0.0003759		-0.00000017
		0.0135891		0.0000213	
350	2.544068		-0.0003546		
		0.0122345			

NEWTON GREGORY BACKWARD (NGB)

$$P_n(x_s) = f_0 + s\Delta f_{-1} + \frac{s(s+1)\Delta^2}{2!} f_{-2} + \frac{s(s+1)(s+2)\Delta^3}{3!} f_{-3} + \frac{s(s+1)(s+2)(s+3)\Delta^4}{4!} f_{-4}$$

where, $s = \frac{x-x_n}{h} = \frac{337.5-360}{10} = -2.25$

$$P_n(337.5) = 2.5563025 + (-2.25)(0.0122345) + \frac{(-2.25)(-1.25)(-0.0003546)}{2!} + \frac{(-2.25)(-0.25)(0.25)(0.0000213)}{3!} + \frac{(-2.25)(-1.25)(-0.25)(0.75))(-0.00000017)}{4!}$$

$$= 2.52827376 \cong 2,5282738$$

GAUSS'S BACKWARD (GB)

$$P_n(x_s) = f_0 + s\Delta f_{-1} + \frac{s(s+1)\Delta^2}{2!} f_{-1} + \frac{s(s^2-1)(s+2)\Delta^3}{3!} f_{-3} + \frac{s(s^2-1)(s+2)\Delta^4}{4!} f_{-2}$$

$$P_4(337.5) = 2.5185139 + 0.75(0.0133639) + \frac{0.75(1.75)(-0.0003989)}{2!} + \frac{0.75(-0.4375)(0.0000255)}{3!} + \frac{0.75(-0.0000025)(-0.4375)}{4!} = 2.528273687 \cong 2.5282737$$

STIRLING'S INTERPOLATION FORMULA

$$P_n(x_s) = f_0 + s(\Delta f_{-1} + \Delta f_0) + \frac{S^2\Delta^2 f_{-1}}{2!} + \frac{S(S^2-1)}{3!}\frac{(\Delta^3 f_{-2} + \Delta^3 f_{-1})}{2} + \frac{S^2(S^2-1)\Delta^4 f_{-2}}{4!}$$

$$P_4(1979) = 2.5185139 + \frac{0.75(0.0133639 + 0.012965)}{2} + \frac{0.5625(-0.0003989)}{2!} + \frac{0.75(-0.4375)}{3!}\frac{(0.0000255 + 0.00023)}{2} + \frac{0.5625(-0.4375)(-0.0000025)}{4!} = 2.5282737$$

BESSEL INTERPOLATION FORMULA

$$P_n(x_s) = \frac{1}{2}(f_0 + f_1) + \left(S - \frac{1}{2}\right)\Delta f_0 + \frac{S(S-1)}{2!}\frac{(\Delta^2 f_{-1} + \Delta^2 f_0)}{2} + \frac{S(S-\frac{1}{2})\Delta^3 f_{-1}}{3!} + \frac{S(S-1)(S-2)}{4!}\frac{(\Delta^4 f_{-2} + \Delta^4 f_{-1})}{2}$$

$$P_4(337.5) = \frac{(2.5185139 + 2.5314739)}{2} + 0.25(0.012965) + \frac{0.75(-0.25)}{2!}\frac{[(-0.0003989) + (-0.0003759)]}{2} + \frac{0.75(0.25)(-0.25)(0.000023)}{3!} + \frac{0.5625(-0.4375)(1.25)}{4!}\frac{[(-0.0000025) + (0.0000017)]}{2} = 25282737$$

LAPLACE-EVERETT'S FORMULA

$$P_n(x_s) = \left[vf_0 + \frac{(v^2-1)\Delta^2 f_{-1}}{3!} + \frac{v(v^2-1)(v^2-2^2)\Delta^4 f_{-2}}{5!}\right] + \left[sf_1 + \frac{s(s^2-1)\Delta^2 f_0}{3!} + \frac{s(s^2-1)(s^2-2^2)\Delta^4 f_{-1}}{5!}\right]$$

$$P_4(337.5) = \left[0.25(2.5185139) + \frac{(-0.9375)(0.25)(-0.0003989)}{3!} + \frac{0.25(-0.9375)(-3.9375)(-0.0000025)}{5!}\right] + \left[0.75(2.5314789) + \frac{(0.4375(0.75)(-0.0003759)}{3!} + \frac{0.75(-0.4375(-3.4375)(-0.0000017)}{5!}\right] = 1.528273754$$

