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Research Paper

An Elementary Proof for Fermat's Last Theorem using an Euler's Equation

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Abstract

Fermat's Last Theorem states that it is impossible to find positive integers A, B and C satisfying the equation

$$A^n + B^n = C^n$$

where n is any integer > 2.
Taking the proofs of Fermat for the index $n = 4$, and Euler for $n = 3$, it is sufficient to prove the theorem for $n = p$, any prime > 3 [1].
We hypothesize that all r, s and t are non-zero integers in the equation

$$r^p + s^p = t^p$$

and establish contradiction.
Just for supporting the proof in the above equation, we have another equation

$$x^3 + y^3 = z^3$$

Without loss of generality, we assert that both x and y as non-zero integers; z^3 a non-zero integer; z and z^2 irrational.
We create transformation equations to the above two equations through parameters, into which we have incorporated an Euler's equation. Solving the transformed equations we prove the theorem.

Keywords: Transformed Fermat's Equations through Parameters.

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I. Introduction

Around 1637, Pierre-de-Fermat, the French mathematician wrote in the margin of his book that the equation $A^n + B^n = C^n$ has no solution in integers A, B and C, if n is any integer > 2. Fermat stated therein the margin of a book that he himself had found a marvelous proof of the theorem, but the margin was too narrow to contain it. His proof is available only for the index $n = 4$, using infinite descent method.

Many mathematicians like Sophie Germain, E.E. Kummer had proved the theorem for particular cases. Number theory has been developed leaps and bounds by the immense contributions by a lot of mathematicians. Finally, after 350 years, the theorem was completely proved by Prof. Andrew Wiles, using highly complicated mathematical tools and advanced number theory[2], [3].

Here we are trying an elementary proof.

II. Assumptions

1) We initially hypothesize that all r, s and t are non-zero integers satisfying the equation

$$r^p + s^p = t^p$$

where p is any prime > 3, with $gcd(r, s, t) = 1$ and establish a contradiction in this proof.

2) Just for supporting the proof in the above equation, we have used another equation

$$x^3 + y^3 = z^3, \qquad \text{with} \quad gcd(x, y, z^3) = 1$$

Without loss of generality, we can have both x and y as non-zero integers, z^3 a non-zero integer; both z and z^2 irrational. Since we intent to prove the theorem only in the equation $r^p + s^p = t^p$ for all possible integral values of r, s and t we have the choice in having $x = 37$; $y = 64$; $z^3 = 64^3 + 37^3 = 19 \times 101 \times 163$. We may choose x and y any number of ways, but one value in sufficient for proving the theorem.

3) By trial and error we have created the transformation equations to $x^3 + y^3 = z^3$ and $r^p + s^p = t^p$

using parameters called $a,\ b,\ c,\ d,\ e$ and f. Creation of such transformation equations could be done in thousands of ways, but finding a proof is most difficult and rare. Every time the rational terms in equation (8) we derived from the transformed equations got cancelled out on both sides. After enormous random trials, the particular formulation of transformed equations was designed to bring out the results for proving the theorem.

4) Into the transformed equations we have incorporated the Euler's equation

$$2^n = 7k^2 + \ell^2,$$

which has solutions for every $n \geq 3$ with k and ℓ odd. Let k be prime to 7, n be odd.[4]

In this proof we are using the solution $2^{19} = (7 \times 271^2) + 101^2$, where $k = 271$ and $\ell = 101$; hence $z^3 = 19 \times 163\ell$.

Proof. By random trials, we have created the following equations

$$\left(a\sqrt{19z^3} + b\sqrt{2^{n/2}}\right)^2 + \left(\frac{c\sqrt{37} + d\sqrt{\ell^{5/3}}}{\sqrt{2^{3n/2}}}\right)^2 = \left(e\sqrt{6} + f\sqrt{\ell}\right)^2$$

and

$$\left(\frac{a\sqrt{\ell} - b\sqrt{st}}{\sqrt{163}}\right)^2 + \left(\frac{c\sqrt{163} - d\sqrt{r}}{\sqrt{7^{5/3}k^{7/3}}}\right)^2 = \left(\frac{e\sqrt{7^{1/3}k^{5/3}} - f\sqrt{19z}}{\sqrt{\ell^{7/3}}}\right)^2 \qquad (1)$$

To represent the equations $x^3 + y^3 = z^3$ and $r^p + s^p = t^p$ respectively through the parameters called $a,\ b,\ c,\ d,\ e$ and f.

