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Research Paper

Graphical Construction of Pi

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ABSTRACT: *An approximation of π can be found using graphical construction using only a few steps. Since π is transcendental, and no straight line of transcendental length can be constructed with a compass and a straight edge alone, we must use estimates. Two simple line segments are constructed with a compass and a straight edge. The first line segment of length $\frac{22}{7}$, a non-transcendental length, has less than 10^{-1} % error. The second line segment of length $\sqrt{9.8696}$, an irrational number, is more accurate and has less than 10^{-5} % error. A third approximation, $\frac{355}{113}$, is also accurate, but is difficult to graphically implement.*

KEYWORDS: *Graphical Construction of Pi, more accurate Pi, graphically implement Pi*

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I. INTRODUCTION

The ratio of a circle's circumference to its diameter, symbolized by the Greek letter π, is not only one of the most important numbers in mathematics, but is also essential to all scientific and technological areas. Due to its importance, graphical approximations of π are often necessary to serve a variety of purposes. If the number π could be expressed as a rational fraction or as the root of a first or second-degree equation, it would be possible to construct a straight line equal to the circumference of a given circle using a compass and a straight edge. Unfortunately, this is not the case. Although the exact length of π can be found by measuring the circumference of a circle with radius 0.5, this length is not a straight line segment. Due to the proof that π is transcendental and that lines of transcendental lengths cannot be constructed, it is impossible to graphically evaluate π using a straight line.

There are several numbers that are commonly used to approximate π, including $\sqrt{10}$, $\frac{22}{7}$, and $\frac{355}{113}$ [1]. Unlike π, they are non-transcendental and could theoretically be exactly constructed using a compass and a straight edge. There are two factors that can be used to compare these numbers: simplicity of construction and accuracy. The lines of length $\sqrt{10}$ and $\frac{22}{7}$ are easily constructed, but are not a very accurate approximation of π. On the other hand, $\frac{355}{113}$ is a very accurate approximation (less than 10^{-5}% error), but is difficult to construct.

One number that excels in both of these factors, however, is the irrational number $\sqrt{9.8696}$[2]. When graphically constructed using a simple set of steps, its length approximates π with great accuracy.

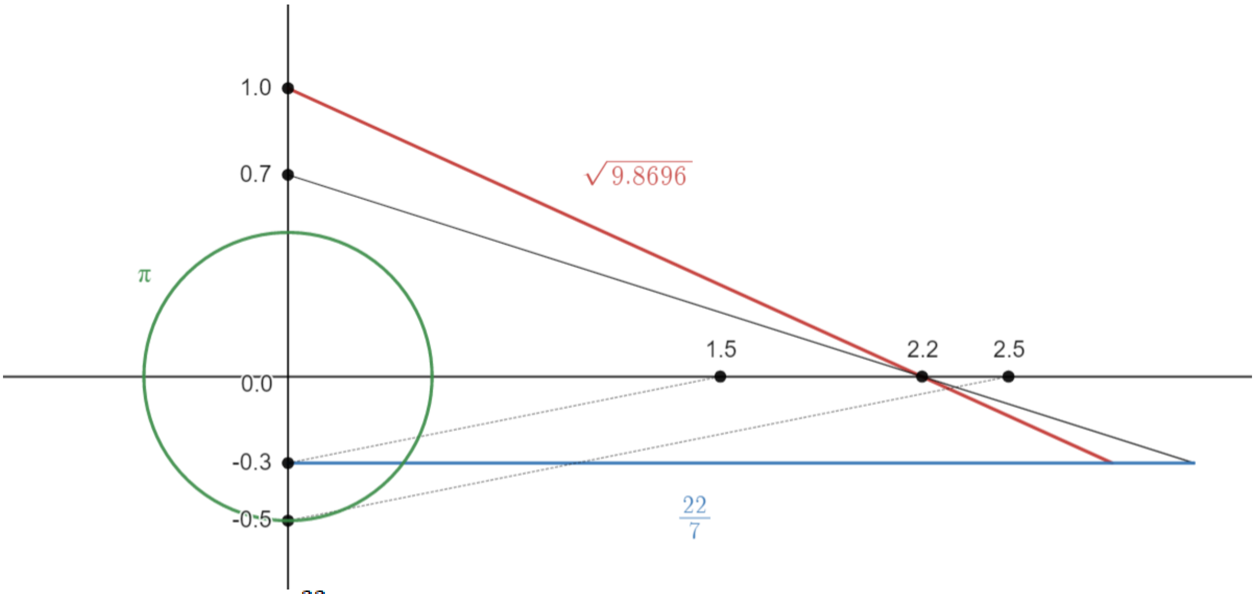
II. FORMULATION

Using the graph drawn below to obtain $\frac{22}{7}$, a line segment of $\sqrt{9.8696}$ can be found.

1) Draw a line from (0, 2.0) through (2.2, 0), and ending at the line $y = -0.3$.

2) The length of this segment is $\sqrt{9.8696}$, found from the following calculation using the Pythagorean theorem:
   a. $\left(\frac{1+0.3}{1}\right) \times \sqrt{1^2 + (2.5 - 0.3)^2} = (1.3) \times \sqrt{5.84}$
   $= \sqrt{9.8696}$

III. GRAPH



To obtain the line segment, $\frac{22}{7}$, refer to the steps below.

1) Draw a circle with a radius of 0.5, centered at (0, 0). The circumference of the circle is equal to π.
2) Find the point (0, 1.0), using a segment of length 1.0 from the origin.
3) Find the point (1.5, 0), using a segment of length 1.0 from the intersection point of the circle and the x-axis.
4) Draw a line, $\bar{B}$, parallel to the line $\bar{A}$. The line crosses through the points (1.5, 0) and (0, -0.3).
5) Find the point (0, 0.7), using a segment of length 0.3 from the point (0, 1.0).
6) Find the point (2.2, 0), using a segment of length 0.3 from the point (2.5, 0).
7) Draw a line from (0, 0.7) through (2.2, 0), and ending at the line $y = -0.3$.
8) Draw a line from (0, -0.3) to the endpoint of the line from the previous step.
9) The length of this segment is $\frac{22}{7}$.

IV. ACCURACY

With $\pi = 3.1415926535\ldots$, the accuracy of the relevant values can be determined:

$$\frac{22}{7} - \pi = +0.0012644892\ldots$$

$$\frac{355}{113} - \pi = +0.0000002668\ldots$$

$$\sqrt{9.8696} - \pi = -0.0000007004\ldots$$

$\sqrt{9.8696}$ and $\frac{355}{113}$ are both better estimations of π than $\frac{22}{7}$, but their accuracy compared to each other are about the same. Despite having similar accuracy, $\frac{355}{113}$ is significantly more difficult to implement than $\sqrt{9.8696}$. The steps used to find $\sqrt{9.8696}$ are simple and easy to replicate, making them a more efficient method to approximate π than using $\frac{355}{113}$.

V. CONCLUSION

The common methods used to approximate π are often tedious and inaccurate. With the methods described in this paper, however, the approximation in the form of a line of length $\sqrt{9.8696}$ can be found with relative ease. The applications of $\sqrt{9.8696}$ are plentiful, and it can be used efficiently for physical graphing and estimating π in a variety of situations. There is an overarching usefulness of the described methods due to their simple replication, therefore making them accessible to everyone.

REFERENCES

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