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Research Paper

Mathematical model formulation for Nigerian Stock Price Returns under Covid-19 and Economic Insurgence induced Volatility Uncertainties

Sunwa Adedoyin Raji, Philip Ajibola Bankole, Timothy Olufemi Olatunde
Mathematics Education Department, Lagos State University of Education, Oto-Ijanikin, Nigeria.
Sunwaraj123@yahoo.com, bankolepa@lasued.edu.ng, olatundeto@lasued.edu.ng
Corresponding Author: Sunwa Adedoyin Raji

Abstract: *This paper presents mathematical model for option valuation on an underlying Nigerian stocks under Covid-19 and Economic insurgence induced volatility uncertainties. The proposed model is referred to as 'Cov-Ins Dual Stochastic Volatility Model (CDSVM)'. The characteristic function of the model CDSVM was derived and a closed-form formula was obtained for option price valuation. A simulation studies was implemented using the model closed-form formula to obtain option prices in comparison with Euler and Alfonsi numerical schemes for the model. The result obtained shows the applicability of the model in an unstable economy for asset price valuation for decision making amidst uncertainties in financial market.*

Keywords: *Covid-19, Economic insurgence, Stochastic volatility, Stock option price*

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I. INTRODUCTION

The economic state of a nation determines the level of wellbeing of the people and the expected development of the nation. Investors and business men has suffered great loss under uncertainties surrounding the returns of their investment in an unstable economy. The various sources of uncertainties affecting assets returns among others include Covid-19 pandemic and Economic Insurgence activities. The fall in the standard of living of many Nigerian, investors and businessmen is on increase nationwide which should not be taking for granted. COVID-19 pandemics outbreak has been reported to have negative impact on Stocks' returns, as well as increase in volatility of the stock price in an unstable economy [1].

In recent time, studies on Covid-19 pandemic effect on the global economy and international financial markets were reported [2]. Sun-Yong Choi also carried out a study on Industry volatility and economic uncertainty in relationship with the Covid-19 pandemic [3]. A canonical epidemiology model methodology was used to study economic decisions in pandemic situation [4]. In the same study in [4], the "existence of some inevitable trade-off between the severity of the short-run recession caused by the pandemic and the health consequences of the Covid-19 spread" were discussed. In recent time, [5] investigated the effect of Covid-19 outbreak on the Nigerian Stock Exchange performance using GARCH Models. It is evident that Covid-19 has some effect on output of an investment. Studies have shown that information flow from a recessed economy impacts investor's decision in uncertain financial markets. Some studies by Bankole and Ugbebor recently have proven that economic state influences asset's price dynamics [6-7]. This motivated Bankole and Adinya to introduce volatility control variable in an asset valuation model with term structure of stochastic interest rate and a single factor stochastic volatility [8]. Asset price dynamics predictive modelling are thus necessary [9]. The asset's price is either on the increase or decrease depending on the information flow from the unstable economy into the uncertain financial market. However, stocks are very risky form of asset which possess high tendency to undergo price fluctuations in response to every filtration of bad news inflow to the market from the economy. The bad news may be due to Covid-19 pandemic outbreak, insurgency attacks on investors, and the likes of that, could negatively cause price fluctuations and decline in investment returns in financial markets.

The key parameter which determines the variance in the output of an asset is tailored to volatility parameter in asset valuation models. Prediction of the exact assets' price (payoffs) especially the risky ones such as stocks becomes uncertain especially in an unstable economy. Options price on an underlying asset (tradeable financial instruments) depends on the market filtration (information) and the news inflow from the economy into the market. Investors' decision in the financial market is dependent on the state of the economy. The behaviour of stock assets price is best described as stochastic process in nature. Some stochastic volatility models have been formulated in the past for valuation of options price on stocks' asset in the financial market. Assets' price becomes more volatile under the exposure of uncertainties. COVID-19 and Economic Insurgence among others are considered to increase the uncertainty level of stock asset returns. Insurgence activities has been seen to affect the supply chain. Insecurity challenge in some part of Northern region of Nigeria has been reported to affect production. As a matter of fact, farmers were mostly affected as they couldn't go to the farm with rest of mind, this invariably affect agricultural produce and the supply chain.

Consequently, the various bad news of economy insurgence contributes to economic instability as well as market price fluctuation of risky assets. When the financial market becomes volatile, the asset price accuracy prediction becomes difficult. That effect has led many researchers to embrace stochastic model formulation incorporating other uncertainty variables in the stochastic volatility models. A good number of stochastic volatility models are reported in literature, such as a single stochastic volatility Heston model [10], Double Heston model [11], Grzelak and Oosterlee [12], Huang S. and Xunxiang, G. [13], and Guohe [14]. Other focus of studies in financial markets includes the addition of jumps in stochastic volatility models, to mention few. [15] modified the Heston model to forecast Stock prices.

However, to the best of our knowledge, the two-fold concept of COVID-19 and Economic Insurgence induced volatility is yet to be incorporated in any stochastic volatility model for asset's value computation. Therefore, this study is geared towards formulation of a new stochastic volatility model incorporating the concept of COVID-19 and Economic Insurgence (EI) induced volatility uncertainties for option pricing on an underlying Nigerian stocks, and the solution of such model for decision making on an investment. The rest part of this study includes: Preliminaries to the Cov-Ins Dual Stochastic Volatility Model (CDSVM) formulation, the Model, Main results, Simulation studies, and Conclusion.