We may have

$$a\sqrt{19z^3} \ + \ b\sqrt{2^{n/2}} = \sqrt{x^3} \tag{2}$$
$$a\sqrt{\ell} - b\sqrt{st} = \sqrt{163r^p} \tag{3}$$
$$c\sqrt{37} + d\sqrt{\ell^{5/3}} = \sqrt{y^3 2^{3n/2}} \tag{4}$$
$$c\sqrt{163} - d\sqrt{r} = \sqrt{s^p 7^{5/3} k^{7/3}} \tag{5}$$
$$e\sqrt{6} + f\sqrt{\ell} = \sqrt{z^3} \tag{6}$$
and
$$e\sqrt{7^{1/3}k^{5/3}} - f\sqrt{19z} = \sqrt{t^p \ell^{7/3}} \tag{7}$$

Solving simultaneously (2) and (3); (4) and (5); (6) and (7), we get

$$a = \left(\sqrt{stx^3} + \sqrt{163 \times 2^{n/2} r^p}\right) \Big/ \left(\sqrt{19stz^3} + \sqrt{\ell \times 2^{n/2}}\right)$$
$$b = \left(\sqrt{\ell x^3} - \sqrt{19 \times 163 r^p z^3}\right) \Big/ \left(\sqrt{19stz^3} + \sqrt{\ell \times 2^{n/2}}\right)$$
$$c = \left(\sqrt{2^{3n/2} y^3 r} + \sqrt{7^{5/3} k^{7/3} \ell^{5/3} s^p}\right) \Big/ \left(\sqrt{37r} + \sqrt{163\ell^{5/3}}\right)$$
$$d = \left(\sqrt{163 \times 2^{3n/2} y^3} - \sqrt{37 \times 7^{5/3} k^{7/3} s^p}\right) \Big/ \left(\sqrt{37r} + \sqrt{163\ell^{5/3}}\right)$$
$$e = \left(z^2\sqrt{19} + \ell^{5/3}\sqrt{t^p}\right) \Big/ \left(\sqrt{6 \times 19z} + \sqrt{7^{1/3} k^{5/3} \ell}\right)$$

and
$$f = \left(\sqrt{7^{1/3} k^{5/3} z^3} - \sqrt{6 \times t^p \ell^{7/3}}\right) \Big/ \left(\sqrt{6 \times 19z} + \sqrt{7^{1/3} k^{5/3} \ell}\right)$$

From (2) and (4), we get

$$\begin{aligned} 2^n &= \left(\sqrt{x^3} - a\sqrt{19z^3}\right)\left(c\sqrt{37} + d\sqrt{\ell^{5/3}}\right) \Big/ \left(b\sqrt{y^3}\right) \\ &= \Big\{(c)\sqrt{37x^3} + (d)\sqrt{\ell^{5/3}x^3} - (ac)\sqrt{19 \times 37z^3} \\ &\qquad -(ad)\sqrt{19z^3\ell^{5/3}}\Big\} \Big/ \left(b\sqrt{y^3}\right) \end{aligned}$$

From (5) and (7), we get

$$\begin{aligned} 7k^2 &= \left(c\sqrt{163} - d\sqrt{r}\right)\left(\sqrt{t^p\ell^{7/3}} + f\sqrt{19z}\right) \Big/ \left(e\sqrt{s^p}\right) \\ &= \Big\{(c)\sqrt{163t^p\ell^{7/3}} + (cf)\sqrt{19 \times 163z} - (d)\sqrt{rt^p\ell^{7/3}} \\ &\qquad -(df)\sqrt{19rz}\Big\} \Big/ \left(e\sqrt{s^p}\right) \end{aligned}$$

From (4) and (7), we get

$$\begin{aligned} \ell^2 &= \left(\sqrt{2^{3n/2}y^3} - c\sqrt{37}\right)\left(e\sqrt{7^{1/3}k^{5/3}} - f\sqrt{19z}\right) \Big/ \left(d\sqrt{t^p}\right) \\ &= \Big\{(e)\sqrt{2^{3n/2}7^{1/3}k^{5/3}y^3} - (f)\sqrt{2^{3n/2} \times 19y^3z} \\ &\qquad -(ce)\sqrt{7^{1/3}k^{5/3} \times 37} + (cf)\sqrt{19 \times 37z}\Big\} \Big/ \left(d\sqrt{t^p}\right) \end{aligned}$$

