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Research Paper

Complete Intuitionistic Fuzzy Inner unitary Subsemigroups on a Regular Semigroup

N.Lalithamani[1], K.Prabakaran*[1], Merga Guluma Dangala[2]
[1](*Department of Mathematics, K.S.Rangasamy College of Technology, Tiruchengode, Tamilnadu, India 637215,*)
[2](*Department of Mahematics,College of Natural and Computational Science, Mettu University, Ethiopia 318.)*
Corresponding Author: K.Prabakaran

ABSTRACT: *In this paper, we introduced the notations to complete intuitionistic inner unitary sub semigroup and intuitionistic fuzzy semigroup. We characterized some definitions and theorem proof in intuitionistic fuzzy complete inner unitary sub semigroup and intuitionistic fuzzy semigroup of these notations.*

KEYWORDS: *Semi group, Intuitionistic fuzzy semigroup, Intuitionistic inner unitary sub semigroup, complete inner unitary sub semigroup, Relations.*

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I. INTRODUCTION

In today's, contemporary world of pure and applied mathematics, the application of algebraic structures is seen in computer science, information technology, image Processing, control theory and Fuzzy automata and there is a great scope for effective research. Algebraic structures, particularly, semigroups play a key role in above mentioned applied branches. Further, the fuzzification's of some sub-systems of semigroups are used in numerous representations containing uncertainties.

Zadeh [15] was the pioneer who introduced fuzzy set, since this concept has been applied to various algebraic structures. Ullahkhan [1] introduced the notion and characterized intuitionistic fuzzy interior ideals and interior ideals of semigroups. Davvaz and Majumder [2] characterized a simple Γ semigroup in terms of intuitionistic fuzzy interior ideals. Sharma [3] derived quotient groups of fuzzy fundamental theorem and Cayley theorem of every intuitionistic fuzzy group.

Yun and Lee [4] proposed intuitionistic fuzzy rough approximation operators and properties of intuitionistic fuzzy investigated fuzzy topology. Maya and Isaac [5] proved the intuitionistic fuzzy continuity of the composition of two intuitionistic fuzzy continuity with example. Mydhily and Natarajan [6] introduced some theorems in intuitionistic fuzzy l sub semi ring of a l semi ring under homomorphism and anti-homomorphism. Eslami [7] demonstrated the some theorem degree of membership and non-membership of intuitionistic fuzzy sets to lattices and fuzzy logic.

Hur *et al.*[8] characterized a regular semigroup , a regular semigroup that is a lattice of left(right) simple semigroups, a semigroup that is semi lattice of groups in terms of intuitionistic fuzzy ideals and intuitionistic fuzzy bi ideals. Khan and Anis [9] characterized right regular semi groups by the properties of their right ideals, bi ideals generalized bi ideal and interior ideal. Mandal [10] introduced fuzzy ideals, some operations ordered semi rings in order to study the structure of fuzzy ordered semi rings. He have studied and characterized in terms of fuzzy k ideal and also in terms of fuzzy h ideal in ordered semi ring and compositions of fuzzy ideals of ordered semi ring.

Gulistan *et al.*[11] generalized cubic soft sets which is the most general approach and characterize the weekly regular semi group in terms of generalized cubic soft ideals with examples and some properties. Elomari *et al.* [12] established intuitionistic fuzzy strongly continuous semi group and uniqueness of solutions for intuitionistic fuzzy equation, some of their properties and some results. Sardar *et al.*[13] characterized intuitionistic fuzzy ideals in semigroups and observed several important properties with examples. Sumathi and Sathappan [14] investigated intuitionistic L-fuzzy bi ideal of semigroup and some of their related properties. In this paper we introduced the some Intuitionistic definitions and theorem proof in intuitionistic fuzzy complete inner unitary sub semigroup and intuitionistic fuzzy semigroup of these notations.

II. PRELIMINARY

In this section we discuss some elementary definitions that we use in the sequel.

