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Research Paper

Large Amplitude Oscillations of Thick Hyperelastic Cylindrical Shell in a Transversely Isotropic Incompressible Material

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ABSTRACT: *-The present work deals with the presentation of an exact numerical solution to the large amplitude oscillations of a thick hyperelastic cylindrical shell of transversely isotropic incompressible material with help of Runge-Kutta method.The equations of motion and time period for the shell walls are obtained under the condition of incompressibility, considering that the applied pressure is constant in time. Here the comparison with isotropic material is also displaced through graphs for the free and forced oscillations due to Heaviside step load.*

Keywords: **-** *Transversely, Cavity, Oscillations, Incompressible, Runge-Kutta method.*

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I. INTRODUCTION

The first dynamic problem in finite elasticity of bounded media found its explicit solution in two research papers of Knowles [1, 2]. In these papers he discussed the solution to the problem of finite amplitude oscillation of an incompressible hyperelastic circular tube and discussed cases of free oscillations and of Heaviside loading. By following the same path Shahinpoor and Nowinski [3], Shahinpoor and Balakrishnan [4] presented some exact solutions in finite hyperelasticity by considering the material to be incompressible, homogeneous and isotropic.

Further the problem of practical importance that is of cavitation or void formation in different materials is studied by many researchers of different fields like mechanics, material science and also applied mathematics, readers can refer to Incompressible elastic material by Balakrishnan and Shahinpoor [5], Dielectric materials by Singh and Verma [6], Shahinpoor [7], Garg [8], Kuldeep and Rajesh [9], Neo-Hookean materials by Chou-Wang and Horgan [10], Compressible materials by Haughton [11], Murphy and Biwa [12], Chun and Jun [13], Anistropic elastic materials by Polignone and Horgan [14], Elastic-Plastic solids by Hou and Abeyaratne [15], also stretches in cavitation are studied by Hou and Zhang [16] and Biwa[17].

The present paper deals with the large amplitude oscillations of the surface of a cylindrical cavity by employing the general theory of finite dynamic deformations of elastic bodies.Recently Buchanan and Ramirez [18] used finite element method to study the vibrations of transversely isotropic solid spheres. Ericksen and Rivlin [19] developed the theory of finite deformations of homogeneous anistropic materials. Huilgol [20] added few more results by considering the special strain energy function. The formulation of the present problem is based on the theory of finite elastic deformations [21-24]. It has been noted that in case of free oscillations the amplitude of oscillations during expansion is less than that during contraction in both isotropic as well as transversely isotropic materials. Also the time duration for contraction is less than that for expansion and the frequency of oscillations increases as we increase the initial velocity in both the cases. In case of forced oscillations the amplitude of oscillations is slightly more than that in the case of free oscillations. This is due to the forcing function.

II. BASIC EQUATIONS

According to Ericksen and Rivlin [19], stress-strain relations for transversely isotropic elastic materials are

$$t_l^k = -p\delta_l^k + 2\frac{\partial\Sigma}{\partial I_1}(c^{-1})_l^k - 2\frac{\partial\Sigma}{\partial I_2}(c)_l^k + 2\frac{\partial\Sigma}{\partial I'}h^k h_l$$

$$+2\frac{\partial\Sigma}{\partial I_2'}[(c^{-1})_m^k h_l+(c^{-1})_{lm}h^k]h^m,(k,l=1,2,3), \quad (2.1)$$

where t_l^k is the symmetric stress-tensor, p is the hydrostatic pressure, $\Sigma=\Sigma(I_1,I_2,I_1',I_2')$ is the strain-energy function and

$$(c^{-1})^{kl}=G^{\alpha\beta}x_{,\alpha}^k x_{,\beta}^l, \quad (2.2)$$

where $x_{,\alpha}^k=\frac{\partial X^k}{\partial x^\alpha}$, x^α and X^k being respectively material and spatial curvilinear coordinates in undeformed and deformed state respectively. Also c is the inverse of c^{-1}.

$$h^k=H^\alpha x_{,\alpha}^k, \quad (2.3)$$

where H^α and h^k are the directions of the an isotropic director in the undeformed and deformed states, respectively.

The principle invariants are

$$I_1=(c^{-1})_k^k, I_2=\frac{1}{2}\{[(c^{-1})_k^k]^2-(c^{-1})_l^k(c^{-1})_k^l\}, \quad I_1'=g_{kl}h^k h^l, \quad I_2'=h_k h^l(c^{-1})^{kl}. \quad (2.4)$$

Equations of motion without body forces are

$$t_{;l}^{kl}=\rho f_l, \quad (2.5)$$

where the semi-colon stands for covariant differentiation, f_l is the acceleration.