THE NEW METHOD

$$P_n(x_s) = \frac{1}{2}(f_0 + f_1) + \frac{s(\Delta f_{-1} + \Delta f_0) - \Delta f_0}{2} + \frac{S^2(\Delta^2 f_{-1} + \Delta^2 f_0) + s(\Delta^2 f_{-1} - \Delta^2 f_0)}{4} + \frac{S(S-1)(2S-1)\Delta^3 f_{-1}}{3!} + \frac{S(S^2-1)(S-2)\Delta^4 f_{-1}}{24!} \text{...(vi)}$$

$$P_4(337.5) = \frac{1}{2}(2.5185139 + 2.5314789) + \frac{0,75(0.0133639 + 0.012965) - 0.012965}{2} + \frac{0.75[(-0.0003989) + (-0.0003759)]}{4} + \frac{0.75(-0,25)(0.5)(0.000023)}{12} + \frac{0.75(-0.4375)(-1.25)(0.0000017)}{24} 2.5282738 \cong 2.5282738$$

Problem 4.4: From the following data, find out the value of sin 45^0

X	20	30	40	50	60	70
$f(x)$	0.34202	0.5	0.64279	0.76604	0.86603	0.93969

Table 4: Difference table for Problem 4.4

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
20	0.34202				
		0.15798			
30	0.5		-0.01519		
		0.14279		0.00435	
40	0.64279		-0.01954		-0.00063
		0.12325		0.00372	
50	0.76604		-0.02326		-0.00065
		0.09999		0.00307	
60	0.86603		-0.02633		
		0.07366			
70	0.93969				

NEWTON GREGORY BACKWARD (NGB)

$$P_n(x_s) = f_0 + s\Delta f_{-1} + \frac{s(s+1)\Delta^2}{2!} f_{-2} + \frac{s(s+1)(s+2)\Delta^3}{3!} f_{-3} + \frac{s(s+1)(s+2)(s+3)\Delta^4}{4!} f_{-4}$$

where, $s = \frac{x-x_n}{h} = \frac{45-70}{10} = -2.5, x = 45$

$$P_4(45) = 0.93969 + (-2.5)(0.07366) + \frac{(-2.5)(-1.5)(-0.02633)}{2!} + \frac{(-2.5)(-1.5)(-0.5)(0.00307)}{3!} + \frac{(-2.5)(-1.5)(-0.5)(0.5)(-0.00065)}{4!} = 0.7071052344 \cong 0.7071052$$

GAUSS'S BACKWARD (GB)

$$P_n(x_s) = f_0 + s\Delta f_{-1} + \frac{s(s+1)\Delta^2}{2!} f_{-1} + \frac{s(s^2-1)(s+2)\Delta^3}{3!} f_{-3} + \frac{s(s^2-1)(s+2)\Delta^4}{4!} f_{-2}$$

$$P_4(45) = 0.64279 + 0.5(0.14279) + \frac{0.5(1.5)(-0.01954)}{2!} + \frac{0.5(-0.75)(0.00435)}{3!} + \frac{0.5(-0.75)(2.5)(0.00063)}{4!} = 0.7071047656 \cong 0.7071048$$

STIRLING'S INTERPOLATION FORMULA

$$P_n(x_s) = f_0 + s(\Delta f_{-1} + \Delta f_0) + \frac{S^2\Delta^2 f_{-1}}{2!} + \frac{S(S^2-1)}{3!}\frac{(\Delta^3 f_{-2} + \Delta^3 f_{-1})}{2} + \frac{S^2(S^2-1)\Delta^4 f_{-2}}{4!}$$

$$P_4(45) = 0.64279 + \frac{0.5(0.14279 + 0.12325)}{2} + \frac{0.25(-0.01954)}{2!} + \frac{0.5(-0.75)}{3!}\frac{(0.00435 + -0.00372)}{2} + \frac{0.25(-0.75)(0.00063)}{4!} = 0.7071047656 \cong 0.7071048$$

BESSEL INTERPOLATION FORMULA

$$P_n(xs) = \frac{1}{2}(f_0 + f_1) + \left(S - \frac{1}{2}\right)\Delta f_0 + \frac{S(S-1)}{2!}\frac{(\Delta^2 f_{-1} + \Delta^2 f_0)}{2} + \frac{S(S-\frac{1}{2})\Delta^3 f_{-1}}{3!} + \frac{S(S-1)(S-2)}{4!}\frac{(\Delta^4 f_{-2} + \Delta^4 f_{-1})}{2}$$

$$P_4(45) = \frac{(0.64279 + 0.76604)}{2} + \frac{0.5(-0.5)}{2!}\frac{[(-0.01954) + (0.02326)]}{2} + \frac{0.5(0)(-0.5)(-0.00372)}{3!} + \frac{0.5(-0.75)(-1.5)}{4!}\frac{90.00063 + 0.00065}{2} = 0.707105$$