Definition: 2.1

An intuitionistic fuzzy set (IFS) $A = <\mu_A, \vartheta_A>$ in T is called an intuitionistic fuzzy ternary sub semi group of T if

$$(i)\ \mu_A(xyz) \geq \min\{\mu_A(x), \mu_A(y), \mu_A(z)\}$$

$$(ii)\ \vartheta_A(xyz) \leq \max\{\vartheta_A(x), \vartheta_A(y), \vartheta_A(z)\} \text{ for all } x, y, z \in T.$$

Definition: 2.2

Let X be a nonempty fixed set. An intuitionistic fuzzy set (IFS) A in X is an object having the form $A = \{< x, \mu_A(x), \vartheta_A(x) > / x \in X\}$, where the function $\mu_A : X \to [0,1]$ and $\vartheta_A : X \to [0,1]$ denote the degree of the membership and non-membership of each element $x \in X$ to the set A, respectively $0 \leq \mu_A(x) + \vartheta_A(x) \leq 1$.

Notation: We shall use the symbol $A = (\mu_A, \vartheta_A)$ for the intuitionistic fuzzy set (IFS) $A = \{< x, \mu_A(x), \vartheta_A(x) > / x \in X\}$.

Definition: 2.3

Let S be any semigroup. An intuitionistic fuzzy set $A = \{< x, \mu_A(x), \vartheta_A(x) > / x \in X\}$ of S is called an intuitionistic fuzzy sub semigroup of S if

(i) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$, for $\forall x, y \in S$

(ii) $\vartheta_A(xy) \leq \vartheta_A(x) \vee \vartheta_A(y)$, for $\forall x, y \in S$

Definition: 2.4

Let S be any semigroup. An intuitionistic fuzzy set A of S is called an intuitionistic fuzzy inner-unitary subset of S, if

$$\mu_A(a) \geq \mu_A(xay) \wedge \mu_A(xy) \quad for\ a, x, y \in S$$

$$\vartheta_A(a) \leq \vartheta_A(xay) \vee \vartheta_A(xy) \quad for\ a, x, y \in S$$

A subsemigroups H of S is called inner unitary, if $xay, xy \in H$ implies $a \in H$ for any $a, x, y \in S.$

Definition: 2.5

Let S be a semigroup. An intuitionistic fuzzy subset A of S is called an intuitionistic fuzzy inner unitary sub semigroup as well as an intuitionistic fuzzy inner unitary subset of S. In other words, an intuitionistic fuzzy subset of S is an intuitionistic fuzzy inner unitary sub semigroup if and only if $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$ and $\vartheta_A(xy) \leq \vartheta_A(x) \vee \vartheta_A(y)$

$\mu_A(a) \geq \mu_A(xay) \wedge \mu_A(xy)$ and $\vartheta_A(a) \leq \vartheta_A(xay) \vee \vartheta_A(xy)$ for any $a, x, y \in S.$

Example: 2.6

Let S be the semigroup of natural numbers with respect to multiplication and p a prime number. Also let the intuitionistic fuzzy subset A of S be defined by

$$\mu_A(x) = \begin{cases} 1, & if \quad x = 1 \\ 0.5 & if \quad x = p^t,\ t = 1,2,.... \\ 0 & ,otherwise \end{cases} \quad \text{and} \quad \vartheta_A(x) = \begin{cases} 1, & if \quad x = 1 \\ 0.5 & if \quad x = p^t,\ t = 1,2,.... \\ 0 & ,otherwise \end{cases}$$

then A is an intuitionistic fuzzy inner unitary subsemigroups of S.

Definition: 2.7

Let S be a semigroup with $E(S) \neq \phi.$ An inner unitary sub semigroup H of S is called complete if $E(S) \subseteq H.$

Definition: 2.8

Let S be a semigroup with $E(S) \neq \phi.$ An intuitionistic fuzzy inner inner unitary sub semigroup A of S is called intuitionistic fuzzy complete, if $\mu_A(e) = 1$ and $\vartheta_A(e) = 0$ for any $e \in E(S).$

III. MAIN THEOREM

Theorem: 3.1

Let S be a semigroup with $\operatorname{Re}g(S)=\{a\in S: there\ exists\ ax\in S$ such that $axa=a\}\neq\phi$ and A be an intuitionistic fuzzy inner unitary subset of S, then

