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Research Paper

Estimation of Population Mean in Poststratified Sampling Schemein the Presence of Measurement Errors

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ABSTRACT: *Measurement errors, if not taken into consideration at the analysis stage, could lead to incorrect results. A number of research works have considered various sampling strategies for the handling of the problem of measurement errors in simple random sampling, systematic sampling and other sampling schemes. However, the focus of the present study is to develop sampling strategies that would address the problem of measurements errors in poststratified sampling scheme. The study intends to utilize auxiliary information in developing the sampling strategies, where there are measurement errors on both the study and auxiliary variables. Based on a poststratified sampling design, the study proposed three (3) separate-type estimators of the population mean of the study variable. First is a customary-type sample mean estimator when there are measurement errors on both the study and auxiliary variables under the poststratified sampling scheme. The second is a difference-type estimator, while the third is a class of ratio/product-type estimators. Both conditional and unconditional properties of the proposed estimators are obtained. Also obtained are the conditions under which the proposed estimators would be more efficient than the usual poststratified sample mean estimator. Again, the best (optimum) estimators among the proposed estimators are also obtained. A numerical illustration was carried out to verify the theoretical results.*
KEYWORDS: *Measurement errors, Auxiliary information, Poststratification, Sampling strategy.*

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I. INTRODUCTION

Most statistical theories are built on the assumption that the observed values in a survey are the same as the true values of the characteristics of interest. In other words, the observed values are assumed to be the same as the true values without making provision for any errors in measurement. However, this is not always the case because there are situations where the observed values might slightly differ from the true values. Consequently, this slight difference between the observed and true values is often referred to as measurement error. Many authors believe that measurement errors should be incorporated in the development of statistical theories. Hence, in the present work, attempts will be made to incorporate measurement errors in developing sampling strategies for parameter estimation in poststratified sampling scheme.

Some authors have considered the problem of measurement errors in surveys under various sampling schemes. Recently, authors like [1] and [2] considered the estimation of the population mean in simple ran- dom sampling scheme when there are measurement errors. Similarly, [3] estimated the population ratio/product, while [4] and [5] estimated the population variance in simple random sampling scheme when there are measure- ment errors. Also, [6] and [7] developed sampling strategies for the handling of measurement errors under the stratified random sampling scheme. However, literature did not reveal any works in the handling of the problem of measurement errors under the poststratified sampling scheme, and this study intends to fill this gap.

II. PROPOSED SAMPLING STRATEGIES

In this section, we develop sampling strategies for handling measurement errors in poststratified sampling scheme, when there are measurement errors on both the study and auxiliary variables. First, we propose the following sampling design:

Consider a population, $\Omega = \{U_1, U_2, \ldots, U_N\}$, of N units, divided into subpopulations or strata of sizes, N_h, $h = 1, 2, \ldots, L$, such that $\sum_{h=1}^{L} N_h = N$. For the purpose of estimating the population mean of a study variable under the poststratified sampling scheme, we use the simple random sampling without replacement (SRSWOR) method to select a sample of n units from the N units in the population and obtain information on the study, auxiliary and stratification variables. We assume that there are measurement errors on both the study and auxiliary variables. Let the pair of values, (y_{hi}, x_{hi}) denote the observed values of the study and auxiliary variables for the i^{th} unit in stratum h, while the pair (Y_{hi}, X_{hi}) denote the corresponding true values. We define "**measurement errors**" under poststratified sampling scheme as the differences between the observed values (y_{hi}, x_{hi}) and the true values (Y_{hi}, X_{hi}). Thus, for the i^{th} population unit in stratum h $(i = 1, 2, \ldots, N_h)$ or the i^{th} sample unit in stratum h $(i = 1, 2, \ldots, n_h)$, the measurement errors on the study and auxiliary variables, under poststratified sampling scheme, are respectively given as

$$U_{hi} = y_{hi} - Y_{hi} \quad \text{and} \quad V_{hi} = x_{hi} - X_{hi} \tag{1}$$

The measurement errors in stratum h, U_{hi} and V_{hi}, are assumed to be stochastic in nature with zero means and stratum population variances, S^2_{uh} and S^2_{vh}, respectively, where

$$S^2_{uh} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h} U^2_{hi} \quad \text{and} \quad S^2_{vh} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h} V^2_{hi} \tag{2}$$