II. Preliminaries to the model formulation

2.1 The Assumption: Let X_t be a stock asset under dual volatility source. That is, $v_{X_t}^{cov}$ and $\sigma_{X_s}^{ins}$, which are respectively the market volatility from covid-19 surge, and the economic insurgence induced market volatility such that

$$v_1(t) = v^{cov} + v^{ins}. \qquad (1)$$

2.2 Covid-19 and Economic insurgence Uncertain variable

Let $\xi(\gamma)$ be Covid-19 and economic insurgence induced variable defined on an uncertain space $(\Gamma, \mathcal{L}, \mathcal{M})$, with $\gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ such that $\mathcal{M}\{\gamma_1\} = \alpha_1$, $\mathcal{M}\{\gamma_2\} = \alpha_2$, $\mathcal{M}\{\gamma_3\} = \alpha_3$.
Then

$$\xi(\gamma) = \begin{cases} 0, & \text{if} \quad \gamma = \gamma_1 \\ \frac{1}{2}, & \text{if} \quad \gamma = \gamma_2 \\ 1, & \text{if} \quad \gamma = \gamma_3 \end{cases} \qquad (2)$$

is an uncertain variable. The following holds

$$\mathcal{M}\{\xi = 0\} = \mathcal{M}\{\gamma \mid \xi(\gamma) = 0\} = \mathcal{M}\{\gamma_1\} = \alpha_1,$$

$$\mathcal{M}\left\{\xi = \tfrac{1}{2}\right\} = \mathcal{M}\left\{\gamma \mid \xi(\gamma) = \tfrac{1}{2}\right\} = \mathcal{M}\{\gamma_2\} = \alpha_2,$$

and

$$\mathcal{M}\{\xi = 1\} = \mathcal{M}\{\gamma \mid \xi(\gamma) = 1\} = \mathcal{M}\{\gamma_3\} = \alpha_3$$

such that $\mathcal{M}\{\gamma_1\} + \mathcal{M}\{\gamma_2\} + \mathcal{M}\{\gamma_3\} \equiv \alpha_1 + \alpha_2 + \alpha_3 = 1.$

III. The Model

Let the price of some selected Nigerian stock assets, X, be defined on a filtered probability space $(\Omega, \mathcal{F}_t, Q, \mathbb{F})$ and assume market filtration is generated by standard Wiener process in a specified time, $t \in [0, T]$. Taking Q to be a risk-neutral probability measure. The model governing the dynamics of the underlying Nigerian stocks price, $S(t)$, is presented as:

$$\begin{cases} \dfrac{dS(t)}{S(t)} = (r-q)dt + \sqrt{v_{1t}}dW_1(t) + \sqrt{v_{2t}}dW_2(t), \qquad S(0) = S_0 > 0 \\ dv_1(t) = \alpha\left(\kappa_1\left(\vartheta^{cov} + \vartheta^{ins} + \vartheta - v_1(t)\right)dt + \sigma_1\sqrt{v_1(t)}d\widehat{W}_1(t)\right), \quad v_1(0) = v_{1,0} > 0, \\ dv_2(t) = \kappa_2(\theta - v_2(t))dt + \sigma_2\sqrt{v_2(t)}d\widehat{W}_2(t), \quad v_2(0) = v_{2,0} > 0. \end{cases} \tag{3}$$

where $v_1(t) = v^{cov} + v^{ins}$, α is a well-defined economic state dependent control variable in:

$$\alpha := \begin{cases} \frac{1}{4}\hat{s} \mid 0 < \hat{s} \le 1, & if\ the\ economy\ is\ in\ Covid-19\ only; \\ \frac{1}{2}\hat{s} \mid 0 < \hat{s} \le 1, & if\ the\ economy\ is\ in\ Insurgence\ state\ only. \\ \frac{3}{4}\hat{s} \mid 0 < \hat{s} \le 1, & if\ the\ economy\ is\ both\ under\ Covid-19\ and\ Insurgence\ state. \\ 1, & if\ the\ economy\ is\ stable. \end{cases}$$

The variable function $S(t)$ denotes the asset value at time, t, the $\sigma_j, j = 1{,}2$ as the volatility of volatility terms for the stochastic volatility processes $v_1(t)$ and $v_2(t)$ respectively, $v_{1,0}$ and $v_{2,0}$ as the initial variance, $\rho_j, j = 1{,}2$ are correlations, and W_1 and W_2 are two Brownian motions driving the system. The model parameters are described as follow: r is the *riskfree interest rate*, q is the *dividend rate*, $\kappa_j, j = 1{,}2$ are the *mean reverting rate*, the long term volatility constant from covid, insurgence and other source is $\vartheta^{cov} + \vartheta^{ins} + \vartheta$ respectively for the stochastic volatility process $v_1(t)$ while θ is the existing *long term volatility constant* in the market for the stochastic volatility process, $v_2(t)$. We refer to this model as Cov-Ins Dual Stochastic Volatility Model (CDSVM).