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Substituting these values in the Euler's equation $2^n = 7k^2 + \ell^2$ after multiplying both sides by $\left\{(bde)\sqrt{y^3 s^p t^p}\right\}$ we get the equation

$$\begin{aligned}
&\left\{(de)\sqrt{s^p t^p}\right\}\left\{(c)\sqrt{37x^3} + (d)\sqrt{\ell^{5/3}x^3} - (ac)\sqrt{19 \times 37z^3} - (ad)\sqrt{19z^3\ell^{5/3}}\right\} \\
&= \left\{(bd)\sqrt{y^3 t^p}\right\}\left((c)\sqrt{163t^p\ell^{7/3}} + (cf)\sqrt{19 \times 163z} - (d)\sqrt{rt^p\ell^{7/3}} - (df)\sqrt{19rz}\right) \\
&\quad + \left\{(be)\sqrt{y^3 s^p}\right\}\left\{(e)\sqrt{2^{3n/2}7^{1/3}k^{5/3}y^3} - (f)\sqrt{2^{3n/2}19y^3z} \right. \\
&\qquad \left. - (ce)\sqrt{37 \times 7^{1/3}k^{5/3}} + (cf)\sqrt{19 \times 37z}\right\} \qquad (8)
\end{aligned}$$

Our aim is to evaluate all rational terms available in equation (8), after multiplying both sides by

$$\left(\sqrt{19stz^3} + \sqrt{\ell \times 2^{n/2}}\right)\left(\sqrt{37r} + \sqrt{163\ell^{5/3}}\right)^2\left(\sqrt{6 \times 19z} + \sqrt{7^{1/3}k^{5/3}\ell}\right)^2$$

for freeing from denominators on the parameters $a,\ b,\ c,\ d;\ e$ and f and again multiplying by

$$\left\{\ell^{2/3}\sqrt{z^5 t}\right\}$$

for getting some rational terms.

I term in LHS of Equation (8), after multiplying by the respective terms, and substituting for $\{(cd)e\}$

$$\begin{aligned}
&= \sqrt{37x^3 s^p t^p}\left(\sqrt{19stz^3} + \sqrt{2^{n/2}\ell}\right)\left(\sqrt{6 \times 19z} + \sqrt{7^{1/3}k^{5/3}\ell}\right) \\
&\times \left(\ell^{2/3}\sqrt{z^5 t}\right)\left(\sqrt{2^{3n/2}y^3 r} + \sqrt{7^{5/3}k^{7/3}\ell^{5/3}s^p}\right)\left(\sqrt{163 \times 2^{3n/2}y^3}\right. \\
&\qquad \left. - \sqrt{37 \times 7^{5/3}k^{7/3}s^p}\right)\left(z^2\sqrt{19} + \ell^{5/3}\sqrt{t^p}\right)
\end{aligned}$$

There is no rational part in this term.

II term in LHS of equation (8), after multiplying by the respective terms, and substituting for $\{d^2 e\}$

$$\begin{aligned}
&= \sqrt{s^p t^p \ell^{5/3}x^3}\left(\sqrt{19stz^3} + \sqrt{2^{n/2}\ell}\right)\left(\sqrt{6 \times 19z} + \sqrt{7^{1/3}k^{5/3}\ell}\right) \\
&\times \left(\ell^{2/3}\sqrt{z^5 t}\right)\left\{\left(163y^3\sqrt{2^{3n}}\right) + \left(37 \times 7^{5/3}k^{7/3}s^p\right)\right. \\
&\qquad \left. - \left(2\sqrt{163 \times 2^{3n/2}y^3}\right)\left(\sqrt{37 \times 7^{5/3}k^{7/3}s^p}\right)\right\}\left(z^2\sqrt{19} + \ell^{5/3}\sqrt{t^p}\right)
\end{aligned}$$

There is no rational part in this term.

