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Research Paper

Functional Random Differential Equation with Random Impulse

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ABSTRACT*: Here, we investigate the neutral functional random differential equation model with random impulses and proved the existence , uniqueness results using Contraction mapping Principle .*
KEYWORDS*: Random differential equation, random impulses, positive solution, contraction principle.*
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I. INTRODUCTION

Many evolution process from various fields are characterized by the fact that they undergo abrupt change of state at certain moments of time between intervals of continuous evolution. The duration of these changes are often negligible compared to the total duration of process act instantaneously in the form of impulses. It is now being recognized that the theory of impulsive differential equations is not only richer than the corresponding theory of differential equations but also represents a more natural frame work for mathematical modeling of many real world phenomena.

The impulses are exist at fixed time or at random time ie., they are deterministic or random. In this paper, we investigate a neutral type of differential equation, its importance in applications is yet to be investigated. So, the problem under study is new to the literature and so are the existence results to the theory of nonlinear problems of ordinary random differential equation.

II. MODEL OF PROBLEM

Consider the neutral functional random differential equation as

$$\left[\frac{p(t,\omega)}{w(t,u(t,\omega),\omega)}\right]' = u(t,u(t,\omega),\omega)+v(t,u(t,\omega),\omega) \quad t\neq\xi_k, t\in[\tau,T], \omega\in\Omega, \tag{1.1}$$

$$p(\xi_k,\omega) = b_k(\tau_k)\,p(\xi_k^-,\omega), k=1,2,\cdots, \tag{1.2}$$

$$p(t_0,\omega) = \varphi, \tag{1.3}$$

Where functionals $u:\Re_\tau\times C\times\Omega\to\Re^n, v:\Re_\tau\times C\times\Omega\to\Re^n-\{0\}, w:\Re_\tau\times C\times\Omega\to\Re^n-\{0\}$ for each $\gamma>0$, define $C(\gamma)=\left\{\varsigma\in C:\|\varsigma\|^2\leq\gamma\right\}$, $C=C([-r,0],\Re^n)$ is the set of piecewise continuous functions from $[-r,0]$ into $\Re^n$ with some given $r>0$ and $p_{(t,\omega)}$ is a function with t is fixed, $\omega\in\Omega$, defined $p_t(s,\omega)=p(t+s+\omega)$ for all $s\in[-r,0]$, $\omega\in\Omega$; $\xi_0=t_0$ and $\xi_k=\xi_{k-1}+\tau_k$ for $k=1,2,\cdots$, here $t_0\in\Re_\tau$ is arbitrary number. Let $\Re^n$ be the $n-$ dimensional Euclidean space . Suppose that τ_k is a random variable defined from Ω to $D_k\stackrel{def.}{=}(0,d_k)$ for all $k=1,2,\cdots$, where $0<d_k<+\infty$ .Let $\tau\in\Re$ be a constant. We denote

$$\Re_{\tau} = [\tau, T], \qquad \Re^{+} = [0, +\infty).$$

Denote $\{B_t, t \geq 0\}$ the simple counting process generated by $\{\xi_n\}$, that is, $\{B_t \geq n\} = \{\xi_n \leq t\}$, and denote F_t the $\sigma -$ algebra generated by $\{B_t, t \geq 0\}$. Then $(\Omega, P, \{F_t\})$ is a probability space.

In this paper, we studied random impulses of neutral type of differential equation and proved the existence, uniqueness results using Contraction Principle .

III. AUXILIARY RESULTS

A random process $\{p(t,\omega),\ t_0 - r \leq t \leq T, \omega \in \Omega\}$ is called a random solution to the equation (1.1)-(1.3), if

(i) $p(t,\omega)$ is $F_t -$ adapted for $\omega \in \Omega$, $t \geq t_0$;