The model has the following stochastic correlation structure:

$$\mathrm{cor}(dW_1, dW_2)_t = \mathrm{cor}(dW_1, d\widehat{W}_2)_t = \mathrm{cor}(dW_2, d\widehat{W}_1)_t = \mathrm{cor}(d\widehat{W}_1, d\widehat{W}_2)_t = 0,$$
$$\mathrm{cor}(dW_1, d\widehat{W}_1)_t = \rho_1 dt, \quad \mathrm{cor}(dW_2, d\widehat{W}_2)_t = \rho_2 dt.$$

IV. MAIN RESULTS

Theorem 4.1: Let a stock asset price, $S(t)$, evolves by the model given in (3), the characteristic function for stock price forecast under recession is of the form:

$$f(i\varphi) = \exp(C_1(T-t) + C_2(T-t) + D(T-t)x_t + E(T-t)v_1(t) + F(T-t)v_2(t) + i\varphi x_t) \tag{4}$$

where $C_1(T-t), C_2(T-t), D(T-t), E(T-t)$, are deterministic constants for the stochastic processes, $x = \ln S(t)$ and $v_1 = v^{cov} + v^{ins}$ well-defined in equation *(1)*. Following the authors [16-17] the characteristic function of the related Partial Integro-Differential Equation (PIDE) evolves in the form given in (4).

Proof:

Let the logarithm stock price, $x = \ln S(t)$, satisfy (4), applying Itô Lemma, the drift term of the model characteristic function is given as:

$$\frac{\partial f}{\partial t} + \left(r - q - \frac{v_1(t)+v_2(t)}{2}\right)\frac{\partial f}{\partial x} + \kappa_1\left(\vartheta^{cov} + \vartheta^{ins} + \vartheta - v_1(t)\right)\frac{\partial f}{\partial v_1} + \kappa_2(\theta - v_2(t))\frac{\partial f}{\partial v_2} + \\ + \frac{1}{2}(v_1(t) + v_2(t))\frac{\partial^2 f}{\partial x^2} + \frac{1}{2}\sigma_1^2 v_1(t)\frac{\partial^2 f}{\partial v_1^2} + \frac{1}{2}\sigma_2^2 v_2(t)\frac{\partial^2 f}{\partial v_2^2} + \rho_1\sigma_1 v_1(t)\frac{\partial^2 f}{\partial x \partial v_1} + \rho_2\sigma_2 v_2(t)\frac{\partial^2 f}{\partial x \partial v_2}. \tag{5}$$

By the fundamental asset valuation theorem, [17-18] the drift term given in equation (5) is set to zero (0), and the partial derivatives are substituted to obtain the following:

$$0 = f[\left(r - q - \frac{v_1(t)+v_2(t)}{2}\right)(D(T-t)+i\varphi) + \kappa_1(\vartheta^{cov} + \vartheta^{ins} + \vartheta - v(t)) \times$$
$$\left(E(T-t) - \frac{\partial C_1(T-t)}{\partial t} - \frac{\partial D(T-t)}{\partial t}x(t)\right) - \frac{\partial E(T-t)}{\partial t}v_1(t) + \frac{1}{2}v_1(t)(D(T-t)+i\varphi)^2$$
$$+ \frac{1}{2}\sigma_1^2 v_1(t)(E(T-t))^2 + \rho_1\sigma_1 v_1(t)(D(T-t)+i\varphi)E(T-t) + \kappa_2(\theta - v_2(t)) \times \tag{6}$$
$$\left(F(T-t) - \frac{\partial C_2(T-t)}{\partial t} - \frac{\partial D(T-t)}{\partial t}x(t)\right) - \frac{\partial F(T-t)}{\partial t}v_2(t) + \frac{1}{2}v_2(t)(D(T-t)+i\varphi)^2$$
$$+ \frac{1}{2}\sigma_2^2 v_2(t)(E(T-t))^2 + \rho_2\sigma_2 v_2(t)(D(T-t)+i\varphi)F(T-t)].$$

Simplifying and arranging in terms of the stochastic processes $x(t)$, $v_j(t), j = 1,2$, and the constant term leads to:

$$0 = [-\frac{\partial D(T-t)}{\partial t}]x(t) + [-\frac{1}{2}D(T-t) - \frac{1}{2}i\varphi - \kappa_1 E(T-t) - \frac{\partial E(T-t)}{\partial t} + \frac{1}{2}(D(T-t))^2$$
$$+ i\varphi\big(D(T-t)\big) - \frac{1}{2}\varphi^2 + \frac{1}{2}\sigma_1^2\big(E(T-t)^2\big) + \rho_1\sigma_1\big(D(T-t)\big)\big(E(T-t)\big) + \rho\sigma i\varphi\big(C(T-t)\big)\big]v_1(t)$$
$$+ \left[(r-q)D(T-t) + (r-q)i\varphi + \kappa_1(\vartheta^{cov} + \vartheta^{ins} + \vartheta)E(T-t) - \frac{\partial C_1(T-t)}{\partial t}\right] \tag{7}$$
$$+ [-\frac{1}{2}D(T-t) - \frac{1}{2}i\varphi - \kappa_2 F(T-t) - \frac{\partial F(T-t)}{\partial t} + \frac{1}{2}(D(T-t))^2 + i\varphi(D(T-t)) - \frac{1}{2}\varphi^2$$
$$+ \frac{1}{2}\sigma_2^2\big(F(T-t)^2\big) + \rho_2\sigma_2\big(D(T-t)\big)\big(F(T-t)\big) + \rho_2\sigma_2 i\varphi\big(C_2(T-t)\big)\big]v_2(t)$$
$$+ \left[(r-q)D(T-t) + (r-q)i\varphi + \kappa_2(\theta)F(T-t) - \frac{\partial C_2(T-t)}{\partial t}\right].$$

Since the stochastic processes, $x(t)$, $v_1(t)$ and $v_2(t)$ cannot be zero, we equate the coefficients to be zero as well as the constant terms in (7).

