*Quest Journals*
*Journal of Research in Applied Mathematics*
*Volume 8 ~ Issue 1 (2022) pp: 10-14*
*ISSN(Online) : 2394-0743 ISSN (Print):2394-0735*
www.questjournals.org

**Research Paper**

# When a blind man is lost in a forest

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***Abstract***: *In this paper we studied a special case of the Lost-in-Forest problem. We established few new theorems based on the condition that the end point of the escape path always lies outside the forest. This condition is similar to the situation when a blind man lost his path in a forest.*

***R**eceived 11 Jan, 2022; **R**evised 23 Jan, 2022; **A**ccepted 25 Jan, 2022 © The author(s) 2022.*
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## I. Introduction:

Bellman's "Lost in a forest" problem is an optimization problem in geometry, which is considered unsolved till date. It was originated in 1955 by the American applied mathematician Richard E. Bellman. Bellman asked, "What is the best path to follow in order to escape a forest of known dimensions?", see[1] .Bellman proposed the problem for two unbounded regions, one for the infinite strip between parallel lines and another one for the half-plane with known distance from the boundary. Later mathematicians showed interest in bounded regions also.

Steven R. Finch and John E. Wetzel described the problem as below:

*A hiker is lost in a forest whose shape and dimensions are precisely known to him. What is the best path for him to follow to escape from the forest?* see[2].

If we have to solve the "Lost in a forest" problem then we have to find the "best" escape path. Bellman expressed two different meanings of "best," one is to minimize the maximum time to escape, and the other one is to minimize the expected time to escape. Croft, Falconer, and Guy gave a third perception which says to maximize the probability of escape within a specified time duration, see [4, p.40].

If we consider 'Best' means to minimize the maximum time to escape, then both of the Bellman's proposed situations have been answered, for the first situation for the minimax or shortest escape path was given by Zalgaller, see [6] and Isbell gave a solution for the second one, see[5]. For both of these two specific cases the shortest escape path is unique up to congruence. There might exist multiple non-unique shortest paths for any forest, Steven R. Finch and John E. Wetzel gave two such examples, see[2]. Much is not known in either case for other meanings of "Best".

In this paper we consider shortestdistance path to be the best path.

Mathematicians managed to find solutions only for classes of shapes, see [3] where John W. Ward collected most of the available solutions related to this problem. No generic solution has been invented till now. A generic solution is considered to be a geometric algorithm which takes the shape of the forest as input and produces the optimal escape route as the output.

If we analyze the solutions for specific shapes then we see that in most of the cases we need not follow the full escape path, after travelling a part of the path we can come out of the forest, the distance we need to travel depends on the relative orientation of the escape path and forest. Let's take Zalgaller's solution for Bellman's first problem of the infinite strip between parallel lines, see [7], where Zalgaller arranged 4 straight line segments and two circular arcs in such a way that guarantees the shortest escape path. Figure 1 shows that if we lose in a forest of an infinite strip of unit width & follow this escape path then we will come out of the forest at any point of the path. Here in Figure 1 we started travelling from point *A* and while travelling (*DE)* part of the path we come out of the forest.

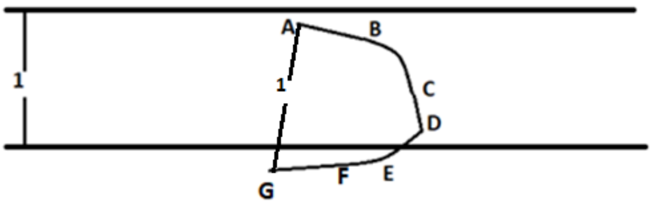


Figure 1.

What will happen for a blind man who is lost in the same forest as in Figure 1? Figure 2 shows the same forest as Figure 1 but the orientation of the forest is changed with respect to the path. If a blind man follows this escape path then he comes out of the forest while travelling the *Arc(BC)* part of the path, but he will be unaware of this fact, and after travelling the full path when he reaches at the end point *G*, he again enters the forest. For a blind man this escape path is obviously of no use. In this case the escape path must ensure that the end point will always lie outside the forest irrespective of the orientation of the path & forest.

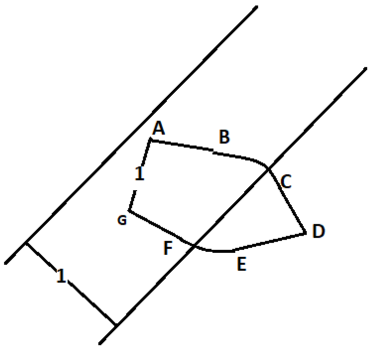


Figure 2.

