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Research Paper

DenoisedLinear Combination of Order Statistics using Different Smoothing Techniques

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Abstract
The major problem in constructing consistent estimator frequently arise because of observed data characterized by measurement errors. In this study, we discussed the branch of statistics known as order statistics in which the sorting of random sample is important. The study suggested denoised L-estimator (DL) which is fundamentally defined and conceptualized as denoised linear combination of order statistic. Applying three different denoising or smoothing (Logistic kernel, Gaussian Kernel and Wavelet) techniques to denoise simulated data of sample size 256, which subjected to noise or measurement errors. The performance and comparisons of the denoised linear combination of order statistics under different smoothers was considered using mean squared error criterion. The result of the study showed that the denoised linear combination of order statistics performed better under Wavelet smoother.
Keyword*: L-estimator, L-moment, L-ratio, Order statistics, Mean squared error.*

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I. Introduction

A branch of statistics known as **order statistics** plays a prominent role in L-moment theory. The study of order statistics is the study of the statistics of ordered (sorted) random variables and samples. A comprehensive exposition on order statistics is provided by David (1981), and an R-oriented approach is described in various contexts by Baclawski (2008).

The random variable X for a sample of size n when sorted creates the order statistics of X: $X_{1:n} \leq X_{2:n} \leq \ldots \leq X_{n:n}$. The sample order statisticsfrom a random sample are created by sorting the sample into ascending order:$x_{1:n} \leq x_{2:n} \leq \ldots \leq x_{n:n}$. As will be seen, the concept and use of order statistics consider both the value (magnitude)and the relative relation (order) to other observations.

Asquith (2011) show that the L-moments, which are based on linear combinationsof order statistics do in fact provide effective and efficient estimators of distributional geometry. In general, order statistics are already a part of the basic summary statistic repertoire possessed by most individuals, including non-scientists and non-statisticians. The minimum and maximum are examples of extreme value order statistics and are defined by the following notation:

$$min\{X_n\} = X_{1:n} \quad (1)$$

$$max\{X_n\} = X_{n:n} \quad (2)$$

The familiar median $X_{0.50}$ by convention is

$$X_{0.50} = \begin{cases} (X_{[n/2]:n} + X_{[(n/2)+1]:n})/2 & \text{if } n \text{ is even} \\ X_{[(n+1)/2]:n} & \text{if } n \text{ is odd} \end{cases} \quad (3)$$

and thus, clearly is defined in terms of one order statistic in the case of odd sample size or a linear combination of two order statistics in the case of even sample sizes. Other order statistics exist and several important interpretations towards the purpose of the study can be made. Hosking (1990) and Hosking and Wallis (1997) provide an "intuitive" justification for L-moments and by association the probability weighted moments Asquith 2011). The justification is founded on order statistics:

• The order statistic X_{11} (a single observation) contains information about the location of the distribution on the real-number line $\mathfrak{R}$;

• For a sample of$n = 2$, the order statistics are $X_{1:2}$ (smallest) and $X_{2:2}$ (largest). For a highly dispersed distribution, the expected difference between $X_{1:2} - X_{2:2}$ would be large,whereas for a tightly dispersed