The following system of ordinary differential equations emerged:

$$\frac{\partial D(T-t)}{\partial t} = 0, \tag{8}$$

$$\frac{\partial C_1(T-t)}{\partial t} = (r-q)i\varphi + \kappa_1(\vartheta^{cov} + \vartheta ins + \vartheta)E(T-t) + (r-q)D(T-t), \tag{9}$$

$$\frac{\partial C_2(T-t)}{\partial t} = (r-q)i\varphi + \kappa_2\theta F(T-t) + (r-q)D(T-t), \tag{10}$$

$$\begin{aligned}\frac{\partial E(T-t)}{\partial t} \;=\; & -\frac{1}{2}D(T-t) - \frac{1}{2}i\varphi - \kappa_1 E(T-t) + \frac{1}{2}(D(T-t))^2 + i\varphi(D(T-t)) \\ & - \frac{1}{2}\varphi^2 + \frac{1}{2}\sigma_v^2(E(T-t))^2 + \rho_1\sigma_1(D(T-t))(E(T-t)) + \rho_1\sigma_1 i\varphi(E(T-t)),\end{aligned} \tag{11}$$

and

$$\begin{aligned}\frac{\partial F(T-t)}{\partial t} \;=\; & -\frac{1}{2}D(T-t) - \frac{1}{2}i\varphi - \kappa_2 F(T-t) + \frac{1}{2}(D(T-t))^2 + i\varphi(D(T-t)) \\ & - \frac{1}{2}\varphi^2 + \frac{1}{2}\sigma_2^2(F(T-t))^2 + \rho_2\sigma_2(D(T-t))(F(T-t)) + \rho_2\sigma_2 i\varphi(F(T-t)).\end{aligned} \tag{12}$$

To this end, at the option maturity time, $t = T$, results to $f(i\varphi) = \exp(i\varphi x(T))$, and the following initial conditions holds: $C(0) = 0, \quad D(0) = 0, \quad E(0) = 0, \quad F(0) = 0.$

Also, from (8),

$$\frac{\partial D(T-t)}{\partial t} = 0 \quad \text{and } D(0) = 0 \;\Rightarrow\; D(T-t) = 0. \tag{13}$$

Using (13) in (11), we have:

$$\frac{\partial E(T-t)}{\partial t} = -\frac{1}{2}i\varphi - \kappa_1 E(T-t) - \frac{1}{2}\varphi^2 + \frac{1}{2}\sigma_1^2(E(T-t))^2 + \rho_1\sigma_1 i\varphi(E(T-t)). \tag{14}$$

Using the initial condition, $C_1(0) = 0$, we have:

$$\frac{\partial E(T-t)}{\partial t} = -\frac{1}{2}\sigma_1^2\left[E^2(T-t) + \left(\frac{2\rho_1 i\varphi}{\sigma_1^2} - \frac{2\kappa_1}{\sigma_1^2}\right)E(T-t) - \frac{i\varphi}{\sigma_1^2} - \frac{\varphi^2}{\sigma_1^2}\right]. \tag{15}$$

The equation (15) is a form of Ricatti differential equation. The reader can see the following references for a related solution of Ricatti differential equation ([7], [10], [17], [20]). We give the solution of the equation (15) as:

$$E(T-t) = \frac{\left(e^{d_j(T-t)}-1\right)\left(\rho_1\sigma_1 i\varphi-\kappa_1-d_j\right)}{\sigma_1^2\left(1-g_j e^{d_j(T-t)}\right)}, \tag{16}$$

where

$$d_j = \sqrt{(\rho_1\sigma_1 i\varphi - \kappa_1)^2 + \sigma_1^2(i\varphi + \varphi^2)}$$

$$g_j = \frac{\rho_1\sigma_1 i\varphi-\kappa_1-d_j}{\rho_1\sigma_1 i\varphi-\kappa_1+d_j}.$$

Setting the time to maturity of the option $\tau = T - t$, the solution (15) is written as:

$$E(\tau) = \frac{\left(\kappa_1-\rho_1\sigma_1 i\varphi+d_j\right)\left(1-e^{d_j\tau}\right)}{\sigma_1^2\left(1-g_j e^{d_j\tau}\right)}, \tag{17}$$

Now, substituting (13) in (12), we have:

$$\frac{\partial F(T-t)}{\partial t} = -\frac{1}{2}i\varphi - \kappa_2 F(T-t) - \frac{1}{2}\varphi^2 + \frac{1}{2}\sigma_2^2(F(T-t))^2 + \rho_2\sigma_2 i\varphi(F(T-t)). \tag{18}$$