III term in LHS of equation (8), after multiplying by the respective terms, and substituting for $\{a(cd)e\}$ is

$$\begin{aligned}
&= \left(-\sqrt{19 \times 37z^3 s^p t^p}\right)\left(\sqrt{6 \times 19z} + \sqrt{7^{1/3}k^{5/3}\ell}\right)\left(\ell^{2/3}\sqrt{z^5 t}\right)\left(\sqrt{stx^3} + \sqrt{163 \times 2^{n/2}r^p}\right) \\
&\times \left(\sqrt{2^{3n/2}y^3 r} + \sqrt{7^{5/3}k^{7/3}\ell^{5/3}s^p}\right)\left(\sqrt{163 \times 2^{3n/2}y^3}\right. \\
&\qquad \left. - \sqrt{37 \times 7^{5/3}k^{7/3}s^p}\right)\left(z^2\sqrt{19} + \ell^{5/3}\sqrt{t^p}\right)
\end{aligned}$$

On multiplying by,

$$\begin{aligned}
&\left\{\left(-\sqrt{19 \times 37z^3 s^p t^p}\right)\sqrt{7^{1/3}k^{5/3}\ell}\left(\ell^{2/3}\sqrt{z^5 t}\right)\sqrt{163 \times 2^{n/2}r^p}\right. \\
&\qquad \left. \times \sqrt{7^{5/3}k^{7/3}\ell^{5/3}s^p}\left(\sqrt{163 \times 2^{3n/2}y^3}\right)\left(z^2\sqrt{19}\right)\right\}
\end{aligned}$$

We get

$$\Big\{ -(2^n \times 7k^2)(19 \times 163\ell^2 z^6)\Big(s^p\sqrt{t^{p+1}}\Big)\sqrt{37y^3r^p}\Big\}$$

where $y = 64$; this term will be discussed later on.

IV term in LHS of equation (8), after multiplying by the respective terms, and substituting for $\{ad^2e\}$ is

$$= \Big(-\sqrt{s^pt^p}\Big)\Big(\sqrt{19z^3\ell^{5/3}}\Big)\Big(\sqrt{6\times 19z}+\sqrt{7^{1/3}k^{5/3}\ell}\Big)\Big(\ell^{2/3}\sqrt{z^5t}\Big)$$
$$\times\Big(\sqrt{stx^3}+\sqrt{163\times 2^{n/2}r^p}\Big)\Big(z^2\sqrt{19}+\ell^{5/3}\sqrt{t^p}\Big)\Big\{\Big(163y^3\sqrt{2^{3n}}\Big)$$
$$+\Big(37\times 7^{5/3}k^{7/3}s^p\Big)-\Big(2\sqrt{163\times 2^{3n/2}y^3}\sqrt{37\times 7^{5/3}k^{7/3}s^p}\Big)\Big\}$$

On multiplying by

$$\Big\{\Big(-\sqrt{s^pt^p}\Big)\sqrt{19z^3\ell^{5/3}}\sqrt{7^{1/3}k^{5/3}\ell}\Big(\ell^{2/3}\sqrt{z^5t}\Big)\sqrt{163\times 2^{n/2}r^p}\Big(z^2\sqrt{19}\Big)$$
$$\Big(-2\sqrt{163\times 2^{3n/2}y^3}\Big)\sqrt{37\times 7^{5/3}k^{7/3}s^p}\Big)\Big\}$$

We get

$$\Big\{\Big(2^{n+1}\times 7\times k^2\ell^2\Big)\Big(19\times 163\times z^6s^p\sqrt{t^{p+1}}\Big)\sqrt{37y^3r^p}\Big\}$$

This term will be discussed later on.

I term in RHS of equation (8), after multiplying by the respective terms, and substituting for $\{b(cd)\}$ is

$$= \Big(t^p\sqrt{163y^3\ell^{7/3}}\Big)\Big(\sqrt{6\times 19z}+\sqrt{7^{1/3}k^{5/3}\ell}\Big)^2\Big(\ell^{2/3}\sqrt{z^5t}\Big)$$
$$\Big(\sqrt{x^3\ell}-\sqrt{19\times 163r^pz^3}\Big)\Big(\sqrt{2^{3n/2}y^3r}+\sqrt{7^{5/3}k^{7/3}\ell^{5/3}s^p}\Big)$$
$$\times\Big(\sqrt{163\times 2^{3n/2}y^3}-\sqrt{37\times 7^{5/3}k^{7/3}s^p}\Big)$$

There is no rational part in this term.

