Quest Journals
Journal of Research in Applied Mathematics
Volume 8 ~ Issue 12 (2022) pp: 44-49
ISSN(Online) : 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

New Forms of Continuous Maps in Topological Spaces

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ABSTRACT: *In this Article, we introduced a new forms of functions called pre-βwg-continuous functions using βwg - closed sets in topological spaces and obtain some of their properties. Further also, we defined and studied the new notions of pre-βwg-open maps and pre βwg closed maps.*

Mathematics Subject Classification (2010): 54C05, 54C08.
KEYWORDS: *βwg-closed sets, βwg-continuous maps, pre-βwg-open maps, βwg-closed maps and contra βwg-irresolute maps.*

***R**eceived 04 Dec., 2022; **R**evised 14 Dec., 2022; **A**ccepted 16 Dec., 2022*

I. INTRODUCTION

In 1970, Levine [9], introduced the concept of g-closed sets and a new class of spaces called $T_{1/2}$ – spaces in topology. Thereafter, many topologists have obtained several interesting results on these g-closed sets. In 1994, Maki et al.,[10] have defined and studied the α-generalised closed sets and α-generalised open sets making use of α-interior and α-closure due to A.S. Mashhour et al.,[12]. Caldas [6] and Balachandran et al.,[3] defined and studied the notion of g-continuous maps by using g-closed sets and discussed some of their properties.Further they have investigated and studied the new concept of gc-irresolute, perfectly g-continuous, strongly g-continuous maps. Recently, Govindappa. Navalagi and Kantappa. M. Bhavikatti [14] introduced and studied new concept of closed sets called βwg-closed sets and in [4, 15 & 16], contra βwg-continuous maps, βwg-continuous maps & βwg-irresolute and strongly βwg-continuous maps were studied. In this paper, we define and study the new concept of pre-βwg-continuous maps and their properties. Further, we also introduce pre βwg-open maps in topological spaces.

II. PRELIMINARIES

Throughout this paper, S, R, and P always denote topological spaces on which no separation axioms are assumed unless otherwise explicitly stated. For a subset A ⊆ S, closure and interior of A is denoted as Cl(A) and Int(A) respectively.

We have the following known definitions and results which are useful in the sequel.

Definition 2.1: A subset A of a space S is known as
(i) Preopen[12] if A ⊆ Int(Cl(A))
(ii) α-open[18] if A⊆ Int(Cl(Int(A)))
(iii) semipreopen[2] (=β-open[1]) if A ⊆ Cl(Int(A))).

The compliments of above open sets are their closed sets.

Definition 2.2: A subset A of S said to be
(i) generalized closed[9] (in brief, g-closed) if cl(A) ⊆ U whenever A⊆ U and U is open in S.
(ii) αg-closed[10] if αcl(A) ⊆ U whenever A⊆U & U is open in S.
(iii) gsp-closed [7] if spcl(A) ⊆ U whenever A⊆ U moreover U is open in S.
(iv) βwg-closed set[14] if βcl(A) ⊆ U whenever A⊆U & U is αg-open in S.

Definition 2.3: A function h:S→R is called a
(i) β-continuous [1] if for each closed V of R, $f^{-1}(V)$ is β-closed in S.
(ii) gc-irresolute[19] if for each g-closed set V of R, $f^{-1}(V)$ is g-closed in S
(iii) βwg-continuous [15] if for each closed set V of R, $f^{-1}(A)$ is βwg-closed in S
(iv) Strongly βwg-continuous [4] if for each βwg-closed V of R, $f^{-1}(V)$ is closed in S.
(v) contra continuous [8] if $f^{-1}(V)$ is closed set in S for each open set V of R.
(vi) contra βwg-continuous [16] if $f^{-1}(V)$ is βwg-closed set in S for every open set V of R.
(vii) Pre gsp-continuous [17] if for each sp-closed set V of R, $f^{-1}(V)$ is gsp-closed in S.
(viii) β-irresolute [11] if for each β –closed set V of R,$f^{-1}(V)$ is β-closed in S.
(ix) βwg-irresolute [15] if for each βwg –closed set V of R, $f^{-1}(V)$ is βwg-closed in S.
(x) quasi sgp-closed[5] if the image of each sgp-closed set of S being closed in R.
(xi) sgp*-closed[5] if the image of sgp-closed subset of S is sgp-closed in R.

