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Research Paper

Effect of Chemical reaction on an MHD heat and mass transfer flow with special reference to Homotopy Perturbation Method

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Abstract: *MHD boundary layer flow over a moving vertical plate with magnetic field and Chemical reaction in presence of heat and mass transfer has been planned. Using He's Homotopy Perturbation Method (HPM), the system of non-linear ordinary differential equations governing the MHD boundary layer equations is solved. The influence of a variety of significant physical parameters on the boundary layer flow is illustrated graphically with the physical interpretation. The obtained results point to the efficiency and convenience of the HPM. Utility of this model has been perceived in diverse industrial and chemical processes.*
Keywords: *MHD; Heat Transfer; Mass Transfer; HPM, Chemical reaction.*
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I. Introduction

Magnetohydrodynamics *(MHD)* concerns with the study of fluids under electro-magnetic effects. Now-a-days applications of MHD principles obtain great importance because of its wide ranging utilities in various fields such as geophysics, astronomical science, space science etc. Because of importance of MHD principle in different field, many researchers give their attentions to do work in the field of MHD. Investigation of MHD boundary layer flow with heat and mass transfer has momentous applications in the fields of aeronautical plasma flows, nuclear reactor, magnetosphere, chemical engineering and electronics. Most of chemical engineering progression like polymer extrusion processes and metallurgical involves cooling of a molten liquid. To improve the quality of the eventual creation, Balla and Naikoti (2015), Islam and Ahmed (2017), Prasad and Reddy (2019) etc. have made astounding contributions in solving various flow problems of assorted geometries.

Due to importance of chemically reactive fluids, several researchers have carried out their studies on the problems of flow under heat and mass transmission. Some of them are Muthucumarswamy (2002), Muthucumarswamy and Meenakshisundaram (2006), Mahapatra *et al.* (2010), Mythreye *et al.* (2015), Mythreye and Balamurugaon (2017), Nisar *et al.* (2021), Haq *et al.* (2021) etc

In this paper, the influence of chemical reaction is adopted to generalize the work of Sarma et. al. (2020). In the process of generalization, almost exact results are drawn which is shown by virtue of comparison graph with the work of Sarma et. al. (2020)

Mathematical Formulation

The present study contemplates an MHD boundary layer flow over a moving vertical plate with heat and mass transfer of viscous in presence of magnetic field. The flow is supposed to be in x-axis which is along the direction of plate and y −axis is taken normal to it. Let u and v be the x −component of fluid velocity and y-component of fluid velocity respectively. The flow formation which describes the physical insight of the problem is given by

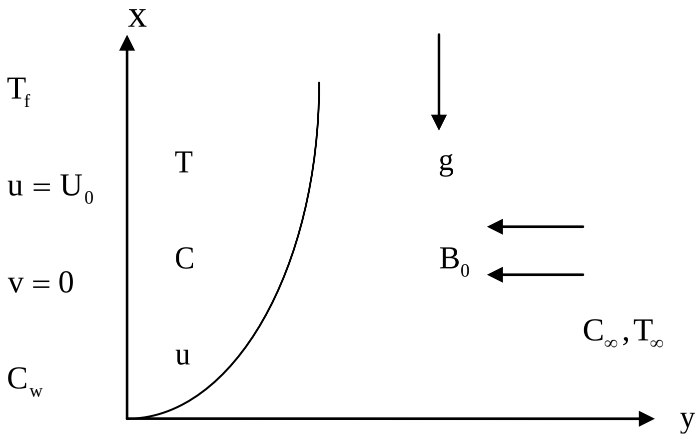

Figure 1

Using boundary layer and *Boussinesq's approximations*, the governing equations for this problem can be formulated as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u + g\beta_T(T - T_\infty) + g\beta_c(C - C_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - Kr'\left(C - C_\infty\right) \tag{4}$$

The boundary conditions for the problem may be written as:

$$u(x,0) = U_0,\ \ v(x,0) = 0, -k\frac{\partial T}{\partial y}(x,0) = h_f\left(T_f - T(x,0)\right), C_w(x,0) = Ax^\lambda + C_\infty$$

$$u(x,\infty) = 0, T(x,\infty) = T_\infty, C(x,\infty) = C_\infty \tag{5}$$

The *Cauchy-Riemann equations* satisfy the continuity equation (1) with:

$u = \frac{\partial \psi}{\partial y}$ and

$$v = -\frac{\partial \psi}{\partial x} \tag{6}$$

$$\eta = y\sqrt{\frac{U_0}{\nu x}}, \psi = \sqrt{\nu x U_0}f(\eta) \tag{7}$$

Where the plate velocity is denoted by U_0, the symbols $\nu, C_\infty, \alpha, D, \beta_T, \beta_c, \rho, g, \sigma, \psi, \eta$ have their appropriate elucidations.

