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Research Paper

Five Dimensional Cosmological Models with Cosmic String and Constant Bulk Viscous Fluid

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ABSTRACT
We discuss the field equation in gravity theory for the general class of Lyra Manifold with bulk viscous fluid together with a five dimensional cosmic strings in the presence of cosmological constant Λ is considered. The physical as well as kinematical behaviors of these models are also discussed. There is a finite expansion in the model. The theory of gravity have been investigated by Kaluza-Klien cosmological models,
KEYWORDS: Cosmic String, Bulk Viscous Coefficient, Cosmological Constant & Lyra Manifold

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I. INTRODUCTION

In the present era Five Dimensional cosmological string model based on Kaluza-Klein Theory is a matter of special interest for astrophysicists as it has elegant presentation internal of geometry. General Relativistic cosmological models provide a frame work for the investigation of evolution of the universe. Present cosmology is based on the Lyra Manifold model. The large scale structure of the universe is completely isotropic which is good with the data and large scale structure.

In this paper the five dimensional cosmic string with constant Bulk viscous fluid in Lyra Manifold in the presence of cosmological constant Λ is considered. The physical as well as kinematical behaviors of these models are also discussed. There is a finite expansion in the model. The expansion in the model expand in the universe.

The analog of Einstein's field equations based on Lyra's manifold as proposed by Sen [1957] and Sen and Dunn [1971] are

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2} \varphi_i \varphi_j - \frac{3}{4} g_{ij} \varphi_m \varphi^m + \Lambda g_{ij} = -\chi T_{ij}. \qquad (1)$$

II. FIELD EQUATION AND THE COSMOLOGICAL MODEL

Here we consider spherically Symmetric metric of five dimensional cosmological model in the form

$$ds^2 = dt^2 - e^{\lambda}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2) - e^{\mu} dy^2 \qquad (2)$$

where λ and μ are the function of time coordinate only.

We assume here that the coordinates to be commoving

$$\text{i.e.,} \quad u^0 = 1 \text{ and } u^1 = u^2 = u^3 = u^4 = 0. \qquad (3)$$

Further we consider the displacement vector ϕ_i in the form

$$\phi_i = (\beta, 0, 0, 0, 0) \qquad (4)$$

Where β is constant

The energy momentum tensor T_{ij} for cosmic string with bulk viscosity is

$$T_{ij} = \rho u_i u_j - \lambda_s x_i x_j - \xi\theta (u_i u_j - g_{ij}) \tag{5}$$

where ρ is the particle density, λ_s is the string tension density, ξ is the bulk viscous coefficient, u^i is the five velocity vector, g_{ij} is the covariant fundamental tensor, x^i is the direction of anisotropy of cosmic string satisfying

$$u_i u^i = -x_i x^i = 1 \tag{6}$$

and $u_i x^i = 0$. (7)

Further the expansion scalar is given by

$$\theta = u^{\ell}{}_{;\ell} \tag{8}$$

Using equation (4) (5) and (6), the explicit form of field equation (1) for the line element (2) are obtained as

$$\frac{3\dot{\lambda}^2}{4} + \frac{3}{4}\dot{\lambda}\dot{\mu} - \frac{3}{4}\beta^2 + \Lambda = \chi\rho \tag{9}$$

$$\frac{3\dot{\lambda}^2}{4} + \ddot{\lambda} + \frac{\dot{\lambda}\dot{\mu}}{2} + \frac{\ddot{\mu}}{2} + \frac{\dot{\mu}^2}{4} + \frac{3}{4}\beta^2 - \Lambda = \chi\xi\theta \tag{10}$$

and

$$\frac{3\ddot{\lambda}}{2} + \frac{3\dot{\lambda}^2}{2} + \frac{3}{4}\beta^2 - \Lambda = \chi(\lambda_s + \xi\theta) \tag{11}$$

where over head dot denotes differentiation w.r.t. ‘t’. In the following section we intend to derive the exact solution of the field equations using β (constant) and ξ (constant) and Λ (constant) in order to overcome the difficulties due to non linear nature of the field equation.