II term in RHS of equation (8), after multiplying by the relevant terms, and substituting for $\{b(cd)f\}$

$$= \sqrt{19\times 163y^3zt^p}\Big(\sqrt{6\times 19z}+\sqrt{7^{1/3}k^{5/3}\ell}\Big)\Big(\ell^{2/3}\sqrt{z^5t}\Big)$$
$$\times\Big(\sqrt{x^3\ell}-\sqrt{19\times 163r^pz^3}\Big)\Big(\sqrt{2^{3n/2}y^3r}+\sqrt{7^{5/3}k^{7/3}\ell^{5/3}s^p}\Big)$$

$$\times\Big(\sqrt{163\times 2^{3n/2}y^3}-\sqrt{37\times 7^{5/3}k^{7/3}s^p}\Big)\Big(\sqrt{7^{1/3}k^{5/3}z^3}-\sqrt{6\times t^p\ell^{7/3}}\Big)$$

Rational part in this term

$$= \Big\{\sqrt{19\times 163y^3zt^p}\sqrt{7^{1/3}k^{5/3}\ell}\Big(\ell^{2/3}\sqrt{z^5t}\Big)\sqrt{x^3\ell}\sqrt{7^{5/3}k^{7/3}\ell^{5/3}s^p}$$
$$\times\Big(-\sqrt{37\times 7^{5/3}k^{7/3}s^p}\Big)\sqrt{7^{1/3}k^{5/3}z^3}\Big\}$$
$$= \Big[-\Big(7^2k^4\ell^2\Big)\Big(z^3s^p\sqrt{t^{p+1}}\Big)\sqrt{37x^3y^3}\sqrt{19\times 163\ell z^3}\Big]$$

$\because\ x = 37;\ y = 64;\ z^3 = 19\times 163\ell$, where $\ell = 101$ in the Euler's equation $2^{19} = (7\times 271^2) + 101^2$.

Also on multiplying by

$$\left\{\sqrt{19\times 163y^3zt^p}\sqrt{7^{1/3}k^{5/3}\ell}\left(\ell^{2/3}\sqrt{z^5t}\right)\left(-\sqrt{19\times 163r^pz^3}\right)\times\sqrt{7^{5/3}k^{7/3}\ell^{5/3}s^p}\left(-\sqrt{37\times 7^{5/3}k^{7/3}s^p}\right)\sqrt{7^{1/3}k^{5/3}z^3}\right\}$$

We get

$$\left\{(7^2k^4\ell^2)\times(19\times 163)z^6s^p\sqrt{t^{p+1}}\sqrt{37y^3r^p}\right\}$$

This term will be discussed later on.

III term in RHS of Equation (8), after multiplying by the respective terms, and substituting for $\{bd^2\}$

$$=-\left(t^p\sqrt{y^3r\ell^{7/3}}\right)\left(\sqrt{6\times 19z}+\sqrt{7^{1/3}k^{5/3}\ell}\right)^2\left(\sqrt{x^3\ell}-\sqrt{19\times 163r^pz^3}\right)\times\left(\ell^{2/3}\sqrt{z^5t}\right)\left\{\left(163y^3\sqrt{2^{3n}}\right)+\left(37\times 7^{5/3}k^{7/3}s^p\right)-2\left(\sqrt{163\times 2^{3n/2}y^3}\times\sqrt{37\times 7^{5/3}k^{7/3}s^p}\right)\right\}$$

There is no rational part in this term.

IV term in RHS of Equation (8), after multiplying by the respective terms, and substituting for $\{bd^2f\}$

$$=\left(-\sqrt{y^3t^p}\right)\sqrt{19rz}\left(\sqrt{6\times 19z}+\sqrt{7^{1/3}k^{5/3}\ell}\right)\left(\ell^{2/3}\sqrt{z^5t}\right)\times\left(\sqrt{x^3\ell}-\sqrt{19\times 163r^pz^3}\right)\left(\sqrt{7^{1/3}k^{5/3}z^3}-\sqrt{6t^p\ell^{7/3}}\right)\left\{\left(163y^3\sqrt{2^{3n}}\right)+\left(37\times 7^{5/3}k^{7/3}s^p\right)-2\left(\sqrt{163\times 2^{3n/2}y^3}\right)\left(\sqrt{37\times 7^{5/3}k^{7/3}s^p}\right)\right\}$$

There is no rational part in this term.