The *temperature* and *concentration* in non-dimensional form are given as

$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_f - C_\infty} \tag{8}$$

And $Kr = \frac{Kr'x}{U_0}$

The non-dimensional ordinary governing differential equations are:

$$f''' + \frac{1}{2}ff'' - Mf' + Gr\theta + Gm\phi = 0 \tag{9}$$

$$\theta'' + \frac{1}{2} Pr f \theta' = 0 \tag{10}$$

$$\phi'' + \frac{1}{2} Sc\, f\, \phi' - Kr\, Sc\, \phi = 0 \tag{11}$$

Applicable boundary conditions are

$$f(0) = 0, f'(0) = 1, \theta'(0) = Bi[\theta(0) - 1], \emptyset(0) = 1 \tag{12}$$

$$f(\infty) = 0, \theta(\infty) = \emptyset(\infty) = 0 \tag{13}$$

Method of Solution:

According to the HPM, the Homotopy form of equations from (9)-(11) can be written as

$$(1-p)(f''' - Mf' + Gr\theta + Gm\phi) + p\left(f''' + \frac{1}{2} ff'' - Mf' + Gr\theta + Gm\phi\right) = 0 \tag{14}$$

$$(1-p)\theta'' + p\left(\theta'' + \frac{1}{2} \Pr f \theta'\right) = 0 \tag{15}$$

$$(1-p)\phi'' + p\left(\phi'' + \frac{1}{2} Sc\, f \phi' - Kr\, Sc\, \phi\right) = 0 \tag{16}$$

Let us consider "f", "θ" and "$\emptyset$" as

$$\left.\begin{aligned} f &= f_0 + pf_1 + p^2 f_2 + \ldots\ldots\ldots\ldots \\ \theta &= \theta_0 + p\theta_1 + p^2\theta_2 + \ldots\ldots\ldots\ldots \\ \phi &= \phi_0 + p\phi_1 + p^2\phi_2 + \ldots\ldots\ldots\ldots \end{aligned}\right\} \tag{17}$$

Using (17) in (14), (15) and (16) and then by simplifying, we obtain:

$$\theta_0 = C_1\eta + C_2 \tag{18}$$

$$\phi_0 = C_3\eta + C_4 \tag{19}$$

$$f_0 = C_5\eta + C_6 e^{-\sqrt{M}\eta} + C_7 e^{\sqrt{M}\eta} + A_3\eta^2 + A_4\eta \tag{20}$$

$$\theta_1 = -\frac{1}{2}\Pr C_1\left(\frac{C_5}{2}\eta^2 + \frac{C_6}{M}e^{-\sqrt{M}\eta} + \frac{C_7}{M}e^{\sqrt{M}\eta} + \frac{A_3}{12}\eta^4 + \frac{A_4}{6}\eta^3\right) + C_8\eta + C_9 \tag{21}$$

$$\phi_1 = -\frac{1}{2}ScC_1\left(\frac{C_5}{2}\eta^2 + \frac{C_6}{M}e^{-\sqrt{M}\eta} + \frac{C_7}{M}e^{\sqrt{M}\eta} + \frac{A_3}{12}\eta^4 + \frac{A_4}{6}\eta^3\right) +$$
$$C_{10}\eta + C_{11} + KrSc\left(\frac{C_3}{6}\eta^3 + \frac{C_4}{2}\eta^2\right) \tag{22}$$

$$\begin{aligned} f_1 = {} & C_{15} + C_{13}e^{\sqrt{M}\eta} + C_{14}e^{-\sqrt{M}\eta} + A_{21}\eta^2 + A_{20}\eta + A_{22}\eta^3 + A_{23}\eta^4 + A_{24}\eta^5 + A_{25}\eta e^{\sqrt{M}\eta} \\ & + A_{26}\eta e^{\sqrt{M}\eta} + A_{27}\eta^2 e^{\sqrt{M}\eta} + A_{28}\eta^2 e^{-\sqrt{M}\eta} + A_{29}\eta^3 e^{\sqrt{M}\eta} + A_{30}\eta^3 e^{-\sqrt{M}\eta} \\ & + A_{31}e^{-2\sqrt{M}\eta} + A_{32}e^{2\sqrt{M}\eta} \end{aligned} \tag{23}$$