distribution, the difference would be small. The expected differences between order statistics of an$n = 2$sample hence can be used to express the variability or scale of a distribution; and
• For a sample of$n = 3$, the order statistics are $X_{1:3}$ (smallest), $X_{2:3}$ (median), and $X_{3:3}$ (largest). For a negatively skewed distribution, the difference $X_{2:3} - X_{1:3}$would be larger(more data to the left) than $X_{3:3} - X_{2:3}$ The opposite (more data to the right) would occur if a distribution were positively skewed.
These interpretations hint towards expression of distribution geometry by select use of intra-sample differences. In fact, various intra-sample differences can be formulated to express fundamental and interpretable measures of distribution geometrically. Intra-sample differences are important link to L-moments, and the link justifies exposition of order statistics. Kaigh and Driscoll (1987) defined $\mathcal{O}$-statistics as "smoothed generalizations of order statistics" and provide hints Kaigh and Driscoll (1987) towards L-moments by suggesting that linear combinations of the order statistics provide location, scale, and "scale-invariant" skewness and kurtosis estimation.
L-moments are summary statistics for probability distributions and data samples. They are analogous to ordinary moments, they provide measures of location, dispersion, skewness, kurtosis, and other aspects of the shape of probability distributions or data samples but are computed from linear combinations of the ordered statistics. Standardised L-moments are called L-moment ratios and are analogous to standardized moments just as for conventional moments; a theoretical distribution has a set of population L-moments. Sample L-moments can be defined for a sample from the population and can be used as estimators of the population of L-moments
A concept regarding order statistics, which will be critically important in the computation of L-moments, is the expectation of order statistic. The expectation is defined in terms of the QDF. The expectation of an order statistic for the ith largest of r values is defined David (1981) in terms of the QDF$x(F)$ as

$$E[X_{i:n}] = \frac{n!}{(i-1)!(n-i)!}\int_0^1 x(F) \times F^{i-1} \times (1-F)^{n-i} dF \quad (4)$$

where the quantity to the left of the integral is$\frac{n!}{(i-1)!(n-i)!} = n\binom{n-1}{i-1}$. Jurecková and Picek (2006) summarize linear statistical estimators known asL-estimatorsand Serfling (1980) considers the asymptotic (very large sample)properties of L-estimators. L-estimators T_n for sample of size n are based on the order statistics and are expressed in a general form as

$$T_n = \sum C_{i:n} h(X_{i:n}) + \sum a_j h^*(X_{[npj+1]:n}) \quad (5)$$

Where $X_{1:n}$are the order statistics,$C_{1:n}, \ldots, C_{n:n}$and$d_1, \ldots, d_n$are given coefficients of weight factors,$0 < P_1 < \cdots < P_k < 1$ and $h(a)$ and $h^*(a)$are given functions for argument a. The coefficients $C_{i:n}$for $1 \leq n$ are generated by a bounded weight function J (a) with a domain [0,1] with a range of the real-number line R by either$C_{1:n} = \int_{(i-1)/n}^{i/n} J(S)ds$ or approximately $C_{1:n} = \frac{J(i/[n+1])}{n}$.Two interesting L-estimators that have immediate connection to the L-moments are sen weighted mean and Gini mean difference statistics, and they would be considered in this study.
It is obvious that the estimators are usually failed to be consistent because of noise measurement errors. Therefore, different smoothers have been introduced in literature to denoise or smoothing data to capture important patterns in the data, while leaving out noise or other fine scale structure or rapid phenomena. These include Epanechnikov kernel, Gaussian, kernel, Polynomial spline, wavelet, etc. See Cai et al., (2000), Cui et al., (2002), You and Zhou, (2007), You et al., (2009), Zhou and Liang (2009), Cui et al., (2010), Fasoranbaku and Soyombo (2015), Fasoranbaku et al., (2016) for details on denoising smoothing approaches.Smoothing extract more information from the data as long as the assumption of smoothing is reasonable and provides flexible and robust analysis. In the direction of the studies, this study considered three different smoothing methods, namely, Logistic kernel, Gaussian kernel, and Wavelet to denoise develop denoised Linear combination of order statistics, termed denoised L-estimator (DL).