From the above case, few questions arise like does there exist any such escape path whose endpoint always lies outside the forest? If such an escape path exists then can we have a generic algorithm to find the shortest path amongst those escape paths? Does all forest have such escape paths? In this paper we studied these questions.

**Definitions:**

**Points and lines:**

- *L(AB)* denotes the distance between points A and B.
- *(AB)* denotes thestraight line segment between points A and B.
- *AB* defines a line.

**Forest**:

- A forest is a planar region with at least one boundary line.
- A bounded forest has an upper bound to the distance between pairs of points in the forest, and diameter is the least upper bound. Bounded forests are closed, so the least upper bound is achieved.
- Unbounded forest does not have an upper bound to the distance between pairs of points. Unbounded forest must have at least one boundary line. Boundary lines may consistof a straight line, or curve or by both.
- Plane is a forest without escape paths.
- Diameter: A diameter of a bounded forest can be defined as any straight line segment between maximally separated points. Clearly for any unbounded forest we will not have any diameter.

**Path**:

- A path is a continuous and correctable planar arc. Straight line segment is considered as a degenerated planner arc.
- *P(AB)* defines the path with starting point *A* and end point *B*. If only a straight line segment becomes a path then we denote this path as a straight line segment *(AB)* as defined in "Points & Lines". *Arc(AB)* defines an arc.
- *L(P(AB))* defines the length of the path *P(AB)*. *L(a)* defines the length of path *a*. *L(AB)* defines the length of the straight line segment path *(AB)* etc.

● “Diameter path” of a bounded forest is defined as a straight line segment whose length is equal to the length of forest diameter.

**Covers & Escapes:**

● A bounded forest ‘covers’ path *P(AB)* if it can encompass a congruent, orientation-preserving copy of *P(AB)*. In other way, a bounded forest covers a path *P(AB)* if *P(AB)* can be accommodated into the forest by translation or rotation or by both.

● *P(AB)* is called an escape path from a forest, if *P(AB)* intersects any of the boundary lines and it does not matter what is the position of *A* with respect to the forest or the relative orientation of *P(AB)* and the forest.

● ‘EP’ path is an escape path whoseendpoint always lies outside the forest irrespective of the orientation of the path & forest. For example *EP(AB)* defines an escape path *P(AB)* which end point *B* always lies outside the forest irrespective of the orientation of *P(AB)* and the forest.

***Lema 1.*** *Diameter path of a bounded forest is an EP path.*

***Proof***. Let *F* be a bounded forest and *d* is its diameter path. , We know that for any bounded forest a diameter path is an escape path,see“Proposition 1” of [3, p.9], and therefore *d* is an escape path of *F*. Let the starting & end points of *d* are *A*&*B* respectively. Let’s consider *d* is not EP path of *F*. Since *d* is not an EP path of *F*,so both *A*&*B* lies inside *F*for fewor all orientations of *F*&*d*. Then we can have a point *C* on *AB*such that *L(AC) > L(AB)* and *C* lies inside *F* or on any boundary lines of *F*.By definition of diameter path it is obvious that *L(AB) > L(AC)*. A contradiction, so the end point of *d*i.e.*B*must lies outside *F* for any relative orientation of *F* and *d*. Therefore *d* is an EP path of *F*.

***Corollary 2.****For any bounded forest there exists at least one EP path.*

***Lema 3.*** *For any unbounded forest there does not exist any EP path.*

***Proof***.Let *F* be an unbounded forest and*EP(AB)* be an EP path for *F*. Connect*A*&*B* by a straight line segment*(AB)*. Since *EP(AB)* is an EP path then *L(EP(AB))* is finite , therefore *L(AB)* is also finite. As *B* is the end point of both the paths *(AB)* and*EP(AB)*, and the later takes us out of the forest for any relative orientation of *EP(AB)*&*F*, so *(AB)* also takes us out of *F* for any relative orientation of *(AB)*&*F*. Therefore *(AB)* is also an EP path.

Since for any unbounded forest, there does not exist an upper bound to the distance between pairs of points, so it is obvious that we can place a straight line segment of finite length in any unbounded region by translation or by rotation or by both, so if we place *(AB)* in *F* then both the starting & end points*A* &*B*respectivelylie inside *F*, which leads to a contradiction that *(AB)* is an escape path. Therefore *F* does not have any EP path.

For example, see Figure 3 which shows a forest *F*with two parallel straight lines named *XY*&*ST*. Let’s consider there exists an EP path *EP(AB)* for which the end point *B* lies outside *F*.Connect *A*&*B*, see Figure 4. By keeping *(AB)* fixed, if we translate *XY*&*ST* in such a way that (*AB)* becomes parallel to *XY*&*ST*. Then point *A*&*B* will always lie inside *XY*&*ST*, this leads to a contradiction that *B* always lies outside *F*. Therefore there does not exist any EP path for *F*.