(i) $\mu_A(ab)\le\mu_A(a^{'}a)\wedge\mu_A(bb^{'})$ and $\vartheta_A(ab)\ge\nu_A(a^{'}a)\vee\vartheta_A(bb^{'})$

(ii) $\mu_A(a^{'}a)=\mu_A(aa^{'})$ and $\vartheta_A(a^{'}a)=\vartheta_A(aa^{'})$

(iii) $\mu_A(a^{'})\ge\mu_A(a)\wedge\mu_A(a^2)$ and $\vartheta_A(a^{'})\le\vartheta_A(a)\vee\vartheta_A(a^2)$

(iv) $\mu_A(aa^{'})=\mu_A(aa^{*})$ and $\vartheta_A(aa^{'})=\vartheta_A(aa^{*})$ for any $a,b\in\operatorname{Re}g(S)$ and $a^{'},a^{*}\in V(a),b^{'},b^{*}\in V(b)$ where $V(a)=\{x\in S: axa=a\ \ and\ \ xax=x\}$.

Proof: (i) Let $a,b\in\operatorname{Re}g(S)$ and $a^{'},a^{*}\in V(a)$ and $b^{'},b^{*}\in V(b)$

Since $a=aa^{'}a$ so $\mu_A(ab)=\mu_A(aa^{'}ab)$ and $\vartheta_A(ab)=\vartheta_A(aa^{'}ab)$ and hence

$$\mu_A(ab)=\mu_A(aa^{'}ab)\wedge\mu_A(ab)$$
$$\le\mu_A(a^{'}a)$$ and
$$\vartheta_A(ab)=\vartheta_A(aa^{'}ab)\vee\vartheta_A(ab)$$
$$\le\vartheta_A(a^{'}a)$$

Similarly, $\mu_A(ab)=\mu_A(bb^{'})$ and $\vartheta_A(ab)=\vartheta_A(bb^{'})$.

Therefore we obtain that $\mu_A(ab)\le\mu_A(a^{'}a)\wedge\mu_A(bb^{'})$ and $\vartheta_A(ab)\ge\vartheta_A(a^{'}a)\vee\vartheta_A(bb^{'})$.

(ii) Evidently, $a^{'}\in\operatorname{Re}g(S)$ and $a\in V(a^{'})$ also hold when $a\in\operatorname{Re}g(S)$ and $a^{'}\in V(a)$. So using (i) we obtain

$$\mu_A(a^{'}a)\le\mu_A(aa^{'})\wedge\mu_A(aa^{'})$$
$$=\mu_A(aa^{'})\le\mu_A(a^{'}a)\wedge\mu_A(a^{'}a)$$
$$=\mu_A(aa^{'})$$ and
$$\vartheta_A(a^{'}a)\ge\vartheta_A(aa^{'})\vee\vartheta_A(aa^{'})$$
$$=\vartheta_A(aa^{'})\ge\vartheta_A(a^{'}a)\vee\vartheta_A(a^{'}a)$$
$$=\vartheta_A(aa^{'})$$

Hence $\mu_A(a^{'}a)=\mu_A(aa^{'})$ and $\vartheta_A(a^{'}a)=\vartheta_A(aa^{'})$

(iii) By Definition 2.3 we have $\mu_A(a^{'})\ge\mu_A(aa^{'}a)\wedge\mu_A(aa)=\mu_A(a)\wedge\mu_A(a^2)$

and $\vartheta_A(a^{'})\le\vartheta_A(aa^{'}a)\vee\vartheta_A(aa)=\vartheta_A(a)\vee\vartheta_A(a^2)$

(iv) By $aa^{*}\in V(aa^{'})$ and $aa^{'}\in V(aa^{*})$, so that

$$\mu_A(aa^{*})\ge\mu_A(aa^{'})\wedge\mu_A((aa^{'}))^2=\mu_A(aa^{'})$$

$\mu_A(aa^{'})\ge\mu_A(aa^{*})$ by using (iii) and

$$\vartheta_A(aa^{*})\le\vartheta_A(aa^{'})\vee\vartheta_A((aa^{'}))^2=\vartheta_A(aa')$$

$\vartheta_A(aa^{'})\le\vartheta_A(aa^{*})$ by using (iii) .Hence $\mu_A(aa^{'})=\mu_A(aa^{*})$ and $\vartheta_A(aa^{'})=\vartheta_A(aa^{*})$.