II. Linear Combination of Order statistics (L-Estimator)

One popular class of estimator is the class of linear combinations of order statistics called L-estimators.Suppose$X_{(1)} \leq \ldots \leq X_{(n)}$ be order statistics of a sample and g be a function mapping the open (0, 1) into the set of real numbers, $\mathcal{R}$, such that $g(t) = g(1-t)$ and $\int_0^1 g(t) = 1$, Friedrich-Wilhelm (1965) gave estimator correspond to (g) function mapping into set of real numbers, $\mathcal{R}$, as

$$L_n = L_n(g) = \frac{1}{n}\sum_{i=1}^{n} g\left(\frac{i}{n+1}\right) X_i \quad (6)$$

Weighted mean (Sen, 1964) statistics is a special L-estimator based on order statistic that has connection to L-moment and it is express as follow.

i. Sen Weighted mean$S_{n:k}$:

This is a robust estimator (Jurecková and Picek, 2006, p. 69) of the mean of a distribution and is defined as

$$S_{n:k} = \binom{n}{2k+1}^{-1} \sum_{i=1}^{n} \binom{i-1}{k}\binom{n-i}{k} X_{i:n} \qquad (7)$$

where $X_{i:n}$are the order statistics and k is a weighting or trimming parameter.A sample version $\hat{S}_{n:k}$ results when $X_{1:n}$are replaced by their sample counterpart$x_{1:n}$.
Note that $S_{n,0} = \mu = \bar{X}_n$or the arithmetic mean, and $S_{n,k}$is the median if either$n$ is even and $K = (n/2) - 1$ or n is odd and $K = (n-1)/2$.

ii. Sample L-moments Estimator by Direct Sample Estimator

Sample L-moments can be computed as the population L-moments of the sample summing over r-element subset of the sample$\{x_1 < .. . < x_i < \ ... \ < x_r\}$, hence averaging by the binomial coefficient.

$$L_n = \frac{1}{n}\binom{n}{r}^{-1} \sum_{x_1<\ldots<x_i<\ldots<x_r}^{n} (-1)^{r-i}\binom{r-1}{i} x_{i:n} \qquad (8)$$

Grouping these by order statistic counts the number of ways an element of an n-element sample can be the ith element of an r-element subset and yields formulas of the form below. Direct estimators for the first four L-moments in a finite sample of n observations are :(Wang, 1996)

$$\lambda_1 = \binom{n}{1}^{-1} \sum_{i=1}^{n} x_{(i)}$$

$$\lambda_2 = \frac{1}{2}\binom{n}{2}^{-1} \sum_{i=1}^{n} \left\{\binom{i-1}{1} - \binom{n-i}{1}\right\} x_{(i)}$$

$$\lambda_3 = \frac{1}{3}\binom{n}{3}^{-1} \sum_{i=1}^{n} \left\{\binom{i-1}{2} - 2\binom{i-1}{1}\binom{n-1}{1} + \binom{n-1}{2}\right\} x_{(i)} \qquad (9)$$

$$\lambda_4 = \frac{1}{4}\binom{n}{4}^{-1} \sum_{i=1}^{n} \left\{\binom{i-1}{3} - 3\binom{i-1}{2}\binom{n-1}{1} + 3\binom{i-1}{1}\binom{n-1}{2} - \binom{n-i}{3}\right\} x_{(i)}$$

where $x_{(i)}$is order statistics and a binomial coefficient, $\lambda_1$is L-mean, λ_2 is L-scale.
L-moments ratios are derived from (7) such that$\tau = \lambda_2/\lambda_1$,$\tau_3 = \lambda_3/\lambda_2$,$\tau_4 = \lambda_4/\lambda_2$are called coefficient of L-variation, L-kurtosis and L-skewness respectively.

III. Denoising Procedures

The basic idea behind smoothing a data set is the creation of an approximating function that attempts to capture important patterns in the data while leaving out the noise and is also referred to as "denoising". There are various methods to help restore a data set from measurement noise. In this study, the following smoothing method are used

I. Kernel smoothing

Given a random sample $X_1 \ldots X_n$ with a continuous, univariate density function $f(.)$,The kernel density estimator is:

$$\hat{f}(x, \hbar) = \frac{1}{n\hbar} \sum_{i=1}^{n} k\left(\frac{x - X_i}{\hbar}\right) \qquad (10)$$