Applying the initial condition, $C_2(0) = 0$, we have:

$$\frac{\partial F(T-t)}{\partial t} = -\frac{1}{2}\sigma_2^2\left[F^2(T-t) + \left(\frac{2\rho_2 i\varphi}{\sigma_2^2} - \frac{2\kappa_2}{\sigma_2^2}\right)F(T-t) - \frac{i\varphi}{\sigma_2^2} - \frac{\varphi^2}{\sigma_2^2}\right]. \tag{19}$$

The equation (19) is also a Ricatti differential equation. The solution of the equation (19) is expressed as:

$$F(T-t) = \frac{\left(e^{d_j(T-t)}-1\right)\left(\rho_2\sigma_2 i\varphi-\kappa_2-d_j\right)}{\sigma_2^2\left(1-g_j e^{d_j(T-t)}\right)}, \tag{20}$$

where

$$d_j = \sqrt{(\rho_2\sigma_2 i\varphi - \kappa_2)^2 + \sigma_2^2(i\varphi + \varphi^2)}$$

$$g_j = \frac{\rho_2\sigma_1 i\varphi-\kappa_2-d_j}{\rho_2\sigma_2 i\varphi-\kappa_2+d_j}.$$

At the time to maturity of the option $\tau = T - t$, the solution (19) is also given as:

$$F(\tau)=\frac{(\kappa_2-\rho_2\sigma_2 i\varphi+d_j)\left(1-e^{d_j\tau}\right)}{\sigma_2^2\left(1-g_je^{d_j\tau}\right)}, \tag{21}$$

Next, we solve for $C_1(T-t)$ in equation (8) in what follows.

$$\frac{\partial C_1(T-t)}{\partial t}=(r-q)i\varphi+\kappa_1(\vartheta^{cov}+\vartheta^{ins}+\vartheta)E(T-t)+(r-q)D(T-t). \tag{22}$$

Substituting (13) into (22), it reduces to:

$$\frac{\partial C_1(T-t)}{\partial t}=(r-q)i\varphi+\kappa_1(\vartheta^{cov}+\vartheta^{ins}+\vartheta)E(T-t). \tag{23}$$

We integrate the both sides of equation (23) as follows:

$$\int_{s=t}^{s=T}\partial C_1(T-s)ds=(r-q)i\varphi ds\int_{s=t}^{s=T}\kappa(\vartheta^{cov}+\vartheta^{ins}+\vartheta)E(T-s)ds. \tag{24}$$

$$\Rightarrow\quad -C_1(T-s)|_{s=t}^{s=T}=(r-q)i\varphi s|_{s=t}^{s=T}+\int_{s=t}^{s=T}\kappa_1(\vartheta^{cov}+\vartheta^{ins}+\vartheta)E(T-s)ds, \tag{25}$$

$$\Rightarrow\quad -C_1(0)+C_1(T-t)=(r-q)i\varphi(T-t)+\kappa_1(\vartheta^{cov}+\vartheta^{ins}+\vartheta)\int_{s=t}^{s=T}E(T-s)ds. \tag{26}$$

Using (21) in (25), we have:

$$C_1(T-t)=(r-q)i\varphi(T-t)+\kappa_1(\vartheta^{cov}+\vartheta^{ins}+\vartheta)\int_{s=t}^{s=T}\frac{\left(e^{d_j(T-s)}-1\right)\left(\rho_1\sigma_1 i\varphi-\kappa_1-d_j\right)}{\sigma_v^2\left(1-g_je^{d_j(T-s)}\right)}ds,$$

$$=(r-q)i\varphi(T-t)+\frac{\kappa_1(\vartheta^{cov}+\vartheta^{ins}+\vartheta)}{\sigma_1^2}\left(\rho_1\sigma_1 i\varphi-\kappa_1-d_j\right)\int_{s=t}^{s=T}\frac{\left(e^{d_j(T-s)}-1\right)}{\left(1-g_je^{d_j(T-s)}\right)}ds,$$

$$=(r-q)i\varphi(T-t)+\frac{\kappa_1(\vartheta^{cov}+\vartheta^{ins}+\vartheta)}{\sigma_1^2}\left(\rho\sigma_1 i\varphi-\kappa_1-d_j\right)\times\left[-\ln\frac{\left(e^{d_j(T-s)}-1\right)}{d_j}+\frac{\ln\left(g_je^{d_j(T-s)}-1\right)}{d_j\cdot g_j}+\frac{\ln\left(e^{d_j(T-s)}\right)}{d_j}d_j\right]_{s=t}^{s=T}.$$

Simplifying further yields:

$$C_1(T-t)=(r-q)i\varphi(T-t)+\frac{\kappa_1(\vartheta^{cov}+\vartheta^{ins}+\vartheta)}{\sigma_1^2}\left(\rho_1\sigma_1 i\varphi-\kappa_1-d_j\right)\times\left[\left(\frac{-\ln(g_j-1)}{d_j}+\frac{\ln(g_j-1)}{d_jg_j}+\frac{\ln 1}{d_j}\right)-\left(-\frac{\ln\left(g_je^{d_j(T-t)}-1\right)}{d_j}+\frac{\ln\left(g_je^{d_j(T-t)}-1\right)}{d_j\cdot g_j}+\frac{\ln\left(e^{d_j(T-t)}\right)}{d_j}\right)\right],$$