(ii) $p(t_0 + s + \omega) = \varphi(s,\omega)$ where $s \in [-r, 0]$, $\omega \in \Omega$ and

$$p(t,\omega) = \sum_{k=0}^{+\infty} \left[\prod_{i=1}^{k} b_i(\tau_i)\, w\left(t, p_{(t,\omega)}, \omega\right) \left[\frac{\varphi(0,\omega)}{w\left(0, \varphi(0,\omega), \omega\right)} \right] \right.$$
$$+ \sum_{i=1}^{k} \prod_{j=1}^{k} b_j(\tau_j)\, w\left(t, p_{(t,\omega)}, \omega\right) \int_{\xi_{i-1}}^{\xi_i} [u\left(s, p_{(s,\omega)}, \omega\right) + v\left(s, p_{(s,\omega)}, \omega\right)] ds$$
$$\left. + w\left(t, p_{(t,\omega)}), \omega\right) \int_{\xi_k}^{t} [u\left(s, p_{(s,\omega)}, \omega\right) + v\left(s, p_{(s,\omega)}, \omega\right)] ds \right] I_{[\xi_k, \xi_{k+1})}(t,\omega), t \in [t_0, T], \omega \in \Omega$$

where $\prod_{j=1}^{k} b_j(\tau_j) = b_k(\tau_k) b_{k-1}(\tau_{k-1}) \cdots b_i(\tau_i)$, $\prod_{j=m}^{n} (\cdot) = 1$ as $m > n$ and $I_A(\cdot)$ is the index function.

(iii) . $\dfrac{p(t,\omega)}{w\left(t, p_{(t,\omega)}, \omega\right)}$ is differentiable and satisfy the equation (1.1) for $t \in [\tau, T]$, $\omega \in \Omega$.

Consider the following hypotheses for proving the main result.

(A_1). The functions v and u satisfies Lipschitz condition and there exists positive constants $L_1, L_2 > 0$ for $\psi, \zeta \in C$ and $t \in [\tau, T]$, $\omega \in \Omega$ such that

$$\left\| w\left(t, \psi_{(t,\omega)}, \omega\right) - w\left(t, \zeta_{(t,\omega)}, \omega\right) \right\|^2 \leq L_1 \left\| \psi - \zeta \right\|_t^2,$$
$$\left\| (u+v)\left(t, \psi_{(t,\omega)}, \omega\right) - (u+v)\left(t, \zeta_{(t,\omega)}, \omega\right) \right\|^2 \leq L_2 \left\| \psi - \zeta \right\|_t^2.$$

(A_2). The functions $w : \Re_{\tau} \times C \times \Omega \to \Re^n - \{0\}$, $u, v : \Re_{\tau} \times C \times \Omega \to \Re^n$ are continuous and t exists a non-negative constant κ such that

$$\left\| v(t, 0, \omega) \right\|^2 \leq \kappa$$
$$\left\| u(t, 0, \omega) \right\|^2 \leq \kappa, \left\| v(t, 0, \omega) \right\|^2 \leq \kappa.$$

(A_3). There exists constants $0 \leq \gamma_1 < 1$ and $\gamma_2 > 0$ for $\varphi(0,\omega) \in C$ such that

$$\left\| w\left(0, \varphi(0,\omega), \omega\right) \right\|^2 \leq \gamma_1 \left\| \varphi(0,\omega) \right\|^2 + \gamma_2$$

(A_4). $E\left[\max_{i,k} \left\{ \prod_{j=1}^{k} \left\| b_j(\tau_j) \right\|^2 \right\} \right] < \infty$.

IV. MAIN RESULT

Suppose the hypotheses $(A_1)-(A_4)$ holds, then there exists a constant $C>0$ such that

$$E\left[\max_{i,k}\left\{\prod_{j=1}^{k}\left\|b_j(\tau_j)\right\|^2\right\}\right]\le C\ .$$

If the inequality $$\frac{2CL_1E\left\|\varphi(0,\omega)\right\|^2}{\gamma_1E\left\|\varphi(0,\omega)\right\|^2+\gamma_2}+2\max\{1,C\}(T-t_0+\omega)^2\left[L_1(L_2\gamma+\kappa)+(L_1\gamma+\kappa)L_2\right]<1$$

(3.1)

hold , then the problem(1.1)-(1.3) has a unique random solution.