Definition 2.4: A space B of S called
(i) a $T_{1/2}$ –space[8] if every g-closed set in it is closed.
(ii) a ${}_{\beta wb}T_b$ – space[14] if every βwg-closed subset in S is closed.

III. Properties of Pre βwg – Continuous Maps

In this present section, we define and obtain the followings

DEFINITION 3.1.

Enable a map h: (S, τ) → (R, σ) & is called pre-βwg-continuous **(**Shortly written, pre βwg-continuous) functions if for each β-closed in set C of S, h(C) is βwg-closed in set R.

THEOREM 3.2.

If h: S →R be pre-βwg-continuous thereupon, it is βwg-continuous.

PROOF:

Allow D is any closed subset of R. So D be β -closed in R. Since h be pre-βwg-continuous map, $h^{-1}(D)$ is βwg – closed set in S. Therefore, h be βwg-continuous.

THEOREM 3.3.

Allow h:S → R be a map, it is Strongly βwg-continuity iff the image of each βwg-closed set of S being βwg-closed in S.

PROOF: Obvious.

PROPOSITION 3.4.

Allow h: S → R be pre- βwg-continuity iff the image of each βwg – closed set in S being β – closed set in R.

PROOF:

Suppose h be pre - βwg-continuity. Allow set Q be β-open in R. Then Q^c is β-closed in R. As h be pre-βwg-continuous, $h^{-1}(Q^c)$ is βwg- closed set in S. Hence $h^{-1}(Q)$ is βwg-open in S.

Conversely, Let D be β – open set in R. Thereupon D^c is β-closed in R. But $h^{-1}(D^c)$ = S- $h^{-1}(D)$ by assumption. Therefore $h^{-1}(D)$ is βwg-closed in S. h is pre-βwg-continuous.

PROPOSITION 3.5.

Allow a map h: S → R be pre-βwg- continuity then h be β-irresolute map.

PROOF: Obvious

THEOREM 3.6.

If **a** function h: S → R be pre-βwg - continuity then h be βwg- irresolute map.

PROOF: Follows by Theorem 3.4.

THEOREM 3.7.

Every is pre-βwg-continuous map being pre – gsp continuity.

PROOF:

Allow set D be β -closed in R. As h be pre-βwg-continuous, $h^{-1}(D)$ is βwg –closed set in S. Then $h^{-1}(D)$ be gsp –closed set of S. As every βwg-closed is gsp-closed. Hence h is pre-gsp-continuity map.

EXAMPLE 3.8.

Take R = P = $\{c_1, c_2, c_3\}$, σ = $\{R, \{c_1\}, \varphi\}$, η = $\{P,\{c_1,c_3\},\varphi\}$. Define a function k: R → P is identity function. Now k be pre-gsp – continuity yet never pre-βwg- continuity. As β-closed subset $\{c_1, c_2\}$ of P, $k^{-1}(\{c_1, c_2\}) = \{c_1,c_2\}$ is gsp-closed but βwg-closed in R.

REMARK 3.9.

The composition of two pre-βwg-continuous functions is not pre- βwg- continuous.

EXAMPLE 3.10.
Allowing S = R ={e, j, t} = P, τ = {φ,{j},{t},{j, t}}, σ ={φ, {e}, R} and η = {P, {t},{e, t},φ}.
Now define h: S → R as h(e) = t, h(j) = e, h(t) = j & k: R →P as k(e) =e, k(j) = t , k(t) = j. Thereupon, both h and k are pre-βwg –continuous maps. Yet their composition koh: S → P be never pre-βwg – continuous function. As β – closed subset {e, j} in P, but $(koh)^{-1}(\{e, j\}) = h^{-1}(k^{-1}\{j, t\})) = \{j, t\}$ be never βwg closed in S.

PROPOSITION 3.11.
If a function h: S → R be pre- βwg - continuous & k: R → P be strongly βwg – continuous map, thereupon the composition koh: S → P be βwg-irresolute.
PROOF:
Let F be βwg-closed set in P. Since k is strongly βwg-continuous, then $k^{-1}(F)$ is closed set and so $k^{-1}(F)$ is β-closed in R. Again since h is pre-βwg-continuous, $k^{-1}(h^{-1}(F)) = (koh)^{-1}(F)$ is βwg-closed in S. Hence koh: S→R is βwg-irresolute.