The above zeroth and first order expressions of velocity, temperature and concentration are found out by using the following restrictions:

$$f_0(0) = 1, f_0'(0) = 1, \qquad \theta_0'(0) = Bi[\theta_0(0) - 1], \emptyset_0(0) = 1$$

$$f_0'(\infty) = \theta_0(\infty) = \phi_0(\infty) = 0$$

&

$$f_1(0) = 0, f_1'(0) = 0, \qquad \theta_1'(0) = Bi[\theta_1(0)], \emptyset_1(0) = 0$$

$$f_1'(\infty) = \theta_1(\infty) = \phi_1(\infty) = 0$$

Neglecting higher order perturbed terms we finally obtain:

$$f = f_0 + pf_1$$

$$\theta = \theta_0 + p\theta_1$$

$$\phi = \phi_0 + p\phi_1$$

II. Results and Discussion

In this study, the numerical results are obtained for different values of parameters Kr, Gr, Gm, Bi_x, Sc, M, Pr with fixed value of *Homotopy Perturbation Parameter* (p=.1) implanted in the flow system.

Figures 2-4 describe the fluid velocity against η. The effects of various values of *magnetic parameter* (M), *Chemical reaction parameter* (Kr) and *Schimdt number* (Sc), on velocity profile are revealed. Figure 2 demonstrates that with the enhancement of *magnetic field parameter*, the fluid velocity moves down monotonically to the free stream value zero far away from the plate satisfying the boundary condition. This happens because the presence of magnetic field in an electrically conducting fluid generates a force called the *Lorentz force* which acts against the flow if the magnetic field is applied in the normal direction. This result clearly interprets the physical behaviour of the magnetic field parameter. From figure 3, it is observed that consumption of chemical species controls the fluid flow. Figure 4 depicts that the velocity transport of the fluid medium is enriched for low mass diffusivity of the species. The fluid motion is controlled and moves towards free stream value on account of the physical parameters involved in the problem.

The concentration profile rises due to the strength of the applied magnetic field which is experienced in Figure 5. The effects of *Chemical reaction parameter* (Kr) and *Schmdit number* (Sc) on species concentration have been incorporated in figures 6-7. It is inferred from these figures that the concentration level of the fluid drops for low *mass diffusivity*, *thermal* and consumption of chemical species.

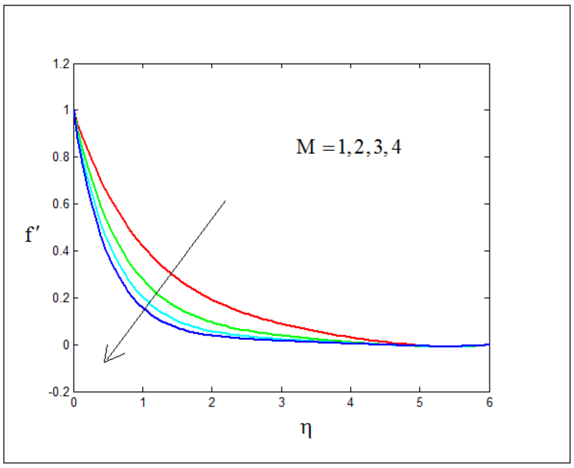

Figure 2: Velocity versus η under Gr=0.1, Gm=0.1, Bi_x=0.1, Sc=0.62, Pr=0.72, Kr=0.1, P=0.1

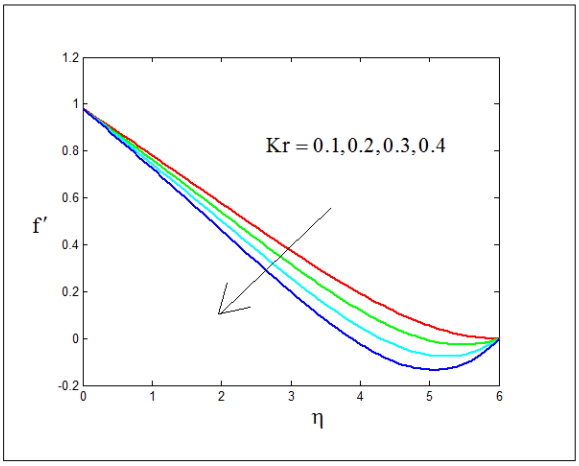

Figure 3: Velocity versus η under Gr=0.1, Gm=0.1, Bi_x=0.1, M=0.1, Pr=0.72, M=0.1, P=.1