Proof. Suppose T be an arbitrary positive number $t_0<T<+\infty$. For apply the principle, we define operator $P:\Phi_T\to\Phi_T$ as

$$(Pp)(t,\omega)=\varphi(t-t_0+\omega),\qquad t\in[t_0-r,t_0],\ \omega\in\Omega\ ,$$

and

$$(Pp)(t,\omega)=\sum_{k=0}^{+\infty}\left[\prod_{i=1}^{k}b_i(\tau_i)\,w\left(t,p_{(t,\omega)},\omega\right)\frac{\varphi(0,\omega)}{g(0,\varphi(0,\omega),\omega)}\right.$$
$$+\sum_{i=1}^{k}\prod_{j=1}^{k}b_j(\tau_j)\,w\left(t,p_{(t,\omega)},\omega\right)\int_{\xi_{i-1}}^{\xi_i}[u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)]ds$$
$$\left.+\ w\left(t,p_{(t,\omega)},\omega\right)\int_{\xi_k}^{t}\{u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)\}ds\right]I_{[\xi_k,\xi_{k+1})}(t,\omega),\qquad t\in[t_0,T]$$

, $\omega\in\Omega$. Then, to show that P maps Φ_T into itself. Obviously$(Pp):\Phi_T\to\Phi_T$ is continuous with $(Pp)_{t_0}=\varphi$ and $\left\|(Pp)(t,\omega)\right\|^2$.

$$=\left\|\sum_{k=0}^{+\infty}\left[\prod_{i=1}^{k}b_i(\tau_i)\,w\left(t,p_{(t,\omega)},\omega\right)\frac{\varphi(0,\omega)}{g(0,\varphi(0,\omega),\omega)}+\sum_{i=1}^{k}\prod_{j=1}^{k}b_j(\tau_j)\,w\left(t,p_{(t,\omega)},\omega\right)\int_{\xi_{i-1}}^{\xi_i}\left[u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)\right]ds\right.\right.$$
$$\left.\left.+w\left(t,p_{(t,\omega)},\omega\right)\int_{\xi_k}^{t}\left[u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)\right]\right]I_{[\xi_k,\xi_{k+1})}(t,\omega)\right\|^2$$
$$\le 2\sum_{k=0}^{+\infty}\left[\prod_{i=1}^{k}\left\|b_i(\tau_i)\right\|^2\left\|w\left(t,p_{(t,\omega)},\omega\right)\right\|^2\left\|\frac{\varphi(0,\omega)}{w(0,\varphi(0,\omega),\omega)}\right\|^2 I_{[\xi_k,\xi_{k+1})}(t,\omega)\right]$$
$$+2\left[\sum_{k=0}^{+\infty}\left[\sum_{i=1}^{k}\prod_{j=1}^{k}\begin{array}{l}\left\|b_j(\tau_j)\right\|\left\|w\left(t,p_{(t,\omega)},\omega\right)\right\|\int_{\xi_{i-1}}^{\xi_i}\left\|u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)\right\|ds\\+\left\|w\left(t,p_{(t,\omega)},\omega\right)\right\|\int_{\xi_k}^{t}\left\|u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)\right\|ds\end{array}\right]I_{[\xi_k,\xi_{k+1})}(t,\omega)\right]^2$$

$$E\left\|(Pp)(t,\omega)\right\|^2$$
$$\le 2E\sum_{k=0}^{+\infty}\left[\prod_{i=1}^{k}\left\|b_i(\tau_i)\right\|^2\left\|w\left(t,p_{(t,\omega)},\omega\right)\right\|^2\left\|\frac{\varphi(0,\omega)}{w(0,\varphi(0,\omega),\omega)}\right\|^2 I_{[\xi_k,\xi_{k+1})}(t,\omega)\right]$$

$$+2E\left[\sum_{k=0}^{+\infty}\left[\sum_{i=1}^{k}\prod_{j=1}^{k}\begin{array}{l}\left\|b_j(\tau_j)\right\|\left\|w\left(t,p_{(t,\omega)},\omega\right)\right\|\int_{\xi_{i-1}}^{\xi_i}\left\|[u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)]\right\|ds\\+\left\|w\left(t,p_{(t,\omega)},\omega\right)\right\|\int_{\xi_k}^{t}\left\|[u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)]\right\|ds\end{array}\right]I_{[\xi_k,\xi_{k+1})}(t,\omega)\right]^2$$

$$\le 2E\left[\max_k\left\{\prod_{j=1}^{k}\left\|b_j(\tau_j)\right\|^2\right\}\right]E\left\|w\left(t,p_{(t,\omega)},\omega\right)\right\|^2 E\left\|\frac{\varphi(0,\omega)}{w(0,\varphi(0,\omega),\omega)}\right\|^2$$