III. FORMULATION OF THE PROBLEM

Let us consider a cylindrical shell made up of an elastic, homogeneous, and incompressible material. Let x_i and X_i to be the rectangular Cartesian co-ordinates of the typical particle at a time t, and r_1 and R_1 be the radii of the cavity in the undeformed and deformed states respectively. Since the motion of the shell is cylindrical symmetric, the cylindrical symmetric motions that we are consider are of the form

$$r=R(r,t). \quad (3.1)$$

We consider the direction of an isotropic director in the undeformed state as

$$H^X=1, H^Y=0, H^Z=0. \quad (3.2)$$

On using (3.1), (3.2) and (2.2) the principal invariants (2.4), and deformed anisotropic director (2.3) are obtained as

$$I_1=I_2=Q^2+Q^{-2}+1, \; I_3=1, I_1'=1, \; I_2'=Q^4, \quad (3.3)$$

$$h^1=Q, h^2=h^3=0, \quad (3.4)$$

where

$$Q(R)=r/R. \quad (3.5)$$

Using (3.3), (3.4) and (2.5), the equation of motion, in the absence of body forces, reduces to

$$\frac{\partial}{\partial Q}[L_1+L_2+L_3Q^2-p]=-[L_2+L_3Q^2]\frac{q}{1-Q^2}+\rho\frac{Q}{1-Q^2}[(R_1\ddot{R}_1+2(1-\frac{R_1^2}{R^2})\dot{R}_1^2], \quad (3.6)$$

where $\ddot{R}_1(\equiv\frac{d^2R_1}{d^2t^2})$ is the acceleration, $\dot{R}_1(\equiv\frac{dR_1}{dt})$ represents the velocity of the particles on the cavity surface and

$$L_1=2Q^{-2}\frac{\partial\Sigma}{\partial I_1}-2Q^2\frac{\partial\Sigma}{\partial I_2}$$

$$L_2=2(Q^2-Q^{-2})\frac{\partial\Sigma}{\partial I_1}+2(Q^2-Q^{-2})\frac{\partial\Sigma}{\partial I_2},$$

$$L_3=2\frac{\partial\Sigma}{\partial I_1'}+4Q^2\frac{\partial\Sigma}{\partial I_2'}. \quad (3.7)$$

Expressing R in terms of Q, we integrate (3.6) to get

$$p=p_0+L_1+L_2+L_3Q^2+2\int_{Q_1}^{Q}[L_2+L_3Q^2]\frac{Q}{1-Q^2}dQ-\rho\frac{R_1}{R}(R_1\ddot{R}_1+\dot{R}_1^2)$$
$$-\frac{\rho}{2(R_1^2-r_1^2)}[1+\frac{r_1^2-R_1^2}{R^2}-\frac{r_1^2}{R_1^2})]\dot{R}_1^2R_1^2, \quad (3.8)$$

where $Q_1=r_1/R_1$ and p_0 is a constant of integration. If $F(t)$ is the uniform pressure applied to the cavity wall $R=R_1$, we have

$$L_1+L_2+L_3Q^2-p=-F(t). \quad (3.9)$$

From (3.8) and (3.9), we get $p_0=F(t)$. Using this relation and (2.1) and (3.8), we obtain

$$t_{kl}=[-F(t)+\rho\ln(\frac{R_1}{R})(R_1\ddot{R}_1+\dot{R}_1^2)]+\frac{\rho}{2(R_1^2-r_1^2)}[1+\frac{r_1^2-R_1^2}{R^2}-\frac{r_1^2}{R_1^2})]\dot{R}_1^2R_1^2\delta_l^k$$
$$+[L_1+L_2+L_3Q^2](1-\delta_l^k)-2\delta_l^k\int_{Q_1}^{Q}[L_2+L_3Q^2]\frac{Q}{1-Q^2}dQ \quad (3.10)$$