Theorem: 3.2

Let S be the semigroup. Then A is intuitionistic fuzzy inner unitary subsemigroups of S if and only if the level subset $A_{<\alpha,\beta>}=\{x\in S:\mu_A(x)\ge\alpha\ \ and\ \ \vartheta_A(x)\le\beta\}$ of A is an inner unitary subsemigroups of S for any $(\alpha,\beta)\in[0,1]$.

Proof: For any $x,y,a\in S$, Let $\mu_A(x)=l_1,\mu_A(y)=l_2$ and $\vartheta_A(x)=p_1,\vartheta_A(y)=p_2$ and let $\alpha=\min\{l_1,l_2\}$ and $\beta=\max\{p_1,p_2\}$.

Then $x, y \in A_{<\alpha,\beta>}$ and so $xy \in A_{<\alpha,\beta>}$ since $A_{<\alpha,\beta>}$ is a sub semigroup of S.
Hence $\mu_A(xy) \geq \alpha = \mu_A(x) \wedge \mu_A(y)$ and $\vartheta_A(xy) \leq \beta = \vartheta_A(x) \vee \vartheta_A(y)$.
Let $\mu_A(xy) = l_1, \mu_A(xay) = l_2$ and $\vartheta_A(xy) = p_1, \vartheta_A(xay) = p_2$ and let $\alpha = \min\{ l_1, l_2\}$ and $\beta = \max\{p_1, p_2\}$. Then $xy, xay \in A_{<\alpha,\beta>}$ consequently $a \in A_{<\alpha,\beta>}$ since $A_{<\alpha,\beta>}$ is an inner unitary sub semigroup of S. Therefore $\mu_A(a) \geq \alpha = \mu_A(xy) \wedge \mu_A(xay)$ and $\vartheta_A(a) \leq \beta = \vartheta_A(xy) \vee \vartheta_A(xay)$.
So that A is an intuitionistic fuzzy inner unitary sub semigroup of S.

Theorem: 3.3
Let S be a semigroup and A an intuitionistic fuzzy inner unitary sub semigroup of S. Then

(i) $\mu_A(xy) = \mu_A(yx)$ and $\vartheta_A(xy) = \vartheta_A(yx)$ $\forall x, y \in S$.

(ii) $\mu_A(a) \geq \mu_A(ax) \wedge \mu_A(x)$, $\mu_A(a) \geq \mu_A(xa) \wedge \mu_A(x)$ and
$\vartheta_A(a) \leq \vartheta_A(axy) \vee \vartheta_A(x)$, $\vartheta_A(a) \leq \vartheta_A(xa) \vee \vartheta_A(x)$ $\forall a, x \in S$

(iii) If $E(S) = \{x \in S : x^2 = x\} \neq \phi,$ then $\mu_A(nk) = \mu_A(n) \wedge \mu_A(k)$ and
$\vartheta_A(nk) = \vartheta_A(n) \vee \vartheta_A(k), \ \forall n, k \in E(S).$

(iv) If $\mathrm{Reg(S)} \neq \phi$, then $\mu_A(a') = \mu_A(a)$ and $\vartheta_A(a') = \vartheta_A(a)$ $\forall a \in \mathrm{Re}\,g(S), a' \in V(a).$

Proof: (i) By Definition 2.3 , we have $\mu_A(xy) \geq \mu_A(y.xy.x) \wedge \mu_A(yx)$
$\mu_A(y.xy.x) = \mu_A((yx)^2) \geq \mu_A(yx) \wedge \mu_A(yx) = \mu_A(yx)$ and
$\vartheta_A(xy) \leq \vartheta_A(y.xy.x) \vee \vartheta_A(yx)$
$\vartheta_A(y.xy.x) = \vartheta_A((yx)^2) \leq \vartheta_A(yx) \vee \vartheta_A(yx) = \vartheta_A(yx)$
So $\mu_A(xy) \geq \mu_A(yx)$ and $\vartheta_A(xy) \geq \vartheta_A(yx)$.
Similarly, we have $\mu_A(yx) \geq \mu_A(xy)$ and $\vartheta_A(yx) \leq \vartheta_A(xy)$
Hence $\mu_A(xy) = \mu_A(yx)$ and $\vartheta_A(xy) = \vartheta_A(yx)$ $\forall x, y \in S$.