$$+2E\left[\max_k\left\{1,\prod_{j=1}^{k}\left\|b_j(\tau_j)\right\|^2\right\}\right]E\left[\sum_{k=1}^{+\infty}\left\|w\left(t,p_{(t,\omega)},\omega\right)\right\|\int_{t_0}^{t}\left\|[u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)]\right\|ds I_{[\xi_k,\xi_{k+1})}(t,\omega)\right]^2$$

$$\le 2CE\left\|w\left(t,p_{(t,\omega)},\omega\right)\right\|^2 E\left\|\frac{\varphi(0,\omega)}{w(0,\varphi(0,\omega),\omega)}\right\|^2$$

$$+2\max\{1,C\}E\left[\left\|w\left(t,p_{(t,\omega)},\omega\right)\right\|\int_{t_0}^{t}\left\|[u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)]\right\|ds\right]^2$$

$$\le 2CE\left\|w\left(t,p_{(t,\omega)},\omega\right)\right\|^2 E\left\|\frac{\varphi(0,\omega)}{w(0,\varphi(0,\omega),\omega)}\right\|^2$$

$$+2\max\{1,C\}E\left\|w\left(t,p_{(t,\omega)},\omega\right)\right\|^2(T-t_0+\omega)\int_{t_0}^{t}E\left\|[u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)]\right\|^2 ds$$

$$\le\left[E\left\|w\left(t,p_{(t,\omega)},\omega\right)\right\|^2\right]\left[2CE\left\|\frac{\varphi(0,\omega)}{w(0,\varphi(0,\omega),\omega)}\right\|^2+2\max\{1,C\}(T-t_0+\omega)\int_{t_0}^{t}E\left\|[u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)]\right\|^2 ds\right]$$

$$\le\left[E\left\|w\left(t,p_{(t,\omega)},\omega\right)-v(t,0,\omega)\right\|^2+E\left\|w(t,0,\omega)\right\|^2\right][2CE\left\|\frac{\varphi(0,\omega)}{w(0,\varphi(0,\omega),\omega)}\right\|^2+4\max\{1,C\}(T-t_0+\omega)$$

$$\times\int_{t_0}^{t}\left\{E\left\|(u\left(s,p_{(s,\omega)},\omega\right)-u(s,0,\omega))+(v\left(s,p_{(s,\omega)},\omega\right)-v(s,0,\omega))\right\|^2+E\left\|u(s,0,\omega)+\right\|v(s,0,\omega)^2\right\}ds$$

$$\le\left[L_1E\left\|p\right\|_t^2+\kappa\right]\left[2C\frac{E\left\|\varphi(0,\omega)\right\|^2}{\gamma_1E\left\|\varphi(0,\omega)\right\|^2+\gamma_2}+4\max\{1,C\}(T-t_0+\omega)\int_{t_0}^{t}\left\{L_2E\left\|p\right\|_s^2+\kappa\right\}ds\right]$$

$$\le\left[L_1E\left\|p\right\|_t^2+\kappa\right]\left[2C\frac{E\left\|\varphi(0,\omega)\right\|^2}{\gamma_1E\left\|\varphi(0,\omega)\right\|^2+\gamma_2}+4\max\{1,C\}(T-t_0+\omega)^2\kappa+4\max\{1,C\}L_2(T-t_0+\omega)\int_{t_0}^{t}E\left\|p\right\|_s^2 ds\right]$$

$$\le\left[L_1E\left\|p\right\|_t^2+\kappa\right]\left[\beta_1+\beta_2E\left\|p\right\|_t^2\right]$$

where

$$\beta_1 = 2C\frac{E\left\|\varphi(0,\omega)\right\|^2}{\gamma_1 E\left\|\varphi(0,\omega)\right\|^2+\gamma_2} + 4\max\{1,C\}(T-t_0+\omega)^2\kappa \text{ and}$$

$$\beta_2 = 4\max\{1,C\}L_2(T-t_0+\omega)^2$$

$$\sup_{t_0\le t\le T} E\left\|(Pp)\right\|_t^2 \le \left[L_1\sup_{t_0\le t\le T}E\left\|p\right\|_t^2+\kappa\right]\left[\beta_1+\beta_2\sup_{t_0\le t\le T}E\left\|p\right\|_t^2\right] \qquad (3.2)$$

Hence, the operator P maps Φ_T into itself.