To this end, let $K(.) \geq 0$ be a symmetric kernel supported on [-1,1] with $\int_{-1}^{1} K(x)dx = 1$ for some smoothing parameter h. Where x is the value of the scalar variable for which one seeks an estimate while X_i are the values of that variable in the data.Kis a function of a single variable called the *kernel*. The kernel determines the *shape* of the function. The parameter *h* is called the *bandwidth* or *smoothing constant*. It controls the degree of smoothing and adjusts the size and form of the function.

$$u = \left(\frac{x - X_i}{\hbar}\right) \qquad (11)$$

For the purpose of this study, the two most used Kernels function are utilized:

a) Logistic Kernel smoothing:

$$K(u) = \frac{1}{e^{u} + 2 + e^{-u}} \qquad (12)$$

b) Gaussian Kernel smoothing:

$$k(u) = \frac{1}{\sqrt{2\pi}} exp\left(-\frac{u^2}{2}\right) \qquad (13)$$

The Choice of Smoothing Parameter h

The problem of selecting the smoothing parameter for kernel estimation has been explored by many authors and no procedure yet been considered the best in every situation. Automatic bandwidth selection methods can be divided into two categories: classical and plug in method.

The accuracy of kernel smoothers is a function of the kernel K and the bandwidth$h$; the accuracy depends mainly on the smoothing parameter h. One of the most frequently used methods of bandwidth selection was introduced. The choice of bandwidth is crucial and is also a challenge. There are various methods for selecting bandwidth but there is no single best method. A common choice is the “Silverman’s rule of thumb” for an optimal bandwidth:

The ideal value of bandwidth h, from the point of view of minimizing the approximate mean integrated square error. From the properties of kernel density estimator, we sum the bias square and the variance to give mean square error as.

$$MSE(\hat{f}(x)) = \frac{R(K)}{nh} f(x) + \frac{h^4}{4} f^{\prime 2 \, 2}_{\;\;2}\left(\frac{1}{nh}\right)(h^4) \quad (14)$$

Integrate the mean square error (14)) over the entire line we find (Parzen(1962))

$$MISE(\hat{f}) = \frac{R(K)}{nh} + \frac{h^4}{4} \mu_2^2(K) R(f'') \quad (15)$$

and the bandwidth h that minimizes MISE is then

$$h_{MISE} = \left(\frac{R(K)}{\mu_2^2(K) R(f'')}\right)^{\frac{1}{5}} n^{-\frac{1}{5}} \quad (16)$$

Using this optimal bandwidth, we have

$$\inf_{h>0} MISE(\hat{f}) \approx \frac{5}{4} [\mu_2^2(K) R^4(K) R(f'')]^{\frac{1}{5}} n^{-\frac{4}{5}} \quad (17)$$

The problem with using the optimal bandwidth is that it depends on the unknown quantity f'', which measures the speed of fluctuations in the density$f$, i.e., the roughness of f. Many methods have been proposed to select a bandwidth that leads to good performance in the estimation, by using an estimate ofσ, one has a data-base estimate of the optimal bandwidth. To have an estimator that is more robust against outliers, the interquartile range R can be used as a measure of spread. This modified version can be written as

$$h_{opt} = 0.9[min(s, lQR)]/1.34 n^{-\frac{1}{5}} \quad (18)$$

where s is the sample standard deviation and IQR is the interquartile range (0.75quartile minus 0.25 quartile).