To incorporate the measurement error in the observed value of the study variable, y, using (1), we write the sample mean of the observed values of y in stratum h, in the presence of measurement errors as

$$\bar{y}''_h = \frac{1}{n_h}\sum_{h=1}^{n_h} y_{hi} = \frac{1}{n_h}\sum_{h=1}^{n_h}\left[Y_{hi} + (y_{hi} - Y_{hi})\right] = \frac{1}{n_h}\sum_{h=1}^{n_h} Y_{hi} + \frac{1}{n_h}\sum_{h=1}^{n_h} U_{hi} \tag{3}$$

Similarly, we write the sample mean of the observed values of x in stratum h, in the presence of measurement errors as

$$\bar{x}''_h = \frac{1}{n_h}\sum_{h=1}^{n_h} x_{hi} = \frac{1}{n_h}\sum_{h=1}^{n_h}\left[X_{hi} + (x_{hi} - X_{hi})\right] = \frac{1}{n_h}\sum_{h=1}^{n_h} X_{hi} + \frac{1}{n_h}\sum_{h=1}^{n_h} V_{hi} \tag{4}$$

Based on the above sampling design and using (3) and (4), we propose three estimators of the population mean of the study variable, y, when there are measurement errors on both the study and auxiliary variables under poststratified sampling scheme. The proposed estimators are as follows:

1. The customary sample mean estimator in the presence of measurement errors given by

$$\bar{y}_{ms1} = \sum_{h=1}^{L} W_h \bar{y}''_h \tag{5}$$

2. A measurement error difference-type estimator given by

$$\bar{y}_{md1} = \sum_{h=1}^{L} W_h[\bar{y}''_h - k_h(\bar{x}''_h - \bar{X}_h)] \tag{6}$$

3. A general class of estimators that makes use of known values of some population parameters of the auxiliary variable in the h^{th} stratum when there are measurement errors given by:

$$\bar{y}_{mc1} = \sum_{h=1}^{L} W_h \bar{y}''_h \left(\frac{a_h \bar{X}_h + b_h}{a_h \bar{x}''_h + b_h}\right)^{\alpha_h} \tag{7}$$

where $k_h(\alpha_h)$ are any real numbers and $a_h(b_h)$ are also any real numbers or known values of population parameters of an auxiliary variable, x, in the h^{th} stratum, like coefficient of variation, skewness, kurtosis, etc. The conditional and unconditional properties of the above proposed estimators shall be obtained accordingly, in Section 3.

III. PROPERTIES OF THE PROPOSED ESTIMATORS

The properties of the above proposed three estimators are obtained under the conditional argument, that is, given an achieved sample configuration, say $\underline{n} = (n_1, n_2, \ldots, n_L)$, as well as under the unconditional argument, that is, for repeated samples of fixed size, n. The properties are as shown in Theorem 3.1.

Therorem 3.1: Under the conditional argument, the properties of the proposed three estimators are obtained as follows:

$$V_c(\bar{y}_{ms1}) = \sum_{h=1}^{L} W^2_h\left(\frac{1}{n_h} - \frac{1}{N_h}\right)(S^2_{yh} + S^2_{uh}) \tag{8}$$

$$V_c(\bar{y}_{md1}) = \sum_{h=1}^{L} W_h^2\Big(\frac{1}{n_h} - \frac{1}{N_h}\Big)(S_{yh}^2 + k_h^2 S_{xh}^2 - 2k_h S_{yxh}) + \sum_{h=1}^{L} W_h^2\Big(\frac{1}{n_h} - \frac{1}{N_h}\Big)(S_{uh}^2 + k_h^2 S_{vh}^2) \quad (9)$$

$$MSE_c(\bar{y}_{mc1}) = \sum_{h=1}^{L} W_h^2\Big(\frac{1}{n_h} - \frac{1}{N_h}\Big)(S_{yh}^2 + \alpha_h^2\lambda_h^2 R_h^2 S_{xh}^2 - 2\alpha_h\lambda_h R_h S_{yxh}) + \sum_{h=1}^{L} W_h^2\Big(\frac{1}{n_h} - \frac{1}{N_h}\Big)(S_{uh}^2 + \alpha_h^2\lambda_h^2 R_h^2 S_{vh}^2) \quad (10)$$