V term in RHS of Equation (8), after multiplying by the respective terms, and substituting for $\{be^2\}$

$$=\left(y^3\sqrt{2^{3n/2}s^p\left(7^{1/3}k^{5/3}\right)}\right)\left(37r+163\ell^{5/3}+2\sqrt{37\times 163\times r\ell^{5/3}}\right)\times\left(\ell^{2/3}\sqrt{z^5t}\right)\left(\sqrt{x^3\ell}-\sqrt{19\times 163r^pz^3}\right)\left(19z^4+\ell^{10/3}t^p+2\ell^{5/3}z^2\sqrt{19t^p}\right)$$

There is no rational part in this term.

VI term in RHS of Equation (8), after multiplying by the respective terms, and substituting for $\{b(ef)\}$

$$=\left(-y^3\sqrt{2^{3n/2}19zs^p}\left(37r+163\ell^{5/3}+2\sqrt{37\times 163r\ell^{5/3}}\right)\times\left(\ell^{2/3}\sqrt{z^5t}\right)\times\left(\sqrt{x^3\ell}-\sqrt{19\times 163r^pz^3}\right)\times\left(z^2\sqrt{19}+\ell^{5/3}\sqrt{t^p}\right)\left(\sqrt{7^{1/3}k^{5/3}z^3}-\sqrt{6t^p\ell^{7/3}}\right)\right.$$

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There is no rational part in this term.

VII term in RHS of Equation (8), after multiplying by the respective terms, and substituting for $\{bce^2)\}$

$$= \left(-\sqrt{37\times 7^{1/3}k^{5/3}}\right)\sqrt{y^3 s^p}\left(\sqrt{37r}+\sqrt{163\ell^{5/3}}\right)\left(\ell^{2/3}\sqrt{z^5 t}\right)$$
$$\left(\sqrt{x^3\ell}-\sqrt{19\times 163 r^p z^3}\right)\left(\sqrt{2^{3n/2}y^3 r}+\sqrt{7^{5/3}k^{7/3}\ell^{5/3}s^p}\right)$$
$$\times\left(z^2\sqrt{19}+\ell^{5/3}\sqrt{t^p}\right)^2$$

On multiplying by

$$\left\{\left(-\sqrt{37\times 7^{1/3}k^{5/3}}\right)\sqrt{y^3 s^p}\sqrt{163\ell^{5/3}}\left(\ell^{2/3}\sqrt{z^5 t}\right)\left(-\sqrt{19\times 163 r^p z^3}\right)\right.$$
$$\left.\times\left(\sqrt{7^{5/3}k^{7/3}\ell^{5/3}s^p}\right)\left(2\ell^{5/3}z^2\sqrt{19t^p}\right)\right\}$$

We get,

$$\left\{\left(2\times 7k^2\ell^4\right)z^6\left(s^p\sqrt{t^{p+1}}\right)(19\times 163)\sqrt{37y^3r^p}\right\}$$

This term will be discussed later on.

VIII term in RHS of Equation (8), after multiplying by the respective terms, and substituting for $\{bc(ef)\}$

$$= \sqrt{19\times 37y^3 z s^p}\left(\sqrt{37r}+\sqrt{163\ell^{5/3}}\right)\left(\ell^{2/3}\sqrt{z^5 t}\right)$$
$$\times\left(\sqrt{x^3\ell}-\sqrt{19\times 163 r^p z^3}\right)\left(\sqrt{2^{3n/2}y^3 r}+\sqrt{7^{5/3}k^{7/3}\ell^{5/3}s^p}\right)$$
$$\times\left(z^2\sqrt{19}+\ell^{5/3}\sqrt{t^p}\right)\left(\sqrt{7^{1/3}k^{5/3}z^3}-\sqrt{6t^p\ell^{7/3}}\right)$$