III. COSMOLOGICAL SOLUTIONS

Here there are five unknowns viz., $\lambda, \mu, \rho, \theta$ and λ_s involved in three field equations (9), (10) and (11). In order to avoid the insufficiency of field equations for solving five unknowns through three field equations,

we consider $\mu = a\lambda$ (12)

where $a(\neq 0)$ is a parameter.

3.1 Case I : cloud string ($\rho + \lambda_s = 0$)

The equation of state is given by

$$\rho + \lambda_s = 0 . \tag{13}$$

Solving the equations (9) and (10), we get

$$\exp(\lambda) = \exp\left[\sqrt{\frac{4\Lambda - 3\beta^2}{3(a+1)}}\left\{t - \frac{2}{A}\log(1 - C_1 e^{At})\right\}\right] \tag{14}$$

$$\text{and } \exp(\mu) = \exp\left[\sqrt{\frac{4\Lambda - 3\beta^2}{3(a+1)}}\left\{t - \frac{2}{A}\log(1 - C_1 e^{At})\right\}\right] \tag{15}$$

$$\text{where } A = \frac{\beta}{(a-1)}\sqrt{3(a+2)(3-a)}\,, \tag{16}$$

$$\rho = \frac{4\Lambda - 3\beta^2}{4\chi}\left(\frac{1 + C_1 e^{At}}{1 - C_1 e^{At}}\right)^2 - \frac{3}{4\chi}\beta^2 - \frac{\Lambda}{\chi}, \tag{17}$$

$$\lambda_s = \frac{3\beta^2}{4\chi} + \frac{\Lambda}{\chi} - \frac{\left(4\Lambda - 3\beta^2\right)}{4\chi}\left(\frac{1 + C_1 e^{At}}{1 - C_1 e^{At}}\right)^2 \tag{18}$$

and $\theta = \left(\frac{3+a}{2}\right)\dot{\lambda} = \left(\frac{3+a}{2}\right)\sqrt{\frac{4\Lambda - 3\beta^2}{3(a+1)}}\left(\frac{1+C_1e^{At}}{1-C_1e^{At}}\right)^2$. (19)

3.2 Case II: Geometric String $\rho = \lambda_s$ (Letelier [1983])

To obtain explicit solutions of the system these unknowns are required. Therefore we take $\mu = a\lambda,\ a \neq 0$ is a parameter and the equation of state i.e. $\rho = \lambda_s$

Solve the equation 9,10 and 11 then we get

$$\lambda = \sqrt{\frac{4\Lambda - 3\beta^2}{3(a+1)}\left\{t - \frac{2}{A}\log\ (1 - C_1e^{At})\right\}}, \tag{20}$$

$$\mu = a\sqrt{\frac{4A - 3\beta^2}{3(a+1)}\left\{t - \frac{2}{A}\log\ \left(1 - C_1e^{At}\right)\right\}}, \tag{21}$$

$$\rho = \frac{4\Lambda - 3\beta^2}{4\chi}\left(\frac{1+C_1e^{At}}{1-C_1e^{At}}\right) - \frac{3\beta^2}{4} - \frac{\Lambda}{\chi} \tag{22}$$

$$\lambda_3 = \frac{3\beta^2}{4\chi} + \frac{\Lambda}{\chi} - \frac{4\Lambda - 3\beta^2}{4\chi}\left(\frac{1+C_1e^{At}}{1-C_1e^{At}}\right) \tag{23}$$

and $\theta = \left(\frac{3+a}{2}\right)\dot{\lambda} = \left(\frac{3+a}{2}\right)\sqrt{\frac{4\Lambda - 3\beta^2}{3(a+1)}}\left\{\frac{1+C_1e^{At}}{1-C_1e^{At}}\right\}$. (24)

3.3 Case III: P-String $\rho = (1+\omega)\lambda_s$

In this case P-String doesn't exist.