Next, show that P is a contraction mapping, we have

$$(Pp)(t,\omega)-(Pq)(t,\omega)$$

$$= \sum_{k=0}^{+\infty}\left[\prod_{i=1}^{k} b_i(\tau_i)\left[w\left(t,p_{(t,\omega)},\omega\right)-w\left(t,q_{(t,\omega)},\omega\right)\right]\left[\frac{\varphi(0,\omega)}{w\left(0,\varphi(0,\omega),\omega\right)}\right]\right.$$

$$+\sum_{i=1}^{k}\prod_{j=1}^{k} b_j(\tau_j)\left\{\begin{array}{l} w\left(t,p_{(t,\omega)},\omega\right)\int_{\xi_{i-1}}^{\xi_i}[u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)]ds \\ -w\left(t,q_{(t,\omega)},\omega\right)\int_{\xi_{i-1}}^{\xi_i}[u\left(s,q_{(s,\omega)},\omega\right)+v\left(s,q_{(s,\omega)},\omega\right)]ds\end{array}\right\}$$

$$\left.+\left\{w\left(t,p_{(t,\omega)},\omega\right)\int_{\xi_k}^{t}[u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)]ds - w\left(t,q_{(t,\omega)},\omega\right)\int_{\xi_k}^{t}[u\left(s,q_{(s,\omega)},\omega\right)+v\left(s,q_{(s,\omega)},\omega\right)]ds\right\}\right]I_{[\xi_k,\xi_{k+1})}(t,\omega)$$

$$\left\|(Pp)(t,\omega)-(Pq)(t,\omega)\right\|^2$$

$$\sum_{k=0}^{+\infty}\left[\prod_{i=1}^{k}\left\|b_i(\tau_i)\right\|^2\left\|w\left(t,p_{(t,\omega)},\omega\right)-w\left(t,q_{(t,\omega)},\omega\right)\right\|^2\left\|\frac{\varphi(0,\omega)}{w\left(0,\varphi(0,\omega),\omega\right)}\right\|^2\right]I_{[\xi_k,\xi_{k+1})}(t,\omega)$$

$$\le 2+2\left[\sum_{k=0}^{+\infty}\left[\sum_{i=1}^{k}\prod_{j=1}^{k}\left\|b_j(\tau_j)\right\|^2\left\|\begin{array}{l} w\left(t,p_{(t,\omega)},\omega\right)\int_{\xi_{i-1}}^{\xi_i}[u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)]ds \\ -w\left(t,q_{(t,\omega)},\omega\right)\int_{\xi_{i-1}}^{\xi_i}[u\left(s,q_{(s,\omega)},\omega\right)+v\left(s,q_{(s,\omega)},\omega\right)]ds\end{array}\right\|\right.\right.$$

$$\left.\left.+\left\|w\left(t,p_{(t,\omega)},\omega\right)\int_{\xi_k}^{t}[u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)]ds - w\left(t,q_{(t,\omega)},\omega\right)\int_{\xi_k}^{t}[u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)]ds\right\|\right]I_{[\xi_k,\xi_{k+1})}(t,\omega)\right]^2$$

$$E\left\|(Pp)(t,\omega)-(Pq)(t,\omega)\right\|^2$$

$$\le 2E\left[\max_k\left\{\prod_{j=1}^{k}\left\|b_j(\tau_j)\right\|^2\right\}\right]E\left\|w\left(t,p_{(t,\omega)},\omega\right)-w\left(t,q_{(t,\omega)},\omega\right)\right\|^2 E\left\|\frac{\varphi(0,\omega)}{w\left(0,\varphi(0,\omega),\omega\right)}\right\|^2$$