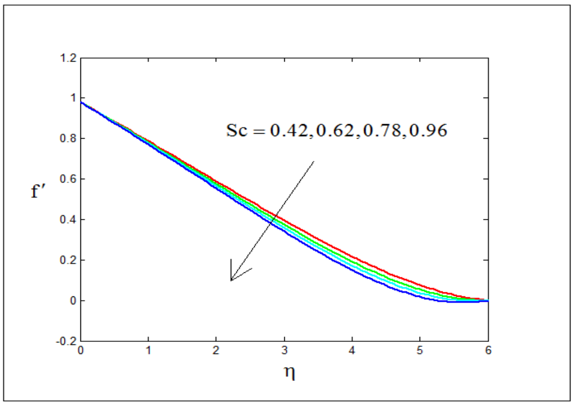

Figure 4: Velocity versus η under Gr=0.1, Gm=0.1, Bi_x=0.1, Kr=0.1, M=0.1, P=.1

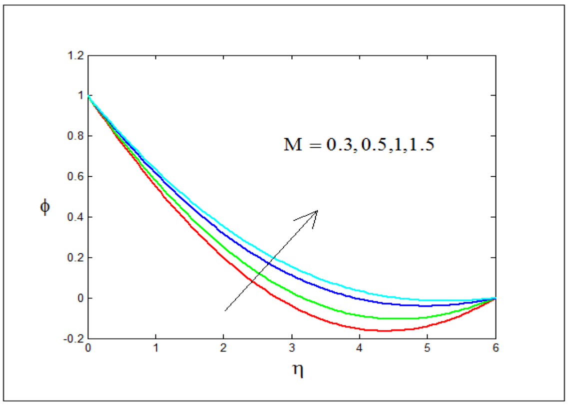

Figure 5: Concentration versus η under Gr=0.1, Gm=0.1, Bi_x=0.1, Sc=0.62, Pr=0.72, Kr=0.1, P=.1

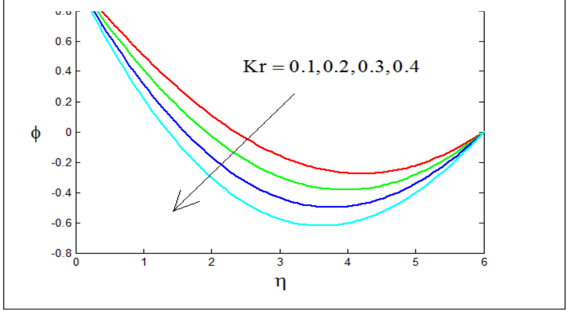

Figure 6: Concentration versus η under Gr=0.1, Gm=0.1, Bi_x=0.1, Sc=0.62, Pr=0.72, M=0.1, P=.1

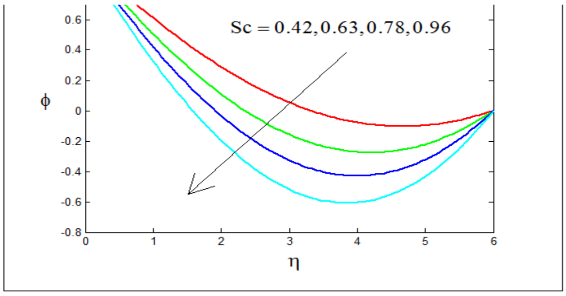

Figure 7: Concentration versus η under Gr=0.1, Gm=0.1, Bi_x=0.1, Kr=0.1, Pr=0.72, M=0.1, P=.1

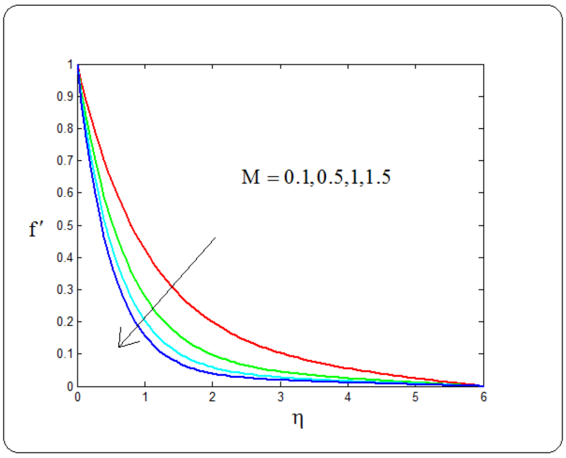

Figure 8: Velocity versus η under Gr=0.1, Gm=0.1, Bi_x=0.1, Sc=0.62, Pr=0.72, Kr=0, P=0.1