For fixed t, t_{kl} should approach to zero as $R \to \infty$from physical considerations. It is noticed from (3.5), (3.7) that$Q \to 1$as $R \to \infty$and $L_2 \to 0$ as $Q \to 1$. Therefore I_1', I_2'also tends to zero as $R \to \infty$and so$L_3 \to 0$. The stresses given by (3.10) satisfy all these conditions if and only if

$$F(t) = \rho \ln(\frac{R_1}{R})(R_1 \ddot{R}_1 + \dot{R}_1^2)] + \frac{\rho}{2(R_1^2 - r_1^2)}[1 - \frac{r_1^2}{R_1^2})]\dot{R}_1^2 R_1^2 - 2\int_{Q_1}^{1} [L_2 + L_3 Q^2] \frac{Q}{1 - Q^2} dQ. \qquad (3.11)$$

This differential equation determines the cavity radius $R_1(t)$ as a function of time.
By introducing the function

$$g(x) = -\frac{1}{\rho r_1^2} \int_{1/x}^{1} [L_2 + L_3 Q^2] \frac{Q}{1 - Q^2} dQ \qquad (3.12)$$

where

$$x(t) = \frac{R_1(t)}{r_1} = \frac{1}{Q_1}, f(t) = \frac{F(t)}{\rho r_1^2}. \qquad (3.13)$$

We rewrite the differential equation (3.11) in the form

$$\ln\left(\frac{R_1}{R}\right)[x\ddot{x} + \dot{x}^2] + \frac{1}{2}\dot{x}^2 + g(x) = f(t). \qquad (3.14)$$

We assume that initially the medium is unstressed and at rest so that$x(0) = 1, \dot{x}(0) = 0$. Thus we consider the motion set up by the sudden application of a constant outward pressure which is maintained at the cavity wall, so that $f(t)$ is positive constant. Under these conditions we integrate (3.14) to obtain

$$\frac{1}{2}x^2\dot{x}^2 \ln\left(\frac{R_1}{R}\right) + \int_1^x x g(x) dx = \frac{1}{2}(x^2 - 1) f(t). \qquad (3.15)$$

This equation represents the trajectory of the motion of the cavity wall in the $(x, \dot{x})$ plane.
It is well known that the motion is periodic if and only if the trajectory (3.15) is a closed curve C in the $(x, \dot{x})$ plane with a finite time period given by

$$T = \oint_C \frac{dx}{\dot{x}}, \qquad (3.16)$$

where$\dot{x}$ is given by (3.15).
If we write

$$G(x) = \int_1^x x g(x) dx, \qquad (3.17)$$

the time period T in (3.16) becomes

$$T = \int_1^x [(x^2 - 1) f(t) - 2(G(x)]^{\frac{-1}{2}} [x^2 \ln\left(\frac{R_1}{R}\right)]^{\frac{1}{2}} dx, \qquad (3.18)$$

where$z$is the maximum dimensionless radius.
Now we shall show that the trajectory (3.15) is a closed curve. This curve is at the initial point $x = 1, v = 0$ at time $t = 0$. The point $(x, \dot{x})$ moves in the region $x > 1, \dot{x} > 0$ if $f(t)$ is positive so that the net pressure on the surface is outward. For positive $f(t), \dot{x}$ passes through a maximum and returns to zero as x increases from unity, thus the curve will be a closed one. According to (3.15) this will happen if there is a root $x = z \neq 1$ of the equation

$$G(x) = \frac{1}{2}(x^2 - 1) f(t), \qquad (3.19)$$

so that $\dot{x} = 0$ for $x = z$.
For a given$f(t)$, we assume that (3.19) possesses such a root.
Since

$$G(x) - \frac{1}{2}(x^2 - 1) f(t) = 0, \qquad (3.20)$$

vanishes both at $x = 1$ and $x = z$, and

$$G'(x) = x g(x) = x f(t), \qquad (3.21)$$

it is clear that the existence of a root $x = z$ of (3.19) implies the existence of a root $x = \bar{z}$ between $x = 1$ and $x = z$ of the equation

$$g(x) = f(t). \qquad (3.22)$$

A solution $x = \bar{z}$ of (3.22) represents a static equilibrium state about which the cavity wall will oscillate; the period of oscillation is given by (3.18).