Plug-in-method makes use of the rule of thumb through the underlying principle: if there is an expression involving an unknown parameter, replace the unknown parameter with an estimate. To apply the plug-in-method in practice, a kernel function will be chosen: In this case, the Logistic and Gaussian Kernel were chosen and the unknown parameter h will be estimated by the optimal bandwidth h_{opt}

IV. Wavelet Smoothing

Wavelets are orthonormal sets of functions whose shape, as the name suggests, is like a little wave. They have compact local support but decay quickly to zero elsewhere. Wavelets can provide approximations of both stationary and non-stationary time series. They are particularly effective for time series characterized by abrupt changes, spikes and periodic cycles. Consumer and business sentiment indexes are characterized by such features. These important properties have inspired several applications of discrete wavelet transforms in economics (see, Crowley 2007). The wavelet approximation of an observed time series is similar to the Fourier transform and has the following form

$$X = \sum_{k \in z} c_{j_0,k} \varphi_{j_0,k}(t) + \sum_{j \in z} \sum_{k \in z} g_{j,k} \psi_{j,k}(t) \quad (19)$$

where Z is the set of integers. This is an orthogonal decomposition that involves J timescales (where, j = 1. . . J) with $k \in Z$coefficients at each scale. The set of father$\varphi$and mother ψwavelets that form an orthonormal basis are defined as

$$\varphi_{j_0,k}(t) = 2^{-j_0/2} \varphi(2^{-j_0} t - k \quad \text{and} \quad \psi_{j,k}(t) = 2^{-j/2} \psi(2^{-j} t - k) \quad (20)$$

and their respective scaling and wavelet coefficients are

$$c_{j_0,k} = \int_R X_t \varphi_{j_0,k}(t) dt \quad \text{and} \quad g_{j,k} = \int_R X_{j,k}(t) dt \quad (21)$$

For a discrete timeseries, the discrete wavelet transform is used. In order to obtain the vector of wavelet coefficients$w$, the $1 \times T$ vector of noisy data X is multiplied by an appropriate $T \times T$wavelet matrix W (whose elements depend on a specific wavelet family)

$$w = WX \quad (22)$$

The vector of wavelet coefficients consists of different sub-vectors, each of length2^j, $(j = 1, \ldots J)$ which represent different resolution levels of the data. For a dyadic length time series with monthly sampling frequency the first resolution level captures frequency variation with duration of 2–4 months. Analogously, the second resolution level captures variation of 4–8 months; the level 3 resolutions capture variation of 8–16 months and so on, up to level J.

Since the data contain measurement errors (noise) this will also be transferred to specific wavelet coefficients. Donoho and Johnston (1994, 1995) proposed a soft thresholding rule to remove the noisy wavelet coefficients associated with the highest frequencies (short-term cyclical fluctuations) and construct noise free estimates of the original data vector$X$. In the first stage, the following thresholding rule is applied to the data

$$\widehat{w} = \begin{cases} sgn(w)(|w| - \tau) if |w| \geq \tau \\ 0 if |w| < \tau \end{cases} \quad \text{where} \quad sgn = \begin{cases} +1 if w > 0 \\ 0 if w = 0 \\ -1 if w < 0 \end{cases} \qquad (23)$$

This rule pushes all coefficients towards zero, but when their magnitude is smaller than the thresholdτ, which defines the level of noise in the data, they are set to zero. The resulting wavelet coefficients are free from noise. In the second stage, an inverse wavelet transform is applied to the vector to obtain noise free estimates of the original data vector X as follows:

$$\hat{X} = W^{-1}\widehat{w} \qquad (24)$$

Obviously, the choice of the threshold is critical, and this subject is extensively researched in the statistics literature. In the empirical applications of this article,we use the universal threshold, $\tau = \hat{\sigma}_\tau\sqrt{2 \log N}$, proposed by Donoho and Johnston (1994), where $\hat{\sigma}_\tau$ is the standard deviation of the wavelet coefficients at the finest level of detail.

Applying the smoothing or denoising techniques discussed above to the variable of interest, we have denoised version of the estimators in section 2 for different techniques of smoothing considered.