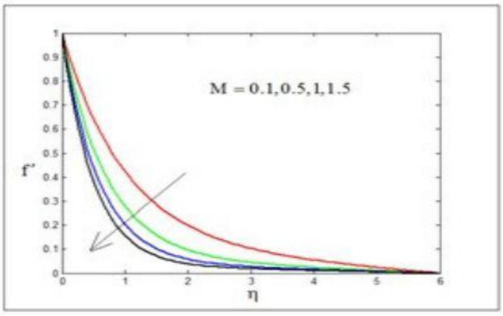

Figure 2: Velocity versus η under $Gr = 0.1$, $Gm = 0.1$, $Bi_x = 0.1$, $Sc = 0.62$, $Pr = 0.72$, $P = 0.1$

III. Comparison Of Results

The work of Sarma et. al. (2020) is considered for comparing the results of the present paper.

Comparing figure 8 with the figure 2 of the work done by Sarma et. al. (2020), we observe the same kind of behaviour due to the implementation of Magnetic intensity in velocity profile for fixed values of Gr = 0.1, Gm = 0.1, Bi_x = 0.1, Sc = 0.62, Pr = 0.72, Kr=0, P = .1 i.e. there is a significant effect of Hartmann number on this profile. Thus, there is an excellent agreement between the results obtained by Sarma et. al. (2020) and those arrived at by the present authors.

Concluding remarks

In this paper, the problem of MHD boundary layer flow over a moving vertical plate in presence of heat and mass transfer with the imposition of chemical reaction is considered by HPM. The obtained results are revealed graphically and are compared with the accurate solutions. The result shows that the estimated solution obtained in this paper has excellent agreement with the work done by Sarma et. al. (2020).

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Appendix

$$a = M,\ b = Gr_x,\ c = Gc_x,\ d = \Pr,\ e = Sc$$

$$C_1 = -\frac{6B_{ix}}{1+6B_{ix}}\text{'}\ C_2 = \frac{6B_{ix}}{1+6B_{ix}}, C_3 = -\frac{1}{6}, C_4 = 1, C_5 = -\frac{2A_2 - A_1(e^{6\sqrt{a}} + e^{-6\sqrt{a}})}{\sqrt{a}(e^{6\sqrt{a}} - e^{-6\sqrt{a}})},$$

$$C_6 = \frac{A_2 - A_1 e^{6\sqrt{a}}}{\sqrt{a}(e^{6\sqrt{a}} - e^{-6\sqrt{a}})}\text{'}\ C_7 = \frac{A_2 - A_1 e^{-6\sqrt{a}}}{\sqrt{a}(e^{6\sqrt{a}} - e^{-6\sqrt{a}})},\ C_8 = B_{ix}C_9 + A_5,$$

$$C_9 = \frac{dC_1}{6B_{ix}+1}\left[9C_5 + \frac{C_6}{2a}e^{-6\sqrt{a}} + \frac{C_7}{2a}e^{6\sqrt{a}} + 54A_3 + 18A_4\right] - \frac{6}{1+6B_{ix}}A_5,$$

$$C_{10} = \frac{eC_3}{12}\left[18C_5 + \frac{C_6}{a}e^{-6\sqrt{a}} + \frac{C_7}{a}e^{6\sqrt{a}} + 108A_3 + 36A_4\right] - C_{11},$$

$$C_{11} = \frac{eC_3}{2}\left(\frac{C_6 + C_7}{a}\right), C_{12} = C_{15} - A_{19}, C_{13} = \frac{B_2 e^{-6\sqrt{a}} - B_3}{e^{6\sqrt{a}} - e^{-6\sqrt{a}}},$$

$$C_{14} = \frac{B_2 e^{6\sqrt{a}} - B_3}{e^{6\sqrt{a}} - e^{-6\sqrt{a}}},\ C_{15} = -2\frac{B_2 e^{-6\sqrt{a}} - B_3}{e^{6\sqrt{a}} - e^{-6\sqrt{a}}} - B_1 - B_2,$$