LAPLACE-EVERETT'S FORMULA

$$P_n(x_s) = \left[vf_0 + \frac{(v^2-1)\Delta^2 f_{-1}}{3!} + \frac{v(v^2-1)(v^2-2^2)\Delta^4 f_{-2}}{5!}\right] + \left[sf_1 + \frac{s(s^2-1)\Delta^2 f_0}{3!} + \frac{s(s^2-1)(s^2-2^2)\Delta^4 f_{-1}}{5!}\right]$$

$$P_4(45) = \left[0.5(0.64279) + \frac{(-0.75)(0.5)(-0.01954)}{3!} + \frac{0.5(-0.75)(-3.75)(-0.00063)}{5!}\right] + \left[0.5(0.76604) + \frac{(0.75(0.5)(0.02326)}{3!} + \frac{0.5(-0.75)(-3.75)(-0.00065)}{5!}\right] = 0.707105$$

THE NEW METHOD

$$P_n(x_s) = \frac{1}{2}(f_0 + f_1) + \frac{s(\Delta f_{-1}+\Delta f_0)-\Delta f_0}{2} + \frac{S^2(\Delta^2 f_{-1}+\Delta^2 f_0)+s(\Delta^2 f_{-1}-\Delta^2 f_0)}{4} + \frac{S(S-1)(2S-1)\Delta^3 f_{-1}}{3!} + \frac{S(S^2-1)(S-2)\Delta^4 f_{-1}}{24!}$$

$$P_4(45) = \frac{1}{2}(0.64279 + 0.76604) + \frac{0.5(0.14279 + 0.12325) - (0.12325)}{2} + \frac{0.25[(-0.01954) + (0.02326)] + 0.5[(-0.01954) - (0.02326)]}{4} + \frac{0.5(0.5)(0)(-0.000372)}{12} + \frac{0.5(-0.75)(-1.5)(0.00065)}{24} = 0.7071052$$

Exact Solution: $sin\ 45^o = 0.7071068$

V. RESULTS

Table 5: Computational Results of the methods

Problems	Exact Solutions	Newton's Gregory Backward	Gauss's Backward	Stirling Method	Bessel's Method	Laplace-Everett's Formula	New Method
4.1.1	0.14956862	0.1495672	0.14959025	0.14959024	0.14957984	0.14957264	0.14956944
4.1.2	421.875	421.875	421.875	421.875	421.875	421.875	421.875
4.1.3	2.5282738	2.5282738	2.5282737	2.5282737	2.5282737	2.5282738	2.5282738
4.1.4	0.7071068	0.7071052	0.7071048	0.7071048	0.707105	0.707105	0.7071052

Table 6: Absolute Errors of the computed results

Problems	Newton's Gregory Backward	Gauss's Backward	Stirling Interpolation	Bessel's Interpolation	Laplace-Everett's Formula	New Method
4.1.1	1.4×10^{-6}	-2.182×10^{-5}	-2.182×10^{-6}	1.12×10^{-5}	-6.644×10^{-6}	-8.2×10^{-6}
4.1.2	0	0	0	0	0	0
4.1.4	0	1×10^{-7}	1×10^{-7}	1×10^{-7}	0	0
4.1.5	1.6×10^{-6}	2×10^{-6}	2×10^{-6}	1.8×10^{-6}	1.8×10^{-6}	1.6×10^{-6}

VI. CONCLUSION

A new interpolation method was developed through the combination of both modified Newton Gregory backward (MNGB) and modified Gauss's backward formula (MGBF). The new method was derived simply by computing the mean of two methods mention above, the newly developed formula was then applied to a variety of real life problems and on comparison with other existing methods (table 5), results from our method shows that the method is very reliable and accurate (table 6).

REFERENCES

[1]. Abramowitz, M. a. (1972). Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, 9th printing. New York: Dover.

[2]. Conte SD, C. d. (1980). . Elementary Numerical Analysis. New York, USA: McGraw-Hill.

[3]. E.Atkinson, K. (1989). An Introduction to Numerical Analysis, (Vol. 2 Ed). New: John Wiley & Sons.

[4]. Gupta, R. C. (1969). Second Order Interpolation in Indian Mathematics up to the fifteenth century. Ind. J. Hist. Sci.

[5]. Kahaner, D. C. (1989). Numerical Methods and Software. Englewood Cliffs, NJ: Prentice Hall.

[6]. Kaw, A. (2009, December 23). INTRO NUMERICAL METHODS. Retrieved from Canvas: http://numericalmethods.eng.usf.edu

[7]. Robert J Schilling, S. L. (2000). Applied Numerical Methods for Engineers. Brooks /Cole, Pacific Grove, CA.

[8]. Scarborough, J. B. (1966). Numerical Mathematical Analysis (Vol. 6 Ed). USA: The John Hopkins Press.

[9]. Sri.Nandakumar M. (2011). Numerical Methods. Malappuram Kerala, India : UNIVERSITY OF CALICUT.

[10]. Waring, E. (1779). Problems concerning interpolations. London: . Philos. Trans. R. Soc.

[11]. Whittaker, E. T. (1967). The Gregory-Newton Formula of Interpolation" and "An Alternative Form of the Gregory-Newton Formula." (Vol. 4th). New York.