Rational part in this term

$$= \left\{\sqrt{19\times 37y^3 z s^p}\sqrt{163\ell^{5/3}}\left(\ell^{2/3}\sqrt{z^5 t}\right)\sqrt{x^3\ell}\sqrt{7^{5/3}k^{7/3}\ell^{5/3}s^p}\right.$$
$$\left.\times\left(\ell^{5/3}\sqrt{t^p}\right)\sqrt{7^{1/3}k^{5/3}z^3}\right\}$$
$$= \left\{\sqrt{37x^3y^3}\times(7k^2\ell^4)\left(z^3 s^p\sqrt{t^{p+1}}\right)\sqrt{19\times 163\ell z^3}\right\}$$

Also on multiplying by,

$$\left\{\sqrt{19\times 37y^3 z s^p}\left(\sqrt{163\ell^{5/3}}\right)\left(\ell^{2/3}\sqrt{z^5 t}\right)\left(-\sqrt{19\times 163 r^p z^3}\right)\right.$$
$$\left.\times\sqrt{7^{5/3}k^{7/3}\ell^{5/3}s^p}\left(\ell^{5/3}\sqrt{t^p}\right)\sqrt{7^{1/3}k^{5/3}z^3}\right\}$$

We get

$$\left\{-(7k^2\ell^4)(19\times 163)\left(z^6 s^p\sqrt{t^{p+1}}\right)\sqrt{37y^3r^p}\right\}$$

This term will be discussed later on.

Case (1):

The rational terms not having the factor $\sqrt{37y^3r^p}$ as a factor. There is no such term on LHS of Equation (8).

Sum of such rational terms on RHS of Equation (8)

$$= -(7^2 \times k^4\ell^2)\sqrt{37x^3y^3}\left(z^3s^p\sqrt{t^{p+1}}\right)\sqrt{19 \times 163\ell z^3} \qquad \text{(vide II term)}$$

$$+ (7k^2\ell^4)\sqrt{37x^3y^3}\left(z^3s^p\sqrt{t^{p+1}}\right)\sqrt{19 \times 163\ell z^3} \qquad \text{(vide VIII term)}$$

$$= -(7k^2\ell^2)\sqrt{37x^3y^3}\left(z^3s^p\sqrt{t^{p+1}}\right)\sqrt{19 \times 163\ell z^3}\left(7k^2 - \ell^2\right)$$

Equating the rational terms on both sides of Equation (8), after dividing both sides by

$$\left\{(-7k^2\ell^2)(z^3)\sqrt{37x^3y^3}\sqrt{19 \times 163\ell z^3}\left(7k^2 - \ell^2\right)\right\}$$

we get

$$\left(s^p\sqrt{t^{p+1}}\right) = 0$$

i.e., Either $s = 0$ or $t = 0$. This contradicts our hypothesis that all r, s and t are non-zero integers in the equation $r^p + s^p = t^p$ and proves that only a trivial solution exists.

Case (2):

Rational terms having $\sqrt{37y^3r^p}$ as a factor,

Sum of all rational terms on LHS of Equation (8) having such terms

$$= (2^n 7k^2\ell^2)\left(19 \times 163z^6\right)\left(s^p\sqrt{t^{p+1}}\right)\sqrt{37y^3r^p}$$

(combining III & IV terms)

Sum of such rational terms on RHS of Equation (8) having such terms

$$= (7^2k^4\ell^2)(19 \times 163z^6)\left(s^p\sqrt{t^{p+1}}\right)\sqrt{37y^3r^p} \qquad \text{(vide II term)}$$

$$+ (7k^2\ell^4)(19 \times 163z^6)\left(s^p\sqrt{t^{p+1}}\right)\sqrt{37y^3r^p} \qquad \text{(combining VII \& VIII terms)}$$

$$= \left\{(7k^2\ell^2)(19 \times 163z^6)\left(s^p\sqrt{t^{p+1}}\right)\sqrt{37y^3r^p}(7k^2 + \ell^2)\right\}$$

$$= (2^n \times 7k^2\ell^2)(19 \times 163z^6)\left(s^p\sqrt{t^{p+1}}\right)\sqrt{37y^3r^p}$$

which gets cancelled with LHS summed up terms.

III. Conclusion

Since Equation (8) in this proof was derived directly from the transformation equations of Fermat's for the index 3 and p where p is any prime > 3, the result $st = 0$, that we obtained on solving the transformed equation should reflect on the Fermat's equation $r^p + s^p = t^p$, thus proving that only a trivial solution exists in the equation $r^p + s^p = t^p$.

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