$$A_1 = 1 - \frac{1}{a}\left(bC_2 + cC_4\right), A_2 = -\frac{6}{a}\left(bC_1 + cC_3\right) - \frac{1}{a}\left(bC_2 + cC_4\right),$$

$$A_3 = \frac{1}{2a}\left(bC_1 + cC_3\right), A_4 = \frac{1}{a}\left(bC_2 + cC_4\right),$$

$$A_5 = -\frac{1}{2}dC_1\left[B_{ix}\left(\frac{C_6}{a} + \frac{C_7}{a}\right) + \left(\frac{C_6}{\sqrt{a}} - \frac{C_7}{\sqrt{a}}\right)\right], \quad A_6 = bC_9 + cC_{11} + C_5A_3 + aC_6C_7,$$

$$A_7 = \frac{b}{2a}dC_1C_7 + \frac{c}{2a}eC_3C_7 - \frac{a}{2}C_5C_7 - A_3C_7, \quad A_8 = \frac{b}{2a}dC_1C_6 + \frac{c}{2a}eC_3C_6 - \frac{a}{2}C_5C_6 - A_3C_6,$$

$$A_9 = bC_8 + cC_{10} + A_3A_4,\ A_{10} = -A_3^{\ 2} + \frac{b}{4}dC_1C_5 + \frac{c}{4}eC_3C_4 + \frac{KrScC_3}{4},$$

$$A_{11} = \frac{b}{12}dC_1A_4 + \frac{c}{12}eC_3A_4 + \frac{KrScC_4}{12},\ A_{12} = \frac{b}{24}dC_1A_3 + \frac{c}{24}eC_3A_3,$$

$$A_{13} = \frac{1}{2}aC_6^{\ 2}, A_{14} = \frac{1}{2}aC_7^{\ 2},\ A_{15} = \frac{1}{2}aC_7A_4,\ A_{16} = \frac{1}{2}aC_6A_4,$$

$$A_{17} = \frac{1}{2}aC_7A_3,\ A_{18} = \frac{1}{2}aC_6A_3,\ A_{19} = -\frac{1}{a}\left(\frac{6A_{11}}{a^2} - \frac{A_9}{a}\right),$$

$$A_{20} = -\frac{1}{a}\left(\frac{24A_{12}}{a^2} + \frac{2A_{10}}{a} - A_6\right),\ A_{21} = -\frac{1}{a}\left(\frac{3A_{11}}{a} - \frac{A_9}{2}\right),$$

$$A_{22} = -\frac{1}{a}\left(\frac{4A_{12}}{a} + \frac{A_{10}}{3}\right),\ A_{23} = -\frac{1}{a}\frac{A_{11}}{4},\ A_{24} = -\frac{1}{a}\frac{A_{12}}{5},$$

$$A_{25} = \frac{A_7}{2a} + \frac{3A_{15}}{4a\sqrt{a}} - \frac{7A_{17}}{2a^2}, A_{26} = \frac{A_8}{2a} - \frac{3A_{16}}{4a\sqrt{a}} - \frac{7A_{18}}{2a^2},$$

$$A_{27} = -\frac{A_{15}}{4a} - \frac{3A_{18}}{4a\sqrt{a}},\ B_1 = A_{31} + A_{32},\ B_2 = \frac{1}{\sqrt{a}}\left(A_{20} + A_{25} + A_{26} - 2\sqrt{a}A_{31} + 2\sqrt{a}A_{32}\right),$$

$$B_3 = \frac{1}{\sqrt{a}}[A_{20} + 12A_{21} + 108A_{22} + 864A_{23} + 6480A_{24} + \{A_{25}\left(1 + 6\sqrt{a}\right)$$

$$+ A_{27}\left(12 + 36\sqrt{a}\right) + A_{29}\left(108 + 216\sqrt{a}\right)\}e^{6\sqrt{a}} + \{A_{26}\left(1 - 6\sqrt{a}\right)$$

$$+ A_{28}\left(12 - 36\sqrt{a}\right) + A_{30}\left(108 - 216\sqrt{a}\right)\}e^{-6\sqrt{a}} - A_{31}2\sqrt{a}e^{-12\sqrt{a}} + A_{32}2\sqrt{a}e^{12\sqrt{a}}]$$