IV. LARGE AMPLITUDE OSCILLATIONS VIBRATION OF THICK HYPERELASTIC CYLINDRICAL SHELL

Let r_1 and $r_2(r_2 > r_1)$ be the radii in the unstressed state and $R_1$and$R_2(R_2 > R_1)$ be the radii in the stressed state at time$t$. We assume that the motions of the shell are finite, of arbitrary magnitude and induced by time dependent hydrostatic surface pressures $F_1(t)$ and $F_2(t)$ on the inner and outer surfaces of the shell respectively. The equation of motion, in this case may be written in the form

$$-\frac{dF}{dQ} = -[L_2 + L_3Q^2]\frac{Q}{1-Q^2} + \rho\frac{Q}{1-Q^2}[(R_1\ddot{R}_1 + 2(1 - \frac{R_1^3}{R^3})\dot{R}_1^2], \tag{4.1}$$

where F is the radial pressure per unit area of the actual surface. Expressing Rin terms of$Q$, we integrate the above equation with respect to Q over the interval $Q_1 = r_1/R_1$to $Q_2 = r_2/R_2$to get

$$F_1(t) - F_2(t) = -\int_{Q_1}^{Q_2}[L_2 + L_3Q^2]\frac{Q}{1-Q^2}dQ + \rho\ln\left(\frac{R_1}{R}\right)\left(R_1\ddot{R}_1 + \dot{R}_1^2\right)$$

$$+\frac{\rho}{2(R_1^2 - r_1^2)}[1 + \frac{r_1^2 - R_1^2}{R^2} - \frac{r_1^2}{R_1^2})]\dot{R}_1^2R_1^2. \tag{4.2}$$

Now the strain energy function is considered to be the form

$$\Sigma = c_1(I_1 - 3) + c_2(I_2 - 3) + c_3(I_1^{'} - 3)^2 + c_4(1 + I_2^{'} - 2I_1^{'}), \tag{4.3}$$

which is a special case of Σ due to Huilgol [20] and is consistent with that for a Neo-Hookean solid in the isotropic case.

Now using (4.3), (3.7), we can rewrite (4.2) after neglecting higher order terms in the form

$$\frac{2}{\rho r_1^2}[F_1(t) - F_2(t)] = x\ddot{x}\ln(1 + \mu x^{-2}) + [\ln(1 + \mu x^{-2}) - \frac{\mu}{\mu + x^2}]\dot{x}^2$$

$$+\frac{c_1 + c_2}{\rho R_1^2}[(1 + x^{-2})(\frac{\mu}{\mu + x^2}) - \ln(\frac{1 + \mu x^{-2}}{1 + \mu})]$$

$$+\frac{c_3}{2\rho R_1^2}[(1 + x^{-2})(\frac{\mu}{\mu + x^2}) - \ln(\frac{1 + \mu x^{-2}}{\sqrt{1 - x^2}})] + \frac{c_4}{4\rho R_1^2}[(\frac{1 + \mu}{\mu + x^2})^2 - \frac{1}{x^4}], \tag{4.4}$$

whereμ is the measure of wall thickness of the shell and defined as

$$\mu = (\frac{r_2}{r_1})^2 - 1. \tag{4.5}$$

For the case of cylindrical cavity in an infinite medium, we let $\mu \to \infty$ as$R_2 \to \infty$, then (4.4) becomes

$$x\ddot{x}\ln(1 + \mu x^{-2}) + [\ln(1 + \mu x^{-2}) - \frac{\mu}{\mu + x^2}]\dot{x}^2$$

$$+\alpha_1[(1 + x^{-2})(\frac{\mu}{\mu + x^2}) - \ln(\frac{1 + \mu x^{-2}}{1 + \mu})]$$

$$+\alpha_2[(1 + x^{-2})(\frac{\mu}{\mu + x^2}) - \ln(\frac{1 + \mu x^{-2}}{\sqrt{1 - x^2}})] + \alpha_3[(\frac{1 + \mu}{\mu + x^2})^2 - \frac{1}{x^4}] = \alpha_4\mathrm{F(t)}. \tag{4.6}$$

where

$$\alpha_1 = \frac{c_1 + c_2}{\rho r_1^2}, \alpha_2 = \frac{c_3}{\rho r_1^2}, \alpha_3 = \frac{c_4}{4\rho r_1^2}, \alpha_4 = \frac{2}{\rho r_1^2}\text{and}F(t) = F_1(t) - F_2(t), \tag{4.7}$$

and we impose the following initial conditions

$$x(0) = x_0, \dot{x}(0) = \dot{x}_0, \tag{4.8}$$

where the dot represents the time derivative, x_0 and$\dot{x}_0$ are the initial displacement and velocity at the inner surface of the cavity.