Setting the time to maturity $T-t:=\tau$, the simplification continue as follows:

$$C_1(\tau)=(r-q)i\varphi\tau+\frac{\kappa_1(\vartheta^{cov}+\vartheta^{ins}+\vartheta)}{\sigma_1^2}\left(\rho_1\sigma_1 i\varphi-\kappa_1-d_j\right)\left[\frac{-\ln(g_j-1)}{d_j}+\frac{\ln(g_j-1)}{d_jg_j}+\frac{\ln\left(g_je^{d_j\tau}-1\right)}{d_j}-\frac{\ln\left(g_je^{d_j\tau}-1\right)}{d_j\cdot g_j}-\frac{\ln\left(e^{d_j\tau}\right)}{d_j}\right],$$

$$= (r-q)i\varphi\tau + \frac{\kappa_1(\vartheta^{cov}+\vartheta^{ins}+\vartheta)}{\sigma_1^2}(\rho_1\sigma_1 i\varphi - \kappa_1 - d_j)\left[\frac{1}{d_j}\left(ln(g_j e^{d_j(T-t)}-1) - \ln(g_j-1)\right) + \frac{1}{d_j\cdot g_j}\left(\ln(g_j-1) - \ln(g_j e^{d_j\tau}-1)\right) - \frac{d_j\tau}{d_j}\right],$$

$$C_1(\tau) = (r-q)i\varphi\tau + \frac{\kappa_1(\vartheta^{cov}+\vartheta ins+\vartheta)}{\sigma_1^2}(\rho_1\sigma_1 i\varphi - \kappa_1 - d_j)\left[\frac{1}{d_j}\ln\left(\frac{g_j e^{d_j\tau}-1}{g_j-1}\right) + \frac{1}{d_j g_j}\ln\left(\frac{g_j-1}{g_j e^{d_j\tau}-1}\right) - \tau\right],$$

$$= (r-q)i\varphi\tau + \frac{\kappa_1(\vartheta^{cov}+\vartheta^{ins}+\vartheta)}{\sigma_1^2}(\rho_1\sigma_1 i\varphi - \kappa_1 - d_j)\left[\frac{1}{d_j}\ln\left(\frac{1-g_j e^{d_j\tau}}{1-g_j}\right) + \frac{1}{d_j g_j}\ln\left(\frac{1-g_j}{1-g_j e^{d_j\tau}}\right) - \tau\right],$$

$$= (r-q)i\varphi\tau + \frac{\kappa_1(\vartheta^{cov}+\vartheta^{ins}+\vartheta)}{\sigma_1^2}(\rho_1\sigma_1 i\varphi - \kappa_1 - d_j)\left[\frac{g_j\ln\left(\frac{1-g_j e^{d_j\tau}}{1-g_j}\right) + \ln\left(\frac{1-g_j}{1-g_j e^{d_j\tau}}\right)}{d_j\cdot g_j} - \tau\right],$$

$$C_1(\tau) = (r-q)i\varphi\tau + \frac{\kappa_1(\vartheta^{cov}+\vartheta^{ins}+\vartheta)}{\sigma_1^2}\left[-(\rho_1\sigma_1 i\varphi - \kappa_1 - d_j)\tau + (\rho_1\sigma_1 i\varphi - \kappa_1 + d_j)\cdot\left(\frac{g_j-1}{d_j}\right)\ln\left(\frac{1-g_j e^{d_j\tau}}{1-g_j}\right)\right],$$

$$= (r-q)i\varphi\tau + \frac{\kappa_1(\vartheta^{cov}+\vartheta^{ins}+\vartheta)}{\sigma_1^2}\left[-(\rho_1\sigma_1 i\varphi - \kappa_1 - d_j)\tau + (\rho\sigma_1 i\varphi - \kappa_1 + d_j)\cdot\left(\frac{\frac{\rho_1\sigma_1 i\varphi-\kappa_1-d_j}{\rho_1\sigma_1 i\varphi-\kappa_1+d_j}-1}{d_j}\right)\ln\left(\frac{1-g_j e^{d_j\tau}}{1-g_j}\right)\right],$$

$$= (r-q)i\varphi\tau + \frac{\kappa_1(\vartheta^{cov}+\vartheta^{ins}+\vartheta)}{\sigma_1^2}\left[(\kappa_1 - \rho_1\sigma_1 i\varphi + d_j)\tau + \left(\frac{\rho_1\sigma_1 i\varphi - \kappa_1 - d_j - \rho_1\sigma_1 i\varphi + \kappa_1 - d_j}{d_j}\right)\ln\left(\frac{1-g_j e^{d_j\tau}}{1-g_j}\right)\right],$$

This is finally given as:

$$C_1(\tau) = (r-q)i\varphi\tau + \frac{\kappa_1(\vartheta^{cov}+\vartheta^{ins}+\vartheta)}{\sigma_1^2}\left[(\kappa_1 - \rho_1\sigma_1 i\varphi + d_j)\tau - 2\ln\left(\frac{1-g_j e^{d_j\tau}}{1-g_j}\right)\right]. \quad (27)$$

Going through the similar steps from the equation (11) through (27), the solution to the Ricatti differential equation (12) is given as:

$$C_2(\tau) = (r-q)i\varphi\tau + \frac{\kappa_2\theta}{\sigma_2^2}\left[(\kappa_2 - \rho_2\sigma_1 2\varphi + d_j)\tau - 2\ln\left(\frac{1-g_j e^{d_j\tau}}{1-g_j}\right)\right]. \quad (28)$$

We have been able to determine the coefficient terms $C_1(\tau), C_2(\tau), D(\tau), E(\tau)$ and $F(\tau)$, for the characteristic function, $f(i\varphi)$, the equation (4) for the model proposed here. The characteristic function derived will be applied in the simulation studies of the option price.