$$+2E\left[\sum_{k=0}^{+\infty}\left[\sum_{i=1}^{k}\prod_{j=1}^{k}\left\|b_j(\tau_j)\right\|\left[\left\|\begin{array}{l} w\left(t,p_{(t,\omega)},\omega\right)\int_{\xi_{i-1}}^{\xi_i}[u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)]ds \\ -w\left(t,q_{(t,\omega)},\omega\right)\int_{\xi_{i-1}}^{\xi_i}[u\left(s,q_{(s,\omega)},\omega\right)+v\left(s,q_{(s,\omega)},\omega\right)]ds\end{array}\right\|\right.\right.\right.$$

$$\left.+\left\|w\left(t,q_{(t,\omega)},\omega\right)\int_{\xi_{i-1}}^{\xi_i}[u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)]ds - w\left(t,q_{(t,\omega)},\omega\right)\int_{\xi_{i-1}}^{\xi_i}[u\left(s,p_{(s,\omega)},\omega\right)+v\left(s,p_{(s,\omega)},\omega\right)]ds\right\|\right]$$

$$+\Big[\Big\|w\left(t, p_{(t,\omega)},\omega\right)\int_{\xi_k}^{t}[u\left(s, p_{(s,\omega)},\omega\right)+v\left(s, p_{(s,\omega)},\omega\right)]ds-w\left(t,q_{(t,\omega)},\omega\right)\int_{\xi_k}^{t}[u\left(s, p_{(s,\omega)},\omega\right)+v\left(s, p_{(s,\omega)},\omega\right)]ds\Big\|$$

$$+\Big\|h\left(t,y_{(t,\omega)},\omega\right)\int_{\xi_k}^{t}[u\left(s, p_{(s,\omega)},\omega\right)+v\left(s, p_{(s,\omega)},\omega\right)]ds-w\left(t,q_{(t,\omega)},\omega\right)\int_{\xi_k}^{t}[u\left(s, p_{(s,\omega)},\omega\right)+v\left(s, p_{(s,\omega)},\omega\right)]ds\Big\|\Big]\Big]I_{[\xi_k,\xi_{k+1})}\left(t,\omega\right)\Big]^2$$

$$\leq 2CE\left\|w\left(t, p_{(t,\omega)},\omega\right)-w\left(t,q_{(t,\omega)},\omega\right)\right\|^2 E\left\|\frac{\varphi\left(0,\omega\right)}{w\left(0,\varphi\left(0,\omega\right),\omega\right)}\right\|^2$$

$$\Big[\sum_{i=1}^{k}\prod_{j=1}^{k}\left\|b_j\left(\tau_j\right)\right\|\Big[\left\|w\left(t, p_{(t,\omega)},\omega\right)-w\left(t,q_{(t,\omega)},\omega\right)\right\|\int_{\xi_{i-1}}^{\xi_i}\left\|[u\left(s, p_{(s,\omega)},\omega\right)+v\left(s, p_{(s,\omega)},\omega\right)]\right\|ds$$

$$+2E\Bigg[\sum_{k=0}^{+\infty}\begin{array}{l}+\left\|w\left(t,q_{(t,\omega)},\omega\right)\right\|\int_{\xi_{i-1}}^{\xi_i}\left\|\left([u\left(s, p_{(s,\omega)},\omega\right)-u\left(s,q_{(s,\omega)},\omega\right)-\left([v\left(s, p_{(s,\omega)},\omega\right)-v\left(s,q_{(s,\omega)},\omega\right)]\right)\right)\right\|ds\Big]\\ +\Big[\left\|w\left(t, p_{(t,\omega)},\omega\right)-w\left(t,q_{(t,\omega)},\omega\right)\right\|\int_{\xi_k}^{t}\left[u\left(s, p_{(s,\omega)},\omega\right)+v\left(s, p_{(s,\omega)},\omega\right)\right]ds\end{array}$$

$$+\left\|w\left(t,q_{(t,\omega)},\omega\right)\right\|\int_{\xi_k}^{t}\left\|\left([u\left(s, p_{(s,\omega)},\omega\right)-u\left(s,q_{(s,\omega)},\omega\right)+\left([v\left(s, p_{(s,\omega)},\omega\right)-v\left(s,q_{(s,\omega)},\omega\right)]\right)\right)\right\|ds$$

$$\leq 2CE\left\|w\left(t, p_{(t,\omega)},\omega\right)-w\left(t,q_{(t,\omega)},\omega\right)\right\|^2 E\left\|\frac{\varphi\left(0,\omega\right)}{w\left(0,\varphi\left(0,\omega\right),\omega\right)}\right\|^2$$