Furthermore, under the unconditional argument or for repeated samples of fixed size, n, the properties of the proposed three estimators were obtained up to first order of approximations as:

$$V(\bar{y}_{ms1}) = \Big(\frac{1-f}{n}\Big)\sum_{h=1}^{L} W_h(S_{yh}^2 + S_{uh}^2) \quad (11)$$

$$V(\bar{y}_{md1}) = \Big(\frac{1-f}{n}\Big)\sum_{h=1}^{L} W_h(S_{yh}^2 + k_h^2 S_{xh}^2 - 2k_h S_{yxh}) + \Big(\frac{1-f}{n}\Big)\sum_{h=1}^{L} W_h(S_{uh}^2 + k_h^2 S_{vh}^2) \quad (12)$$

$$MSE(\bar{y}_{mc1}) = \Big(\frac{1-f}{n}\Big)\sum_{h=1}^{L} W_h(S_{yh}^2 + \alpha_h^2\lambda_h^2 R_h^2 S_{xh}^2 - 2\alpha_h\lambda_h R_h S_{yxh}) + \Big(\frac{1-f}{n}\Big)\sum_{h=1}^{L} W_h(S_{uh}^2 + \alpha_h^2\lambda_h^2 R_h^2 S_{vh}^2) \quad (13)$$

where $V_c(V)$ and $MSE_c(MSE)$ respectively denote conditional (unconditional) variance and mean squared error.

Proof: First we obtain the properties of the customary sample mean estimator, $\bar{y}_{ms1}$. The proposed estimator, $\bar{y}_{ms1}$, given in (5), can be written, using (3), as

$$\bar{y}_{ms1} = \sum_{h=1}^{L} W_h\Big[\frac{1}{n_h}\sum_{i=1}^{n_h} Y_{hi} + \frac{1}{n_h}\sum_{i=1}^{n_h} U_{hi}\Big] \quad (14)$$

Therefore, given an achieved sample configuration, say $\underline{n} = (n_1, n_2, \ldots, n_L)$, and using (14), we obtain the conditional expectation (E_c) of $\bar{y}_{ms1}$, as

$$E_c(\bar{y}_{ms1}) = E_c\Big\{\sum_{h=1}^{L} W_h\Big[\frac{1}{n_h}\sum_{i=1}^{n_h} Y_{hi} + \frac{1}{n_h}\sum_{i=1}^{n_h} U_{hi}\Big]\Big\} = \sum_{h=1}^{L} W_h\Big[E_c\Big(\frac{1}{n_h}\sum_{i=1}^{n_h} Y_{hi}\Big) + E_c\Big(\frac{1}{n_h}\sum_{i=1}^{n_h} U_{hi}\Big)\Big] = \sum_{h=1}^{L} W_h\bar{Y}_h = \bar{Y} \quad (15)$$

The expression (15) shows that the estimator, $\bar{y}_{ms1}$ is unbiased for $\bar{Y}$ under the conditional argument. To obtain the conditional variance of the estimator using (14) we have

$$V_c(\bar{y}_{ms1}) = \sum_{h=1}^{L} W_h^2\Big[V_c\Big(\frac{1}{n_h}\sum_{i=1}^{n_h} Y_{hi}\Big) + V_c\Big(\frac{1}{n_h}\sum_{i=1}^{n_h} U_{hi}\Big)\Big] = \sum_{h=1}^{L} W_h^2\Big(\frac{1}{n_h} - \frac{1}{N_h}\Big)[S_{yh}^2 + S_{uh}^2] \quad (16)$$

as stated in the theorem, under the conditional argument, noting that the true stratum values, Y_{hi} of the study variable, y, are uncorrelated with the associated measurement errors, U_{hi}. The unconditional expectation of the estimator, $\bar{y}_{ms1}$, is obtained by taking the expectation of (15) for repeated samples of fixed size n. This gives the unconditional expectation of $\bar{y}_{ms1}$ as