V. Simulation Study

Here, the performance of the linear combination of order statistics known as L- estimator are considered based on different smoothing techniques i.e logistic kernel, Guassian kernel and wavelet. Our interest is to draw a sample integrated with error from pseudo-population. The true variable f_0 and error ε_0 are generated from sample size $n = 256$as $g = f_0 + \varepsilon_0$to provide noisy data,where $f_o \sim N(n, mean = 10, sd = 2)$ and $\varepsilon_0 \sim N(n, mean = 0, sd = 0.25)$ is independent variate of normal distribution.. Thus, the process is repeated for 1000 times. The choiceof the smoothing parameters for the kernel and wavelet smoothing techniques are selected by plug-in ($\hbar_{opt} = 0.9[min(s, lQR)]/1.34n^{-\frac{1}{5}}$) and universal threshold ($\tau = \hat{\sigma}_\tau\sqrt{2 \log N}$) methods respectively, we have $\hbar = 0.7690 \; and \; \tau = 6.1271$. Further, we applied the smoothers to denoise the noisy data g.Using *lmomco*library from Rpackage,we compute the Sen Weighted mean, its bias variance, standard deviation and mean squared error (MSE). and presented in Table 1. The L-moment and L-ratio are as well computed and presented in Table 2.

Table 1:Sen Weighted under Logistic, Gaussian and Wavelet Smoothing Methods

L-estimator	Undenoised	Logistic Kernel	Gaussian Kernel	Wavelet
$S_{n:k}$	10.0331	9.8978	9.9661	10.0331
Bias	0.0331	-0.1022	-0.0339	0.0331
MSE	3.6951	3.7050	3.6785	0.1282
RMSE	1.9222	1.9248	1.9179	0.3580
SD	1.9220	1.9221	1.9176	0.3565

Table2: L-moments and L-ratios

L-estimators	Undenoised	Logistic Kernel	Gaussian Lernel	Wavelet
λ_1= First L-moment (L-mean)	10.0331	9.8978	9.9661	10.0331
λ_2= Second L-moment (L-scale)	1.0882	1.0877	1.0854	0.2046
λ_2/λ_1= L-Coefficient of variation (L-CV)	0.1085	0.1099	0.1089	0.0204
λ_3 = Third L-moment	-0.0005	0.0204	0.0001	-0.0053
λ_4 = Fourth L-moment	0.1480	0.1488	0.1484	0.0123
λ_3/λ_2= L-Skweness (τ_3)	-0.0004	0.0187	0.0001	-0.0258
λ_4/λ_2= L-Kurtosis (τ_4)	0.1360	0.1368	0.1368	0.0601

From the results in Table 1 and two, it is clear that there is a special connection between the sen weighted mean and the L-moment estimate i.e; $S_{n:k} = \lambda_1$. The relationship between L-scale and pie π provides the SD of the data distribution. i.e$\lambda_2 \times \sqrt{\pi} = SD$. For fair comparisons of the effectiveness of the smoothing methods (denoised L-estimators) considered, we use the MSE computed in Table 1. Examine the MSEs, it can be observed that the denoised L-estimator under wavelet smoothing has the smallest MSE, follow by denoised L-estimator under Gaussian Kernel, Undenoised L-estimator and lastly the denoised L-estimator under Logistic Kernel. Also, it can be seen in Table 2 that the L-skewness under the different smoothers considered are closer to zero, which show that the data is almost symmetric, and the L-kurtosis shows that the probability density function of the data distribution for the different smoothers has no flatter tail.

VI. Conclusion

This study estimating parameters which are based on denoised linear combination of order statistics. The Logistic Kernel, Gaussian Kernel and Wavelet smoothers are used to denoise the variable of interest. the bandwidth of the smoothers was selected by plug-in and universal threshold methods for kernel and wavelet smoothers respectively. The performance of the denoised linear combination of order statistics is compared based on mean squared error (MSE) criterion to determine the effectiveness of the smoothing methods considered. The simulation study carried out for sample size 256 with 1000 Monte Carlo samples, show that denoised linear combination of order statistic under wavelet smoother which has the smallest MSE is the most efficient smoother and suitable for smoothing.

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