Theorem 4.2: Let the stock asset price, $S(t)$, evolve under the dynamics expressed in *(3)*. Then an analytic formula for the call option price following European-type is expressed as:

$$C(\tau, K) = S_t \exp(-qT) P_1 - K\exp(-rT) P_2 \tag{29}$$

where P_1 and P_2 denote two probabilities given respectively as:

$$P_1 = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} \Re\left(\frac{e^{-i\varphi k} f(\varphi - i; x_t, v_{1,t}, v_{2,t})}{i\varphi S_t e^{(r-q)\tau}}\right) d\varphi, \quad P_2 = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} \Re\left(\frac{e^{-i\varphi k} f(\varphi\,; x_t, v_{1,t}, v_{2,t})}{i\varphi}\right) d\varphi,$$

$k = \ln K$.

4.1 Simulation of the model CDSVM

The implementation of the proposed model is synonymous to the implementation of the double Heston model but with more variables and parameters. This is carried out by writing MATLAB code. The model parameters are ordered as follow:

$$\Theta = \left(\kappa_1, \theta_1, \sigma_1, v_{1,0}, \rho_1, \kappa_2, \varpi_2, \sigma_2, v_{2,0}, \rho_2\right) \tag{30}$$

where $\varpi_2 = \vartheta^{cov} + \vartheta^{ins} + \vartheta$.

The following starting values were used to obtain an estimates for the Risk Neutral Density for the model CDSVM. Initial stock price $S = 132.14$, $r = 0.0020$, $q = 0.0078$, Time range $T = [35, 70, 130, 220, 300, 450]$.

Table 1: The Model Risk Neutral Density (RND) Estimates

Maturity days	*RND Area value*
35	
70	
130	
220	
300	
450	

The Table 1 above shows the estimated value for the Risk Neutral Densities for an underlying stock S(t) with respect to the specified data using the Cov-Ins Dual stochastic Volatility Model (CDSVM). We tested the model for an assumed maturity days as shown in the Table 1. The Figure 1 below shows the visualization of the Risk Neutral Densities estimates using the CDSVM.