$$E(\bar{y}_{ms1}) = EE_c(\bar{y}_{ms1}) = E(\bar{Y}) = \bar{Y} \quad (17)$$

which shows the unbiasedness of the estimator, $\bar{y}_{ms1}$ under the unconditional argument, as stated in the theorem. The associated unconditional variance is obtained by taking the expectation of (16) for repeated samples of fixed size n, and this gives the unconditional variance of the estimator, $\bar{y}_{ms1}$ as

$$V(\bar{y}_{ms1}) = EV_c(\bar{y}_{ms1}) = \sum_{h=1}^{L} W_h^2\Big[E\Big(\frac{1}{n_h}\Big) - \frac{1}{N_h}\Big](S_{yh}^2 + S_{uh}^2) \quad (18)$$

Following [8],

$$E\Big(\frac{1}{n_h}\Big) = \frac{1}{nW_h} \quad (19)$$

Substituting (19) into (18) gives the unconditional variance of the estimator, y_{ms1} as

$$V(\bar{y}_{ms1}) = \Big(\frac{1-f}{n}\Big)\sum_{h=1}^{L} W_h(S_{yh}^2 + S_{uh}^2) \quad (20)$$

as stated in the theorem. Similarly, the conditional and unconditional properties of the difference-type estimator and the class of estimators are obtained by following the same procedure. And this completes the proof.

In summary, the study proposes three sampling strategies/estimators for the handling of the problem of measurement errors in poststratified sampling scheme. These are the adopted customary sample mean estimator when there are measurement errors on both the study and auxiliary variables, the difference-type estimator and a class of ratio/product-type estimators. Recall, in particular, that the proposed class of estimators of the population mean of the study variable (y) when there are measurement errors on both the study and auxiliary variables under the poststratified sampling scheme is given by

$$\bar{y}_{mc1} = \sum_{h=1}^{L} W_h \bar{y}_h'' \Big(\frac{a_h \bar{X}_h + b_h}{a_h \bar{x}_h'' + b_h} \Big)^{\alpha_h} \tag{21}$$

and can generate a wide range of ratio-type and product-type estimators by making appropriate choices of the constants, α_h, a_h and b_h in stratum h. Table 3.1 shows some special cases of the proposed class of estimators, $\bar{y}_{mc1}$.

Table 3.1: Special Cases of Proposed Class of Estimators

Estimator	α_h	a_h	b_h
$\bar{y}_{mc1}(1) = \sum_{h=1}^{L} W_h \bar{y}_h''(\bar{X}_h/\bar{x}_h'')$, Customary ratio-type estimator	1	1	0
$\bar{y}_{mc1}(2) = \sum_{h=1}^{L} W_h \bar{y}_h''(\bar{X}_h + C_{xh})/(\bar{x}_h'' + C_{xh})$, Sisodia-Dwivedi estimator [9]	1	1	C_{xh}
$\bar{y}_{mc1}(3) = \sum_{h=1}^{L} W_h \bar{y}_h''(\bar{X}_h + \beta_{2xh})/(\bar{x}_h'' + \beta_{2xh})$, Singh-Kakran estimator I [10]	1	1	β_{2xh}
$\bar{y}_{mc1}(4) = \sum_{h=1}^{L} W_h \bar{y}_h''(\bar{X}_h C_{xh} + \beta_{2xh})/(\bar{x}_h'' C_{xh} + \beta_{2xh})$, Upadhyaya and Singh estimator I [11]	1	C_{xh}	β_{2xh}
$\bar{y}_{mc1}(5) = \sum_{h=1}^{L} W_h \bar{y}_h''(\bar{X}_h \beta_{2xh} + C_{xh})/(\bar{x}_h'' \beta_{2xh} + C_{xh})$, Upadhyaya and Singh estimator II [11]	1	β_{2xh}	C_{xh}
$\bar{y}_{mc1}(6) = \sum_{h=1}^{L} W_h \bar{y}_h''(\bar{x}_h''/\bar{X}_h)$, Customary product-type estimator	−1	1	0
$\bar{y}_{mc1}(7) = \sum_{h=1}^{L} W_h \bar{y}_h''(\bar{x}_h'' + C_{xh})/(\bar{X}_h + C_{xh})$, Pandey and Dubey estimator [12]	−1	1	C_{xh}
$\bar{y}_{mc1}(8) = \sum_{h=1}^{L} W_h \bar{y}_h''(\bar{x}_h'' + \beta_{2xh})/(\bar{X}_h + \beta_{2xh})$, Singh-Kakran estimator II [10]	−1	1	β_{2xh}
$\bar{y}_{mc1}(9) = \sum_{h=1}^{L} W_h \bar{y}_h''(\bar{x}_h'' C_{xh} + \beta_{2xh})/(\bar{X}_h C_{xh} + \beta_{2xh})$, Upadhyaya and Singh estimator III [11]	−1	C_{xh}	β_{2xh}
$\bar{y}_{mc1}(10) = \sum_{h=1}^{L} W_h \bar{y}_h''(\bar{x}_h'' \beta_{2xh} + \sigma_{xh})/(\bar{X}_h \beta_{2xh} + \sigma_{xh})$, Singh estimator [13]	−1	β_{2xh}	σ_{xh}