$$+2\max\{1,C\}\,E\Big[\sum_{k=0}^{+\infty}\Big(\left\|w\left(t, p_{(t,\omega)},\omega\right)-w\left(t,q_{(t,\omega)},\omega\right)\right\|\int_{t_0}^{t}\left[u\left(s, p_{(s,\omega)},\omega\right)+v\left(s, p_{(s,\omega)},\omega\right)\right]ds$$

$$+\left\|w\left(t,q_{(t,\omega)},\omega\right)\right\|\int_{t_0}^{t}\left\|\left([u\left(s, p_{(s,\omega)},\omega\right)-u\left(s,q_{(s,\omega)},\omega\right)-\left([v\left(s, p_{(s,\omega)},\omega\right)-v\left(s,q_{(s,\omega)},\omega\right)]\right)\right)\right\|ds\Big)I_{[\xi_k,\xi_{k+1})}\left(t,\omega\right)\Big]^2$$

$$\leq 2CE\left\|w\left(t, p_{(t,\omega)},\omega\right)-w\left(t,q_{(t,\omega)},\omega\right)\right\|^2 E\left\|\frac{\varphi\left(0,\omega\right)}{w\left(0,\varphi\left(0,\omega\right),\omega\right)}\right\|^2$$

$$+\max\{1,C\}\Big(E\left\|w\left(t, p_{(t,\omega)},\omega\right)-w\left(t,q_{(t,\omega)},\omega\right)\right\|^2\left(T-t_0+\omega\right)\int_{t_0}^{t}E\left\|[u\left(s, p_{(s,\omega)},\omega\right)+v\left(s,q_{(s,\omega)},\omega\right)]\right\|^2 ds$$

$$+E\left\|w\left(t,q_{(t,\omega)},\omega\right)\right\|^2\left(T-t_0+\omega\right)\int_{t_0}^{t}E\left\|\left([u\left(s, p_{(s,\omega)},\omega\right)-u\left(s,q_{(s,\omega)},\omega\right)]+\left([v\left(s, p_{(s,\omega)},\omega\right)-v\left(s,q_{(s,\omega)},\omega\right)]\right)\right)\right\|^2 ds\Big)$$

$$\leq 2CL_1E\left\|\frac{\varphi\left(0,\omega\right)}{w\left(0,\varphi\left(0,\omega\right),\omega\right)}\right\|^2 E\left\|p-q\right\|_t^2+\max\{1,C\}\left(T-t_0+\omega\right)$$

$$\times\Big[L_1E\left\|p-q\right\|_t^2\int_{t_0}^{t}\left(L_2E\left\|p\right\|_s^2+\kappa\right)ds+\left(L_1E\left\|p\right\|_t^2+\kappa\right)L_2\int_{t_0}^{t}E\left\|p-q\right\|_s^2 ds\Big].$$

Then,

$$\sup_{t_0\leq t\leq T}E\left\|\left(Pp\right)-\left(Pq\right)\right\|_t^2$$

$$\leq 2CL_1E\left\|\frac{\varphi(0,\omega)}{w(0,\varphi(0,\omega),\omega)}\right\|^2 \sup_{t_0\leq t\leq T} E\|p-q\|_t^2 + 2\max\{1,C\}(T-t_0+\omega)\left[L_1 \sup_{t_0\leq t\leq T} E\|p-q\|_t^2 (T-t_0+\omega)\right.$$

$$\left.\times\left(L_2 \sup_{t_0\leq t\leq T} E\|p\|_t^2+\kappa\right)+\left(L_1 \sup_{t_0\leq t\leq T} E\|p\|_t^2+\kappa\right)L_2(T-t_0+\omega)\sup_{t_0\leq t\leq T} E\|p-q\|_t^2\right]$$

$$\leq 2CL_1E\left\|\frac{\varphi(0,\omega)}{w(0,\varphi(0,\omega),\omega)}\right\|^2 \sup_{t_0\leq t\leq T} E\|p-q\|_t^2 + 2\max\{1,C\}(T-t_0+\omega)\left[L_1 \sup_{t_0\leq t\leq T} E\|p-q\|_t^2 (T-t_0+\omega)\right.$$