(ii) It is straight forward proved.
Let $\forall n, k \in E(S)$; then clearly $n \in V(n)$ and $k \in V(k)$.
So we have $\mu_A(nk) \leq \mu_A(n.n) \wedge \mu_A(k.k) = \mu_A(n) \wedge \mu_A(k)$ and
$\vartheta_A(nk) \leq \vartheta_A(n.n) \wedge \vartheta_A(k.k) = \vartheta_A(n) \wedge \vartheta_A(k)$ by theorem 3.1(i)
On the other hand $\mu_A(nk) \geq \mu_A(n) \wedge \mu_A(k)$ and $\vartheta_A(nk) \leq \vartheta_A(n) \vee \vartheta_A(k)$.
Since A is an intuitionistic fuzzy sub semigroup.
Therefore $\mu_A(nk) = \mu_A(n) \wedge \mu_A(k)$ and $\vartheta_A(nk) = \vartheta_A(n) \vee \vartheta_A(k)$.

(iii) Let $a \in \mathrm{Re}\,g(S)$ *and* $a' \in V(a).$ by using theorem 3.1 (iii), we have
$\mu_A(a^{'}) \geq \mu_A(a) \wedge \mu_A(a^2)$ and $\vartheta_A(a^{'}) \leq \vartheta_A(a) \wedge \vartheta_A(a^2)$.
But $\mu_A(a^2) \geq \mu_A(a) \wedge \mu_A(a)$ and $\vartheta_A(a^2) \geq \vartheta_A(a) \wedge \vartheta_A(a)$
Since A is an intuitionistic fuzzy sub semigroup.
Similarly $\mu_A(a\ \ ) \geq \mu_A(a'aa') \wedge \mu_A(a'.a') \geq \mu_A(a')$ and $\vartheta_A(a\ \ ) \geq \vartheta_A(a'aa') \wedge \vartheta_A(a'.a') \geq \vartheta_A(a')$
Hence $\mu_A(a') = \mu_A(a)$ and $\vartheta_A(a') = \vartheta_A(a)$.

Theorem: 3.4
Let S be a semigroup. Then an intuitionistic fuzzy sub semigroup of A of S is an intuitionistic fuzzy inner unitary sub semigroup if and only if

(i) $\mu_A(xy) = \mu_A(yx)$ and $\vartheta_A(xy) = \vartheta_A(yx)$

(ii) $\mu_A(a) \geq \mu_A(ax) \wedge \mu_A(x)$ or $\mu_A(a) \geq \mu_A(xa) \wedge \mu_A(x)$ and
$\vartheta_A(a) \leq \vartheta_A(ax) \vee \vartheta_A(x)$ or $\vartheta_A(a) \geq \vartheta_A(xa) \wedge \vartheta_A(x)$ for any $a, x, y \in S$.

Proof: This follows immediately from Theorem 3.3
Conversely, if the condition (i) and (ii) hold, then we have

$$\mu_A(a) \geq \mu_A(a.yx) \wedge \mu_A(yx) = \mu_A(ay.x) \wedge \mu_A(yx)$$
$$= \mu_A(xay) \wedge \mu_A(xy) \text{ and}$$
$$\vartheta_A(a) \leq \vartheta_A(a.yx) \vee \vartheta_A(yx) = \vartheta_A(a.yx) \vee \vartheta_A(yx)$$
$$= \vartheta_A(xay) \vee \vartheta_A(xy)$$

Hence A is also an intuitionistic fuzzy inner unitary subset of S. Therefore A is an intuitionistic fuzzy inner unitary sub semigroup of S.

Theorem: 3.5

Let S be a semigroup and A an intuitionistic fuzzy inner unitary sub semigroup of S. Then

(i) If $e \in E(S)$ and $a \in S$ such that $a\Re e$ then $\mu_A(a) \leq \mu_A(e)$ and $\vartheta_A(a) \leq \vartheta_A(e)$

(ii) If $a \in \mathrm{Re}\,g(S)$ and $a' \in V(a),$ then $\mu_A(a) \leq \mu_A(aa')$, $\mu_A(a) \leq \mu_A(a'a)$ and $\vartheta_A(a) \geq \vartheta_A(aa')$, $\vartheta_A(a) \geq \vartheta_A(a'a)$.