Obviously, the best estimators in the proposed class of estimators are those with the best choices of α_h, a_h and b_h. These are the estimators whose values of α_h, a_h and b_h minimize the unconditional mean square error of $\bar{y}_{mc1}$ given as:

$$MSE(\bar{y}_{mc1}) = \Big(\frac{1-f}{n}\Big) \sum_{h=1}^{L} W_h (S_{yh}^2 + \alpha_h^2 \lambda_h^2 R_h^2 S_{xh}^2 - 2\alpha_h \lambda_h R_h S_{yxh}) + \Big(\frac{1-f}{n}\Big) \sum_{h=1}^{L} W_h (S_{uh}^2 + \alpha_h^2 \lambda_h^2 R_h^2 S_{vh}^2) \tag{22}$$

where

$$\lambda_h = \Big(\frac{a_h \bar{X}_h}{a_h \bar{X}_h + b_h}\Big) \tag{23}$$

Applying the least squares method, it follows that the best choices of α_h, a_h and b_h that minimize the unconditional mean square error of $\bar{y}_{mc1}$ satisfy the expression:

$$\theta_{h4} = \frac{\alpha_h a_h \bar{X}_h R_h}{a_h \bar{X}_h + b_h} = \frac{\alpha_h a_h \bar{Y}_h}{a_h \bar{X}_h + b_h} = \frac{S_{yxh}}{S_{xh}^2 + S_{vh}^2} \tag{24}$$

with the associated optimum unconditional mean square error of $\bar{y}_{mc1}$ given by

$$MSE_0(\bar{y}_{mc1}) = \Big(\frac{1-f}{n}\Big) \sum_{h=1}^{L} W_h (S_{yh}^2 + \theta_{h4}^2 S_{xh}^2 - 2\theta_{h4} S_{yxh} + S_{uh}^2 + \theta_{h4}^2 S_{vh}^2) \tag{25}$$

Consequently, for the purpose of numerical illustration of results, we shall compare the efficiency of the usual poststratified sample mean estimator when there are no measurement errors with those of the optimum estimators, the special cases of estimators listed in Table 3.1, the adopted customary sample mean estimator, and the adopted difference-type estimator.

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IV. NUMERICAL ILLUSTRATION

Australian Survey data (Telford and Cunningham (1991)) shall be used for the numerical illustration of the theoretical results, with sum of skin folds (ssf) as the study variable, y, while the percentage Body fat (pcBfaT) is considered as the auxiliary variable, x. The study and auxiliary variables are positively correlated and are only observed values, which might not necessarily be the true values. Theoretically, the difference between the observed values and the true values constitutes the measurement errors. However, measurement errors are not usually observed in the field or included alongside the observed values. Thus, we shall simulate the measurement errors associated with the observed values of the various study and auxiliary variables.

The data simulation shall be carried out according to the assumptions that the measurement errors have zero means and they are uncorrelated with the associated study and auxiliary variables. Secondly, the measurement errors for the study variable are equally uncorrelated with the measurement errors for the auxiliary variable. The required measurement errors, satisfying these assumptions, shall be simulated using the multivariate normal (mvrnorm) procedure in R software. The respondents are stratified according to their sex with females as Stratum 1 and males as Stratum 2. It is assumed that a simple random sample of $n = 60$ respondents is taken from the population of $N = 202$ respondents. The summary of some relevant statistics is given in Table 3.2.