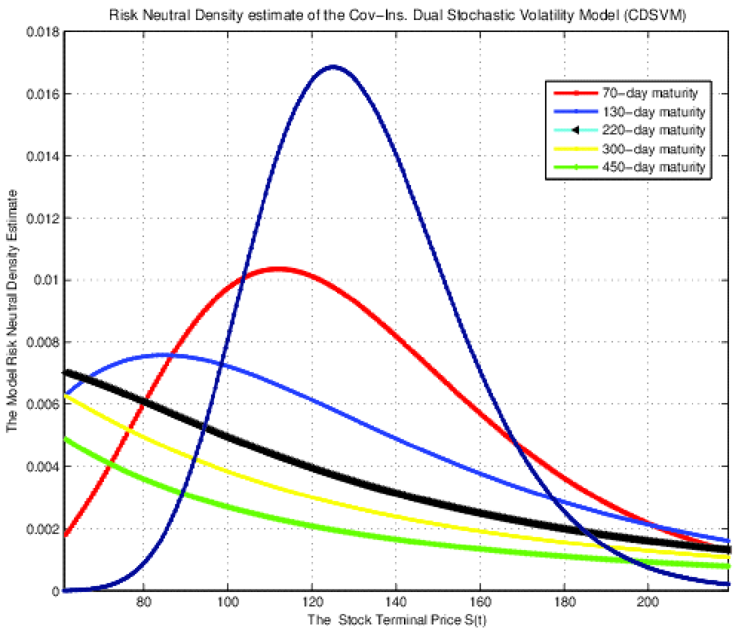


Figure 1: The CDSVM Risk Neutral Densities Graph

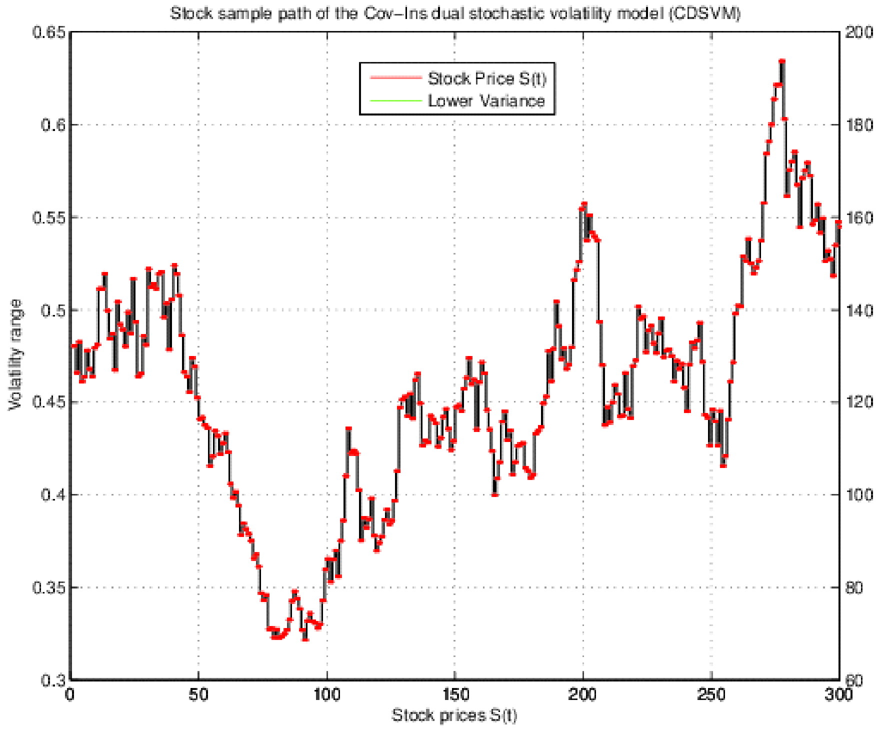
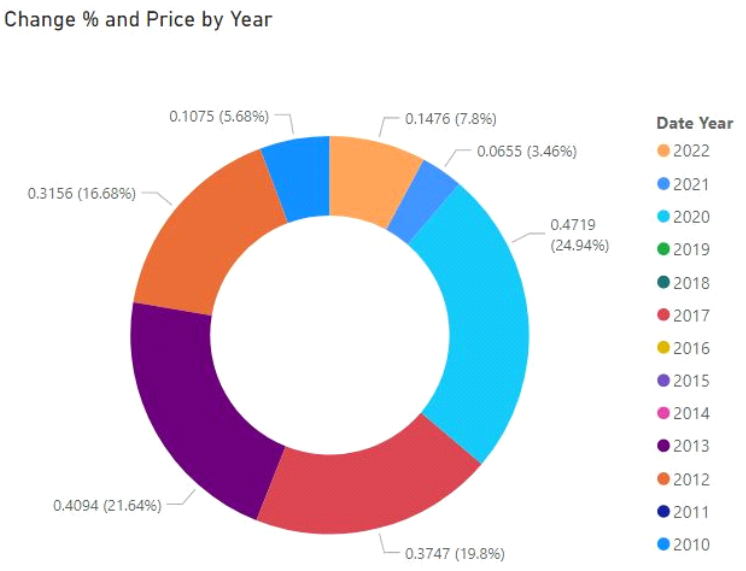


Figure 2: Stocks asset sample paths using the model CDSVM



Year	Change %
2010	0.11
2011	-0.16
2012	0.32
2013	0.41
2014	-0.16
2015	-0.16
2016	-0.03
2017	0.37
2018	-0.17
2019	-0.15
2020	0.47
2021	0.07
2022	0.15
Total	**1.05**

Figure 3: Percentage change in Nigeria Stock Exchange indices from November 2010 - October 2022

Figure 4: NSE price changes from year November 2010 - October 2022

4.2 Options prices obtained from the model CDSVM Simulation

4.2.1 Table of Result

The parameter's value *used for simulation are given below:*

$S_0 = 132.14, K = 80, \quad r = 0.02; \quad q = 0.00; \quad \tau = 0.5; \quad v_{01} = 0.6^2, \; v_{02} = 0.7^2, \; \sigma_1 = 0.1, \quad \sigma_2 = 0.2, \; \rho_1 = 0.5, \rho_2 = 0.9, \; \kappa_1 = 1.2, \; \kappa_2 = 0.6, \; \vartheta^{cov} = 0.05, \vartheta^{ins} = 0.05, \vartheta = 0.05, \; \theta_2 = 0.1$

Table 2: Options prices from some selected simulation schemes for the model CDSVM

Technique	Call option Prices	% Error estimates	Simulation time (secs)
CDSVM closed form formula (Theorem 4.2)	66.3890		
Euler scheme	66.5306	-0.213	7.265
Alfonsi scheme	65.6760	2.580	32.719

In the Table 2 above, the option prices obtained has slight variation in value based on technique (method). Nevertheless, the option price obtained from the model in closed-form, that is, using the Theorem still conforms with the other two numerical schemes considered for the model. On Euler scheme and Alfonsi scheme, the reader could see the references [18-19]. Rouah [20] provided more illustration on simulation studies in a related problem. However, the present study encompasses covid and economic insurgence as it affects stock prices and supply chain.

IV. CONCLUSION

In this study, we considered the notion of uncertainties in option price arising from Covid- and economic insurgence filtration into the financial market amidst other uncertainties source in the financial market. A mathematical form of double stochastic volatility model was formulated in which its characteristic function was derived and applied to obtain a formula for valuation of an option price in the sense of European - type option. The proposed analytic formula for option prices was implemented on Nigerian stocks data gotten from the website of www.investing.com. The results obtained were proven to be applicable in option valuation with respect to the economy states studied in this paper.

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