Proof: (i) Since $a\Re e$, so $a = e$ or there are $x, y \in S$ such that $e = ax$ and $a = ey$.

If $a = e$, then $\mu_A(a) = \mu_A(e)$ and $\vartheta_A(a) = \vartheta_A(e)$.

If $a \neq e$, then $e = ax$ and $a = ey$ where $x, y \in S$

Thus we have $\mu_A(a) \geq \mu_A(ea) \wedge \mu_A(a)$

$$= \mu_A(e.ey) \wedge \mu_A(a)$$
$$= \mu_A(ey) \wedge \mu_A(a)$$
$$= \mu_A(a) \wedge \mu_A(a) = \mu_A(a) \text{ and}$$
$$\vartheta_A(a) \geq \vartheta_A(ea) \wedge \vartheta_A(a)$$
$$= \vartheta_A(e.ey) \wedge \vartheta_A(a)$$
$$= \vartheta_A(ey) \wedge \vartheta_A(a)$$
$$= \vartheta_A(a) \wedge \vartheta_A(a) = \vartheta_A(a)$$

(ii) Here note that $a\Re aa'$ therefore we have the following results exits on from (i) becomes

$$\mu_A(a) \leq \mu_A(aa'),\ \mu_A(a) \leq \mu_A(a'a) \text{ and } \vartheta_A(a) \geq \vartheta_A(aa'),\ \vartheta_A(a) \geq \vartheta_A(a'a).$$

Theorem: 3.6

Let S be a semigroup with $E(S) \neq \phi.$ Then an intuitionistic fuzzy subset A of S is an intuitionistic fuzzy complete inner unitary sub semigroup of S if and if the (l, p) level subset $A_{<l,p>}$ of A is a complete inner unitary sub semigroup of S for any $(l, p) \in [0,1].$

Proof: If subset A is an intuitionistic fuzzy complete inner unitary sub semigroup, then $A_{<l,p>}$ is inner unitary sub semigroup of S for any $(l, p) \in [0,1]$, by Theorem 3.2 and for any $e \in E(S)$ we have $\mu_A(e) = 1 \geq l$ and $\vartheta_A(e) = 0 \leq p$, that is $e \in A_{<l,P>}$. Hence $A_{<l,P>}$ is a complete inner unitary sub semigroup of S.

Conversely, $A_{<l,P>}$ is a complete inner unitary sub semigroup of S for any $(l, p) \in [0,1]$, then A is an intuitionistic fuzzy inner unitary sub semigroup of S by Theorem 3.3. Taking $l = 1$ and $p = 0$, we have that $e \in A_{<1,0>}$ for any $e \in E(S)$ since $A_{<1,0>}$ is complete inner unitary. So $\mu_A(e) \geq 1$ and $\vartheta_A(e) \leq 0$ forces that $\mu_A(e) = 1$ and $\vartheta_A(e) = 0$. Therefore, A is an intuitionistic fuzzy complete inner unitary sub semigroup of S.

Theorem : 3.7

Let S be a regular semigroup and A an intuitionistic fuzzy inner unitary sub semigroup of S. Then A intuitionistic fuzzy complete if and only if

(i) $\mu_A(ea) = \mu_A(a)$ and $\vartheta_A(ea) = \vartheta_A(a)$ for any $e \in E(S)$ and $a \in S$

(ii) For each $a \in S,$ there is an $x \in S$ such that $\mu_A(ax) = 1$ and $\vartheta_A(ax) = 0.$