V. NUMERICAL SOLUTIONS TO THE PROBLEM

The numerical solutionto the large amplitude oscillations of a thick hyperelastic cylindrical shell of transversely isotropic incompressible material with help of Runge-Kutta method.algorithm for solving a system of n first-order differential equations [25].

$$y_{j,i+1} = y_{ji} + h(k_{j1} + 2k_{j2} + 2k_{j3} + k_{j4})/6$$

$$k_{j1} = f_j(x_i, y_{1i}, y_{2i}, \ldots, y_{ni}), y*_{ji} = y_{ji} + \frac{1}{2}hk_{j1}$$

$$k_{j2} = f_j(x_i + \frac{1}{2}n, y*_{1i}, y*_{2i}, \ldots, y_{ni}), \bar{y}_{ji} = y_{ji} + \frac{1}{2}hk_{j2}$$

$$k_{j3} = f_j(x_i + \frac{1}{2}h, \bar{y}_{1i}, \bar{y}_{2i}, \ldots, \bar{y}_{ni}), \bar{y}*_{ji} = y_{ji} + hk_{j3}$$

$$k_{j4} = f_j(x_i + h, \bar{y}*_{1i}, \bar{y}*_{2i}, \ldots, \bar{y}*_{ni}), \tag{5.1}$$

which is the algorithm for solving a system of n first-order differential equations, where $j = 1,2,\ldots\ldots n$ number of first-order differential equations $i = i$thstep of integration.
First we rewrite (4.6) in the following

$$y = \dot{x}, \quad (5.2)$$

$$x\dot{y}\ln(1+\mu x^{-2}) + [\ln(1+\mu x^{-2}) - \frac{\mu}{\mu+x^2}]y^2$$
$$+\alpha_1[(1-x^{-2})(\frac{\mu}{\mu+x^2}) - \ln(\frac{1+\mu x^{-2}}{1+\mu})]$$
$$+\alpha_2[(1-x^{-2})(\frac{\mu}{\mu+x^2}) - \ln(\frac{1+\mu x^{-2}}{\sqrt{1-x^2}})] + \alpha_3[(\frac{1+\mu}{\mu+x^2})^2 - \frac{1}{x^4}] = \alpha_4 \mathrm{F(t)}. \quad (5.3)$$

The initial conditions are

$$x(0) = x_0, \qquad y(0) = \dot{x}_0 \quad (5.4)$$

The numerical solutions are then presented graphically for different forcing functions.

Case-1. Free Oscillations

$$FT = 0, t \geq 0. \quad (5.5)$$

Fig.-1 shows the numerical solution for the case of free oscillations. The curves are plotted for $x_0 = 1.0$with $\dot{x}_0$ as a parameter. It has been noted that the amplitude of oscillations during expansion is less than that during contraction in both isotropic as well as transversely isotropic materials but the amplitude of oscillations in transversely isotropic is less as compared to that in case of isotropic materials.

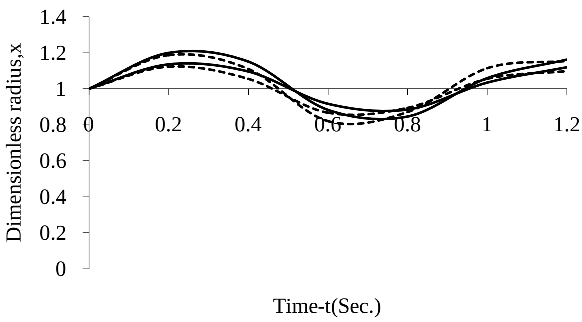

Fig-1 Free Oscillations

Case-2. Forced oscillation-Heaviside step load

$$FT = \begin{cases} 0, \text{for} t \leq 0, \\ F_0, \text{for} t > 0. \end{cases} \quad (5.6)$$

The numerical solutions for this case are given in Fig-2. This shows the same trend as Fig.-1, but the amplitude of oscillations is slightly more than that in the case of free oscillations. This is due to the forcing function.

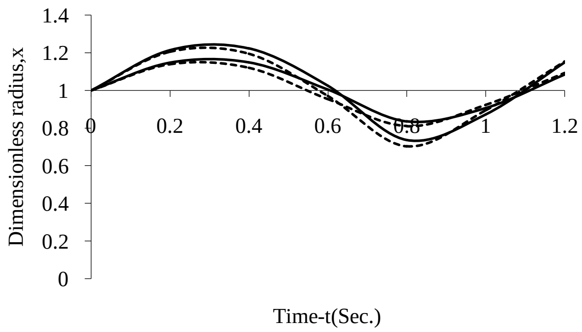

Fig-2 Forced Oscillations – Heaviside Step Loading

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