Table 3.2: Summary Statistics for Australian Survey data

Parameters	Stratum 1 (Females)	Stratum 2 (Males)
N_h	100	102
W_h	0.495050	0.504951
$\bar{Y}_h$	17.849100	9.250882
$\bar{X}_h$	86.973000	51.422549
R_h	0.205226	0.179899
S^2_{yh}	29.734018	10.142270
S^2_{xh}	1145.860577	355.476219
S_{yxh}	178.960117	58.079227
C_{xh}	0.389208	0.366650
β_{2xh}	0.775909	1.385599

Source: Australian Survey Data on sex, sport and body-size dependency of haematology in highly trained athletes by [14] Telford and Cunningham (1991)

Using the summary statistics in Table 3.2 and the theoretical results obtained earlier, the Percentage Relative Efficiency (PRE) of the proposed estimators over the usual poststratified sample mean estimator ($\bar{y}_{ps}$) when there are measurement errors on both the study and auxiliary variables are shown in Table 3.3. The table shows that the proposed ratio-type estimators, as expected, performed better than the proposed product-type estimators, in terms of having higher percentage relative efficiencies, since the data consists of positively correlated study and auxiliary variables. Considering the ten (10) special cases of the proposed class of estimators, ($\bar{y}_{mc1}$), when there are measurement errors on both the study and auxiliary variables, Table 3.3 indicates that the proposed estimator (utilizing auxiliary information) that has the highest efficiency is the adopted Upadhyaya-Singh (1999) estimator I, $\bar{y}_{mc1}(4) = \sum_{h=1}^{L} W_h \bar{y}''_h \left(\bar{X}_h C_{xh} + \beta_{2xh}\right) / \left(\bar{x}''_h C_{xh} + \beta_{2xh}\right)$, followed by the adopted Singh-Kakran (1993) estimator, $\bar{y}_{mc1}(3) = \sum_{h=1}^{L} W_h \bar{y}''_h \left(\bar{X}_h + \beta_{2xh}\right) / \left(\bar{x}''_h + \beta_{2xh}\right)$, while the estimator with the least efficiency among the ten (10) special cases under consideration is the customary product-type estimator, $\bar{y}_{mc1}(6) = \sum_{h=1}^{L} W_h \bar{y}''_h \left(\bar{x}''_h / \bar{X}_h\right)$, when there are measurement errors on both the study and auxiliary variables.

Table 3.3: Percentage Relative Efficiency (PRE) of Some Proposed Estimators over the Usual Poststratified Mean Estimator ($\bar{y}_{ps}$) when there are measurement errors on both the study and auxiliary variables for positively correlated study and auxiliary variables