Proof:
(i) For any $e \in E(S)$ and $a \in S$, since $\mu_A(ea) \geq \mu_A(e) \wedge \mu_A(a)$ and $\vartheta_A(ea) \leq \vartheta_A(e) \vee \vartheta_A(a)$ and $\mu_A(e) = 1$ and $\vartheta_A(e) = 0$, so $\mu_A(ea) \geq \mu_A(a)$ and $\vartheta_A(ea) \leq \vartheta_A(a).$ on other hand, $\mu_A(a) \geq \mu_A(ea) \wedge \mu_A(e)$ and $\vartheta_A(a) \leq \vartheta_A(ea) \vee \vartheta_A(e)$ by Theorem 3.4 and $\mu_A(e) = 1$ and $\vartheta_A(e) = 0$, so we have $\mu_A(a) \geq \mu_A(ea)$ and $\vartheta_A(a) \leq \vartheta_A(ea)$.
Therefore $\mu_A(ea) = \mu_A(a)$ and $\vartheta_A(ea) = \vartheta_A(a).$ Here, we know the condition (i) holds here. Since S is regular, so $V(a) \neq \phi$ for any $a \in S.$ Let $x \in V(a),$ then $ax \in E(S)$ and so $\mu_A(ax) = 1$ and $\vartheta_A(ax) = 0.$ This shows that (ii) holds.
Conversely, we need only to show that $\mu_A(e) = 1$ and $\vartheta_A(e) = 0$ for any $e \in E(S)$.
By Condition (ii), there is an $x \in S$ such that $\mu_A(ex) = 1$ and $\vartheta_A(ex) = 0$, so we have $\mu_A(e) \geq \mu_A(ex) \wedge \mu_A(x) = \mu_A(x)$ and $\mu_A(x) \geq \mu_A(ex) \wedge \mu_A(e) = \mu_A(e)$ $\vartheta_A(e) \leq \vartheta_A(ex) \vee \vartheta_A(x) = \vartheta_A(x)$ and $\vartheta_A(x) \leq \vartheta_A(ex) \vee \vartheta_A(e) = \vartheta_A(e)$ by Theorem 3.4 and therefore we obtain that $\mu_A(e) = \mu_A(x)$ and $\vartheta_A(e) = \vartheta_A(x)$. But $\mu_A(ex) = \mu_A(x)$ and $\vartheta_A(ex) = \vartheta_A(x)$ by the condition (i). Consequently $\mu_A(e) = 1$ and $\vartheta_A(e) = 0.$

Theorem: 3.8
Let S be a regular semigroup and A an intuitionistic fuzzy complete inner unitary sub semigroup of S. Then $\mu_A(a'b) = \mu_A(a^*b)$ and $\vartheta_A(a'b) = \vartheta_A(a^*b)$ for $a,b \in S$ and $a', a^* \in V(a).$

Proof: Let $a,b \in S$, $a', a^* \in V(a)$ and $b' \in V(b).$

Then $\mu_A(a'b) \geq \mu_A(a.a'b.b') \wedge \mu_A(ab')$
$= \mu_A(aa') \wedge \mu_A(bb') \wedge \mu_A(ab')$ by Theorem 3.3(iii)
$= 1 \wedge 1 \wedge \mu_A(ab')$
$\geq \mu_A(a^*.ab'.b) \wedge \mu_A(a^*b) = \mu_A(a^*b)$

And $\vartheta_A(a'b) \leq \vartheta_A(a.a'b.b') \vee \vartheta_A(ab')$
$= \vartheta_A(aa') \vee \vartheta_A(bb') \vee \vartheta_A(ab')$ by Theorem 3.3(iii)
$= 0 \vee 0 \vee \vartheta_A(ab')$
$\leq \vartheta_A(a^*.ab'.b) \vee \vartheta_A(a^*b) = \vartheta_A(a^*b)$

Similarly, we prove that $\mu_A(a^*b) \geq \mu_A(a'b)$ and $\vartheta_A(a^*b) \leq \vartheta_A(a'b).$
Hence $\mu_A(a^*b) = \mu_A(a'b)$ and $\vartheta_A(a^*b) = \vartheta_A(a'b).$

IV. CONCLUSION

It is obvious that if we introduced some operator on complete intuitionistic fuzzy of an inner unitary sub semigroup in order to study the structure of intuitionistic fuzzy sets. As a continuation of this research characterized some intuitionistic definitions and their theorem were proved in the intuitionistic fuzzy complete inner unitary sub semigroup and intuitionistic fuzzy semigroup.

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