THEOREM 3.12:
If a function h: S→R is βwg-continuous and k: R→P is strongly βwg-continuous. Thereupon the composition hok: S→P is βwg-irresolute.
PROOF:
Let F be βwg-closed set in P. Since k is strongly βwg-continuous, then $k^{-1}(F)$ is closed in R. Again since h is pre-βwg-continuous, $h^{-1}(k^{-1}(F)) = (koh)^{-1}(F)$ is βwg-closed in S. Hence koh: S→P is βwg-irresolute.

PROPOSITION 3.13.
Allow a function h:S →R being βwg-irresolute & k: R → P be pre-βwg-continuous map, their composition koh: S →P be pre - βwg – conti. map.
Easy Proofs & follows by Theorem 3.11.

THEOREM 3.14.
If h: S→R and k: R→P are pre-βwg-continuous functions and R is $_{\beta wg}T_b$ –space. Then the composition koh : S→P is pre-βwg-continuous.
PROOF:
Let F be β-closed set in P. Then $k^{-1}(F)$ is βwg-closed in R as k is pre-βwg-continuous. Since, R is $_{\beta wg}T_b$ –space, $h^{-1}(F)$ is closed set and so β-closed in R. Again since h is pre-βwg-continuous, $h^{-1}(k^{-1}(F)) = (koh)^{-1}(F)$ is βwg-closed in S.
THEOREM 3.15.
Allowing both functions h: S →R be βwg – continuous, k:R →P be pre-βwg continuous functions & R be $_{\beta wg}T_b$ –space. Thereupon their composition koh: S → P being is pre-βwg-continuous.
***PROOF*:**
Let F be β-closed set in P. Then $k^{-1}(F)$ is βwg-closed in R as k is pre-βwg-continuous. Since, R is $_{\beta wg}T_b$ -space, $h^{-1}(F)$ is closed set in R. Again since h is βwg-continuous, $h^{-1}(k^{-1}(F)) = (koh)^{-1}(F)$ is pre-βwg-closed in S.

DEFINITION 3.16.
A function h:S → R is said to be contra strongly βwg-continuous if the inverse image of each βwg-open set of R is closed in S.
Clearly it is easy to see that a map h: S→R is contra strongly βwg-continuous if and only if inverse image of each wg-closed set of R is open in S.

PROPOSITION 3.17.
If a function h:S → R is contra strongly βwg-continuous & k:R → P is βwg-continuous function then koh :S→P is contra continuous.
PROOF:
Let Q be an open set in P. Since k is βwg-continuous, $k^{-1}(Q)$ is βwg-open in R. Therefore, $h^{-1}(k^{-1}(Q))$ is closed in S. Since, as h is contra strongly βwg-continuous. So, $(koh)^{-1}(Q) = h^{-1}(k^{-1}(Q))$ is closed in S. Hence koh is contra continuous.

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IV.Pre-βwg-closed Functions and Pre-βwg-open Functions

In this section, we define the followings

DEFINITION 4.1.

A map h: (S, τ) → (R, σ) is termed as **Pre-βwg-closed** (resp. Pre-βwg-open) if for each β-closed(resp. β-open) subset N of S, h(N) is βwg-closed set(resp. βwg- open set) of R.

DEFINITION 4.2.

A map h:S → R being said to be strongly βwg-open if for each βwg-open set D of S,then h(D) is open in R.

THEOREM 4.3.

If function h: (S,τ) →(R, σ) being said to be Pre-βwg-open then it is βwg-open.

PROOF:

Let V is open set of S. Thereupon, V is β - open set in S. As h is pre-βwg - open, h(V) is βwg – open set of R. So it shows that f is βwg-open map.

THEOREM 4.4.

If a function h: (S, τ) → (R, σ) being Pre βwg-open iff for each β-closed subset of S is βwg-closed in R.

PROOF:

Suppose h be pre βwg – closed. As Q is β – open set of S. Thereupon, Q^c is β – open subset of S. Again h be pre βwg – open, h(Q^c) = S- h(Q) being βwg – open set of R . Hence h(Q) is βwg-close in R.

Conversely, allow D is β-closed in S. Thereupon, D^c be β – open subset of X. But h(R- D) = R-h(D) being βwg – closed set of R by assumption. Therefore h(D) be pre-βwg-open subset of R. So h being Pre-βwg-open.

