Quest Journals
Journal of Research in Applied Mathematics
Volume 7 ~ Issue 8 (2021) pp: 17-19
ISSN(Online) : 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

Trivial – Zero of Riemann zeta function

Shivprakash singh

ABSTRACT:
Here we will see a new formula for Riemann zeta function ζ(s) which have trivial-zeros= -2,-4,-6,-8,-10,-12,… in Euler formula trivial-zero of zeta function depend on Bernoulli number but in this paper we can see that trivial-zero of zeta function doesn't depend on Bernoulli number.

Received 02 August, 2021; Revised: 12 August, 2021; Accepted 16 August, 2021

Euler –Riemann zeta function defined as

$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = 1^{-s} + 2^{-s} + 3^{-s} + 4^{-s} + \ldots$ where s is any complex number

Euler (1707-1783) find this sum in form of prime product formula

$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = 1^{-s} + 2^{-s} + 3^{-s} + 4^{-s} + \ldots = \prod_{p} [\mathbf{1}/(\mathbf{1} - p^{-s})]$; p= prime

Bernhard Riemann (1826-1866) gave following functional equation for zeta function
For: Re(s)<1

$\zeta(s) = 2^{s} \pi^{s-1} \text{Sin}(\pi s/2)\Gamma(1-s)\zeta(1-s)$

Euler formula in form of Bernoulli number

$\boldsymbol{\zeta(2n) = [(-1)^{n+1}(2\pi)^{-2n} B_{2n}]/[\,2(2n)!]}$ **for n=0,1,2,3,…**

$\boldsymbol{\zeta}(\mathbf{n}) = [B_{1-n}^{+}]/[n-1]$ for n=0,-1,-2,-3,…

Here zeta function can be written as

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = \sum_{k=1}^{\infty} (2k)^{-s} + \sum_{k=1}^{\infty} (2k-1)^{-s}$$

$$\zeta(s)(1-2^{-s}) = \sum_{k=1}^{\infty} (2k-1)^{-s}$$

Now using Gama function

$$\Gamma(s) = \int_0^{\infty} e^{-t} t^{s-1}\, dt$$

Put

t=(2k-1)x Limit t→0 ⇒ Limit x→0

Limit t→∞ ⇒ Limit x→∞

$$\Gamma(s)(2k-1)^{-s} = \int_0^\infty e^{-(2k-1)x} x^{s-1}\, dx$$

Taking summation over k both sides

$$\sum_{k=1}^{\infty} \Gamma(s)\ (2k-1)^{-s} = \sum_{k=1}^{\infty} \int_0^\infty e^{-(2k-1)x} x^{s-1}\, dx$$

We get

$$\zeta(s)(1-2^{-s})\ \Gamma(s) = \int_0^\infty [\frac{1}{e^x - e^{-x}}] x^{s-1}\, dx$$

$$\zeta(s)(1-2^{-s})\ \Gamma(s) = 1/2 \int_0^\infty [cosechx] x^{s-1}\, dx$$

Expanding $cosechx$ in form of Bernoulli number we get

$$\zeta(s)(1-2^{-s})\ \Gamma(s) = 1/2 \int_0^\infty [\sum_{n=0}^{\infty}\ [\ 2(1 - 2^{2n-1}) B_{2n} x^{2n-1}]/2n!] x^{s-1}\, dx$$

$$\zeta(s)(1-2^{-s})\ \Gamma(s) = \int_0^\infty [\sum_{n=0}^{\infty}\ [(-1)^n\ |(2^{2n-1}-1) B_{2n}|\ x^{2n}]/2n!] x^{s-2}\, dx$$

Using Ramanujan (1887-1920) integral formula
(Ramanujan- master theorem)

If a complex –valued function f(x) has an expansion of the form

$$\mathrm{f(x)} = \sum_{k=0}^{\infty}\ [\varphi(k)\ (-x)^k]/k!$$

Then

$$\int_0^\infty [\mathrm{f(x)}] x^{s-1}\, dx = \Gamma(s)\varphi(-s)$$

Corollary:[2]-Ramanujan-note book

$$\int_0^\infty [\varphi(0) - \frac{\varphi(2)x^2}{2!} + \frac{\varphi(4)x^4}{4!} - \&c] x^{s-1}\, dx = \Gamma(s)\varphi(-s)\cos(\frac{\pi s}{2})$$

Here I am going to use mod value

$$\zeta(s)(1-2^{-s})\ \Gamma(s) = \int_0^\infty [\sum_{n=0}^{\infty}\ [(-1)^n\ |(2^{2n-1}-1) B_{2n}|\ x^{2n}]/2n!] x^{s-2}\, dx$$

$$\varphi(2n) = |(2^{2n-1}-1) B_{2n}|$$

Hence

$$\varphi(n) = |(2^{n-1}-1) B_n|$$

Applying above result we get

$$\zeta(s)(1-2^{-s})\ \Gamma(s) = \Gamma(s-1)\varphi(1-s)\cos(\frac{\pi[s-1]}{2})$$

$$\zeta(s)(1-2^{-s})\ \Gamma(s) = \Gamma(s-1)|(2^{-s}-1) B_{1-s}|\cos(\frac{\pi[s-1]}{2})$$

$$\zeta(s)(1-2^{-s})\ \Gamma(s) = \Gamma(s-1)|(2^{-s}-1) B_{1-s}|\sin(\frac{\pi s}{2})$$

$\Gamma(s)/\Gamma(s-1)$=[(s-1) $\Gamma(s)]/[(s-1)\Gamma(s-1)]= s-1$

Hence

ζ(s)(1-2^{-s})$(s-1)$ = $|(2^{-s}-1)B_{1-s}|\sin(\frac{\pi s}{2})$

ζ(s)(1-2^{-s})$(s-1)$ = $|(2^{-s}-1||B_{1-s}|\sin(\frac{\pi s}{2})$

For negative integral value of s

$$|(2^{-s}-1)| = 2^{-s}-1$$

Hence

ζ(s) = $[|\boldsymbol{B_{1-s}}|\boldsymbol{sin}\left(\frac{\pi s}{2}\right)]/[\mathbf{1}-\boldsymbol{s}]$

This is Riemann zeta function for negative integral value of s.

For example;
Put s=-1 in above formula

We get
ζ(-1) = $[|\mathbf{B_2}|\mathbf{sin}\left(\frac{-\pi}{2}\right)]/[\mathbf{2}]$; $\mathbf{B_2} = \mathbf{1/6}$

ζ(-1)= -1/12

Calculating other values of zeta-function
ζ(-2)= 0

ζ(-3)= 1/120

ζ(-4)= 0

&c.

REFERENCES:

[1]. Ramanujan note-book.
[2]. Abramowitz,M.;Stegun,I.A. (1972),Bernoulli and Euler polynomials and the Euler-Maclaurin Formula";Handbook Mathematical Functions with formulas,Graph, and Mathematical Tables (9th printing ed.), New York:Dover Publications, pp.804-806.
[3]. Arfken,George (1970).Mathematical method for physicists(2nd ed.).Academic Press.ISBN 978-0120598519.

*Corresponding Author: Shivprakash singh