IV. DISCUSSION

Some physical as well as geometrical properties of the models are :-

(a) The anisotropy σ is defined as (Raychaudhuri, 1955)

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{g_{oo,o}}{g_{oo}} - \frac{g_{11,o}}{g_{11}}\right)^2 + \left(\frac{g_{11,o}}{g_{11}} - \frac{g_{22,o}}{g_{22}}\right)^2 + \left(\frac{g_{22,o}}{g_{22}} - \frac{g_{33,o}}{g_{33}}\right)^2 + \left(\frac{g_{33,o}}{g_{33}} - \frac{g_{oo,o}}{g_{oo}}\right)^2\right]$$

i.e., $\sigma^2 = \dot{\lambda}^2$

i.e., $\sigma = \dot{\lambda} = \sqrt{\frac{4\Lambda - 3\beta^2}{3(a+1)}}\left(\frac{1+C_1e^{At}}{1-C_1e^{At}}\right).$

(b) Spatial Volume

$$V = (-g)^{1/2} = \left(-e^{3\lambda}r^4\sin^2\theta\, e^{\mu}\right)^{1/2}$$

$$= -e^{\frac{3}{2}\lambda}\, r^2 \sin\theta\, e^{\mu/2}.$$

where $\lambda = \sqrt{\frac{4\Lambda - 3\beta^2}{3(a+1)}\left[t - \frac{2}{A}\log\ \left(1 - C_1e^{At}\right)\right]}.$

(c) Expansion Scalar

$$\theta = u^{\ell}_{;\ell} = \left(\frac{3+a}{2}\right)\dot{\lambda} = \left(\frac{3+a}{2}\right)\sqrt{\frac{4\Lambda - 3\beta^2}{3(a+1)}}\left(\frac{1+C_1e^{At}}{1-C_1e^{At}}\right).$$

(d) $$\frac{\sigma}{\theta} = \frac{\dot{\lambda}}{\left(\frac{3+a}{2}\right)\dot{\lambda}} = \frac{2}{3+a}.$$

The model does not approaches to isotropy as $\sigma \neq 0$ and $\frac{\sigma}{\theta} = \text{constant}\ (\neq 0)$, for all t

Thus in this case the anisotropy exists throughout the evolution.

when $t \to 0, \sigma = \text{constant}$

when $t \to \infty, \sigma = \text{constant}$

So the shape of the universe does not change during evolution. In case $C_1 = \frac{1}{2}$, $\lim\ t \to \infty\ \sigma = 0$ and the model approaches to isotropy.

(e) The deceleration parameter of q is given

$$q = -3\theta^2\left[\theta;_\alpha \upsilon^\alpha + \frac{1}{3}\theta^2\right]$$

$$= -\frac{3}{4}\left\{(3+a)^2\sqrt{\frac{4\Lambda - 3\beta^2}{3(a+1)}\ \frac{(1+C_1e^{At})^4}{(1+C_1e^{At})^6}}\left[2C_1Ae^{4t} + \frac{(1+C_1e^{At})^2}{3}\right]\right\} \tag{25}$$

for $A > 0,\ C > 0$, the value of the deceleration parameters is negative, which indicates inflation in the model.

V. CONCLUSION

In this paper it is shown that the cosmological model for Takabayasi String with cosmological constant does exist in five dimensional string cosmological model with bulk viscous fluid in Lyra manifold, the equation of state for Takabayasi String viz., $\rho = (1+\omega)\lambda_s$ admits model only when $\omega = 0$ **.** In case of constant bulk viscous coefficient, at the initial epoch $t = 0, \theta$ is finite and θ decreases when t increases and $\theta \to 0$ when $t \to \infty$ and a = **-** 3. Thus there is finite expansion in the model. The matter density $\rho \to \infty$ when t=0 and $C_1 = 1$ and $\rho \to 0$ when $t \to \infty, \chi = 1$. ρ is decreasing function of time. The models stop at infinite time. The model starts with a big-bang at t=0 and the expansion in the model decreases as time increases. Also, the declaration parameter q as given in equation (25) implies an accelerating model of the universe. Recent observations by S. Perlmutter [1998] reveals that the present universe is accelerating phase and deceleration parameter lies somewhere in the range $1 \leq q \leq 0$. It also follows that our model of the universe consistent with recent observations. The parameter q approaches the value zero (when a= -3) as in the case of universe.

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