DEFINITION 4.5.

If a function k:(R,σ) → (P,η) is known as always βwg – closed map if for each βwg – closed set D of R, k(D) being βwg-closed in P.

DEFINITION 4.6.

Allowing k: (R,σ) → (P, η) is known as completely βwg – closed map if for each βwg – closed set M of R, k(M) be regular closed set of P..

Now We prove the followings

PROPOSITION 4.7.

Allow a function h: (S,τ) → (R, σ) being completely βwg-open and k: (R, σ) → (P,η) being pre-βwg-open. Then their composition koh: S → R is always βwg – open function.

PROOF:

Take set V is any βwg – open subset of S. Since h be completely βwg – open set, h (V) be βwg –regular open set of R. Hence k(h(V)) = koh(V) being βwg – open in P, as h be pre βwg-open function. Therefore koh: S→ P is always βwg-open function.

REMARK 4.8.

Clearly, note that composition of two pre-βwg - closed functions, again not being pre-βwg - closed functions true as seen from below example.

EXAMPLE 4.9.

Allowing S = {1,3,5}= R, τ ={φ,{1},{1,5}, S}, σ ={φ,{1},R}, P={φ,{1},{1,3},P}. Now a function k:R→P be defined as by k(1) = 5, k3) = 1, k(5) = 3 & let h:S →R be identity map, both h , k are pre-βwg-closed maps. Yet koh being never pre-βwg – closed map. As β - closed set {3, 5} of S.

Now koh ({3,5}) = k (h ({3,5})) = k({3,5})={1,3} be never βwg – closed set of P.

PROPOSITON 4.10.

If function h:S →R being always βwg open, k: R → P being completely βwg-open. Thereupon, their composition hof: S→P is completely βwg-open.

PROOF:

Follows by Theorem 4.7.

Easy proofs of the following results omitted

PROPOSITION 4.11.
Authorize the functions h:S →R be βwg-open and k:R → P being pre-βwg-open, koh: S → P be βwg-open.

PROPOSITION 4.12.
Allow a function h: S → R be always β-open and k: R → P be pre- βwg-open, thereupon koh being pre-βwg-open.

THEOREM 4.13.
Allow a function h: S→R be sgp*-closed & k:R→P be strongly βwg-closed, thereupon koh:S→P being quasi sgp-closed.

PROPOSITION 4.14.
Allowing a function h:S →R be completely βwg-open and k: R→ P being pre- βwg-open, koh: S→P be always βwg-open.

THEOREM 4.15.
If a function h: S→R is βwg-closed and R is $_{\beta wg}T_b$ -space. Then f is closed map.

We define the following

DEFINITION 4.16.
A map h: S→R is called contra strongly βwg-open if the image of each βwg-open set of S is closed in R.
Clearly, it is easy to see that a map h:S→R is contra strongly βwg-open if and only if image of each βwg-closed set of S is open in R.

THEOREM 4.17.
If h: S→R is contra strongly βwg-open & k: R→P is contra-closed map then koh:S→P is contra strongly βwg-open map.
PROOF: Obvious.

DEFINITION 4.18.
A map h:S→R is said to be contra βwg-open if the image of every open set of S is βwg -closed in R.

THEOREM 4.19: If h: S→R & k:R→P be two maps, then the following statements holds:
- **(i)** If h is pre-βwg-open & k is strongly βwg-open, then koh is β-open.
- **(ii)** If koh is always βwg-open and h is βwg-irresolute surjection, then k is always βwg-open.
- **(iii)** If koh is pre-βwg-open and h is completely βwg-continuous, then k is always βwg-open.
- **(iv)** If koh is strongly βwg-open and h is βwg-continuous surjection, then k is an open.
- **(v)** If koh is always βwg-open and k is βwg-irresolute injection, then h is always βwg-open.
- **(vi)** If koh is contra βwg-open & k is βwg- irresolute injection, then h is βwg-open.
- **(vii)** If koh is βwg-open & k is strongly βwg-continuous, then h is open.

V. CONCLUSION

In this article, We introduced and studied a new class of maps termed as pre-βwg-continuous maps using βwg-closed sets in topological spaces and obtain some of their properties. Further also, we defined and studied the new notions of pre-βwg-open maps and pre βwg closed maps.

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