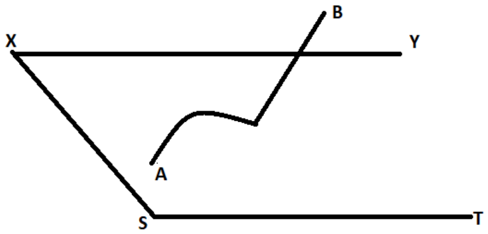


Figure 3.

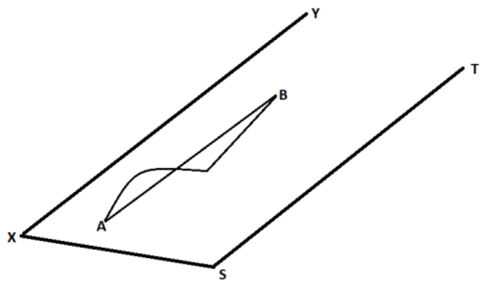


Figure 4.

***Theorem 4.*** *For any bounded forest there exists a shortest EP path.*

*Proof:* Let *F* bea forest where an initial starting point *A* is given.By Corollary 2,*F* has at least one EP path.

There may be paths of finite or infinite lengths (we may have paths of infinite length in unbounded forest), but it’s evident that any escape path can’t be of infinite length because a path of infinite length can never take us out of the forest, so every escape path must be of finite length and so EP paths are also of finite lengths.

If there exists only one EP path then that will be the shortest one. If multiple such paths exist then we proceed below.

Let's say for *F* there exist multiple EP paths from *A*, and those EP paths are *EP(AB)*, *EP(AC)*,*EP(AD)* and so on. Note that the length of multiple EP paths may be the same.
Consider a set S such that *S = {L(EP(AB)),L(EP(AC)),L(EP(AD)).....}*. Clearly set *S* is nonempty. Since length is always positive and real numbers so the elements of set *S* are positive real numbers. Hence we get a nonempty set *S* of positive real numbers. *S* may be a finite or an infinite set.
Using "Axiom of choice" theory we can say that there exists a least number in set S, let's say the least number is *L(EP(AX))* which is the length of path *EP(AX)*. So *EP(AX)* is the shortest EP path. There may exist multiple non-unique shortest EP paths, length of any path may equal to any other path, but the length of the shortest EP path must be equal to the length of path *EP(AX)*.

***Theorem 5. The shortest EP path of a bounded forest is a straight line segment.***
*Proof.* Let*F* be a bounded forest. By Corollary 2, we know that every bounded forest has at least one EP path. Theorem 4 confirms that there exists a shortest EP path for *F*.
Note: In this prove we included only convex forest's figure for reader's easiness. Same steps are applicable for concave shaped forest also. Readers can try the same.
Let's say a straight line segment can't be the shortest EP path. Therefore the shortest EP path shouldbe a continuous rectifiable path consists of:
a. Only curves or,
b. Curves along with Straight linesegments or,
c. Only straight line segments which do not form a single straight line segments
Case 'a' & 'b': Figure 5 & Figure 6 shows that *EP(AD)* as the shortest EP path consists of curves, or curves along with straight lines as in Case 'a' and Case 'b' accordingly.
Connect *A*&*D*, see Figure 7 & 8. Since the end point of the paths *(AD)*&*EP(AD)* is *D* and *EP(AD)* always takes us out of the forest then path *(AD)* also leads us out of the forest irrespective of the orientation of *(AD)*&*F*. Hence *(AD)*is also an EP path.
We know that the shortest distance between two points on a plane is the straight line segment between the points, hence $L(AD) < L(EP(AD))$.This is a contradiction that *EP(AD)* is the shortest EP path.