Estimator	PRE
$\bar{y}_{ps} = \sum_{h=1}^{L} W_h \bar{y}_h$	100.00
$\bar{y}_{ms1} = \sum_{h=1}^{L} W_h \bar{y}_h''$	95.20
$\bar{y}_{md1} = \sum_{h=1}^{L} W_h[\bar{y}_h'' - k_h(\bar{x}_h'' - \bar{X}_h)]$, $k_h = S_{yxh}/S_{xh}^2 \; \forall h$	886.35
$\bar{y}_{mc1}(1) = \sum_{h=1}^{L} W_h \bar{y}_h'' \left(\bar{X}_h / \bar{x}_h''\right)$	541.57
$\bar{y}_{mc1}(2) = \sum_{h=1}^{L} W_h \bar{y}_h'' \left(\bar{X}_h + C_{xh}\right)/\left(\bar{x}_h'' + C_{xh}\right)$	550.29
$\bar{y}_{mc1}(3) = \sum_{h=1}^{L} W_h \bar{y}_h'' \left(\bar{X}_h + \beta_{2xh}\right)/\left(\bar{x}_h'' + \beta_{2xh}\right)$	560.59
$\bar{y}_{mc1}(4) = \sum_{h=1}^{L} W_h \bar{y}_h'' \left(\bar{X}_h C_{xh} + \beta_{2xh}\right)/\left(\bar{x}_h'' C_{xh} + \beta_{2xh}\right)$	588.60
$\bar{y}_{mc1}(5) = \sum_{h=1}^{L} W_h \bar{y}_h'' \left(\bar{X}_h \beta_{2xh} + C_{xh}\right)/\left(\bar{x}_h'' \beta_{2xh} + C_{xh}\right)$	552.18
$\bar{y}_{mc1}(6) = \sum_{h=1}^{L} W_h \bar{y}_h'' \left(\bar{x}_h'' / \bar{X}_h\right)$	20.35
$\bar{y}_{mc1}(7) = \sum_{h=1}^{L} W_h \bar{y}_h'' \left(\bar{x}_h'' + C_{xh}\right)/\left(\bar{X}_h + C_{xh}\right)$	20.46
$\bar{y}_{mc1}(8) = \sum_{h=1}^{L} W_h \bar{y}_h'' \left(\bar{x}_h'' + \beta_{2xh}\right)/\left(\bar{X}_h + \beta_{2xh}\right)$	20.63
$\bar{y}_{mc1}(9) = \sum_{h=1}^{L} W_h \bar{y}_h'' \left(\bar{x}_h'' C_{xh} + \beta_{2xh}\right)/\left(\bar{X}_h C_{xh} + \beta_{2xh}\right)$	21.08
$\bar{y}_{mc1}(10) = \sum_{h=1}^{L} W_h \bar{y}_h'' \left(\bar{x}_h'' \beta_{2xh} + \sigma_{xh}\right)/\left(\bar{X}_h \beta_{2xh} + \sigma_{xh}\right)$	29.33
Optimum estimators	886.37

We also observe from Table 3.3 that the efficiency of the proposed difference-type estimator, $\bar{y}_{md1} = \sum_{h=1}^{L} W_h[\bar{y}_h'' - k_h(\bar{x}_h'' - \bar{X}_h)]$, $k_h = S_{yxh}/S_{xh}^2 \; \forall h$, is close to the efficiency of the optimum estimators in the proposed class of estimators, ($\bar{y}_{mc1}$), since the choice of the constant, $k_h = S_{yxh}/S_{xh}^2 \; \forall h$, is close to the optimum value that minimizes the mean square error of the proposed class of estimators, $\bar{y}_{mc1}$. Furthermore, we observe that when there are measurement errors on both the study and auxiliary variables, the customary sample mean estimator, $\bar{y}_{ms1} = \sum_{h=1}^{L} W_h \bar{y}_h''$, is less efficient than the ratio-type estimators in the proposed class of estimators, $\bar{y}_{mc1}$, but more efficient than the suggested product-type estimators. This confirms that the use of auxiliary information is most likely to result in an improved efficiency over the customary sample mean estimators when estimators that utilize auxiliary information are appropriately chosen, according to the nature and direction of the relationship between the study and auxiliary variables. Table 3.3 specifically reveals that the ratio-type estimators are preferable when there is a positive correlation between the study and auxiliary variables.

V. CONCLUDING REMARKS

The study identified a gap in terms of non-availability of sampling strategies for the handling of the prob- lem of measurement errors in poststratified sampling scheme and consequently proposed three sampling strate - gies/estimators as a solution. These are adopted customary-type sample mean estimator, the difference-type es- timator and a class of ratio/product type estimators. The usual poststratified sample mean estimator, $\bar{y}_{ps}$, when there are no measurement errors was found to be more efficient than the proposed customary-type sample mean estimator. However, the proposed difference-type estimator, as well as the best estimators in the proposed class of estimators were found to perform better than the usual poststratified sample mean estimator in terms of having smaller variance/mean square errors. For the positively correlated study and auxiliary variables used for numerical illustration of results, the proposed ratio-type estimators performed better than the proposed product-type estima- tors, as expected. Among the ten (10) special cases of estimators considered in the proposed class of estimators, the proposed estimator with the highest efficiency was the adopted Upadhyaya-Singh (1999) estimator I, while the estimator with the least efficiency was the customary product-type estimator. Consequently, we recommend the proposed adopted (ratio-type) Upadhyaya-Singh (1999) estimator I, for the handling of measurement errors in poststratified sampling scheme when the study and auxiliary variables are positively correlated.

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