$$\left.\times(L_2\gamma+\kappa)+(L_1\gamma+\kappa)L_2(T-t_0+\omega)\sup_{t_0\leq t\leq T} E\|p-q\|_t^2\right]$$

$$\leq\left\{2CL_1\frac{E\|\varphi(0,\omega)\|^2}{\gamma_1E\|\varphi(0,\omega)\|^2+\gamma_2}+2\max\{1,C\}(T-t_0+\omega)^2\left[L_1(L_2\gamma+\kappa)+(L_1\gamma+\kappa)L_2\right]\right\}\sup_{t_0\leq t\leq T} E\|p-q\|_t^2$$

We have

$$\|(Pp)-(Pq)\|_{\Phi_T}^2$$

$$\leq\left\{2CL_1\frac{E\|\varphi(0,\omega)\|^2}{\gamma_1E\|\varphi(0,\omega)\|^2+\gamma_2}+2\max\{1,C\}(T-t_0+\omega)^2\left[L_1(L_2\gamma+\kappa)+(L_1\gamma+\kappa)L_2\right]\right\}\|p-q\|^2 .$$

Hence, from (3.1), P is a contraction mapping. Therefore, mapping P has a unique random fixed point p in Φ_T which is a random solution of (2.1) with $\varphi\in C(\gamma)$. For $t\in[t_0,T]$, $\omega\in\Omega$ and $t\neq\xi_k$, there exists (ξ_k,ξ_{k+1}) such that $t\in(\xi_k,\xi_{k+1})$, which is

$$p(t,\omega)=\prod_{i=1}^{k} b_i(\tau_i)\,w(t,p_{(t,\omega)},\omega)\left[\frac{\varphi(0,\omega)}{w(0,\varphi(0,\omega),\omega)}\right]$$

$$+\sum_{i=1}^{k}\prod_{j=1}^{k} b_j(\tau_j)\,w(t,p_{(t,\omega)},\omega)\int_{\xi_{i-1}}^{\xi_i}\left[u(s,p_{(s,\omega)},\omega)+v(s,p_{(s,\omega)},\omega)\right]ds+w(t,p_{(t,\omega)},\omega)\int_{\xi_k}^{t}\left[u(s,p_{(s,\omega)},\omega)+v(s,p_{(s,\omega)},\omega)\right]ds$$

therefore, ,

$$\left[\frac{p(t,\omega)}{w(t,x_{(t,\omega)},\omega)}\right]'=u(t,p_{(t,\omega)},\omega)+v(t,p_{(t,\omega)},\omega),\text{ as } t\neq\xi_k ,\ \omega\in\Omega .$$

Furthermore,

$$p_{\xi_k}=\prod_{i=1}^{k} b_i(\tau_i)\,w(t,p_{(t,\omega)},\omega)\left[\frac{\varphi(0,\omega)}{w(0,\varphi(0,\omega),\omega)}\right]+\sum_{i=1}^{k}\prod_{j=1}^{k} b_j(\tau_j)\,w(t,p_{(t,\omega)},\omega)\int_{\xi_{i-1}}^{\xi_i}[u(s,p_{(s,\omega)},\omega)+v(s,p_{(s,\omega)},\omega)]ds$$

and $$p_{\xi_k}^{-}=\prod_{i=1}^{k-1} b_i(\tau_i)\,w(t,p_{(t,\omega)},\omega)\left[\frac{\varphi(0,\omega)}{w(0,\varphi(0,\omega),\omega)}\right]$$

$$+\sum_{i=1}^{k-1}\prod_{j=1}^{k-1} b_j(\tau_j)\, w(t, p_{(t,\omega)}, \omega)\int_{\xi_{i-1}}^{\xi_i} [u(s, p_{(s,\omega)}, \omega) + v(s, p_{(s,\omega)}, \omega)]ds$$

$$+ w(t, p_{(t,\omega)}, \omega)\int_{\xi_{k-1}}^{t} [u(s, p_{(s,\omega)}, \omega) + v(s, p_{(s,\omega)}, \omega)]ds$$

imply that $p_{\xi_k} = b_k(\tau_k)\, p_{\xi_k}^{-}$. Thus, $p(t,\omega)$ is a random solution to system (2.1).

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