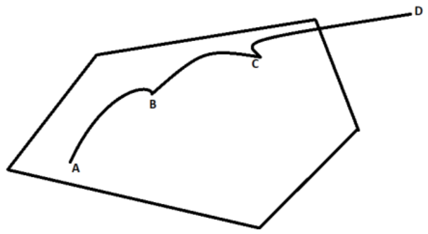

Figure 5

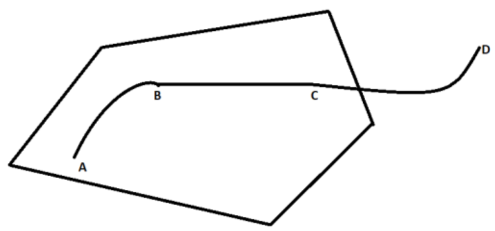

Figure 6

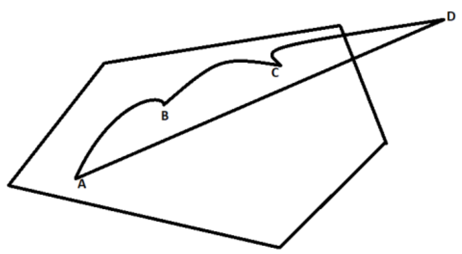

Figure 7

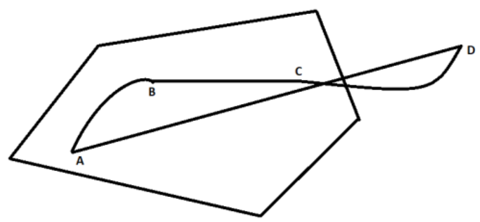

Figure 8

Case 'c': If F has the shortest EP path consisting of only straight line segments, then there might be two cases:
1. Shortest EP path consists of straight line segments which do not form a single straight line segment (this is case 'c'),
2. Shortest EP path will be only a single straight line segment.
If we can discard the case '1' then we are done.
Figure 9 shows that *EP(AE)* as the shortest EP path consists of straight line segments. Connect *A*&*E*, see Figure 10. Since the end point of the paths *(AE)*&*EP(AE)* is *E* and *EP(AE)* always takes us out of the forest then path *(AE)* also leads us out of the forest irrespective of the relative orientation of *(AE)*&*F*. Hence *(AE)* is also an EP path.

We know that the shortest distance between two points on a plane is the straight line segment between the points, hence $L(AE) < L(EP(AE))$.This is a contradiction that *EP(AE)* is the shortest path.

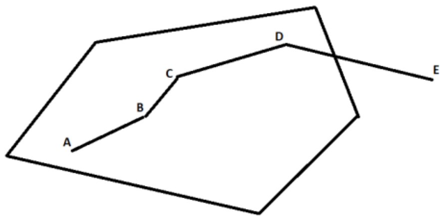


Figure 9

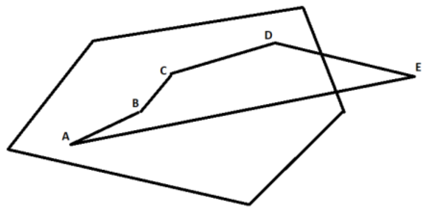


Figure 10

So we are left with only the last option that the shortest EP path will be a straight line segment which can't be further minimized. Therefore the shortest EP path for a bounded forest will be a straight line segment.

***Theorem 6. The shortest EP path for a bounded convex forest is a diameter path & this path is unique.***
*Proof.* Let*F* be a bounded convex forest. By Corollary 2, we know that every bounded forest has at least one EP path. Lema 1 gives that diameter path is an EP path. So *F* has a diameter EP path. We denote this diameter EP path by *d*. We have to prove that *d* is the shortest EP path & there does not exist any other EP path of same length.
Theorem 4 confirms that there exists a shortest EP path for *F*. If *F* has no other EP path then *d* will be the unique shortest EP path. If *F* has multiple EP paths then we proceed below.
Theorem 5 gives that the shortest EP path of*F* will be a straight line segment.Let's say straight line segment *e* is the shortest EP path of *F*. Let's consider $L(d) \neq L(e)$ , since we assume that *e* is the shortest EP path then obviously $L(e) < L(d)$ .By definition *d* is the straight line segment between maximally separated points of *F*. Let's say the maximally separated points of *F* are *A*&*B*respectively i.e. *d= (AB)*. Let *e= (XY)*. Since $L(e) < L(d)$ i.e.$L(XY) < L(AB)$, so we can have two points *J*&*K* on *(AB)* such that *L(JK) = L(XY)*. By translation we can place *(XY)* on *(JK)*. Since *L(JK) < L(AB)* so the interior of *F* can cover *(JK)*, therefore can cover *(XY)*also. So it is clear that *(XY)=e* can't be an EP path, a contradiction. So *d* is the shortest EP path.
Now we have to prove the uniqueness of *d*. Let's say there exists another EP path*f*, where *L(f) =L(d)*. By Theorem 5, *f* must be a straight line segment. Since *L(d) = L(f)* and *d*&*f* are both straight line segments so by definitions of "Diameter path", *d =f*. Therefore the shortest EP path for a bounded convex forest is a diameter path & this path is unique.

***Open Questions:***
1. Does there exist any generic algorithm which takes the shape and dimensions of a forest and determinesthe shortest EP path?
2. Does there exist any generic algorithm for concave forest to find out the shortest EP path?

Acknowledgement: I want to thank Dr. Aritra Mukherjee(kalpurush1601@gmail.com) for his valuable feedback and suggestions to enhance this paper.

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