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Research Paper

On Sombor indices of $VC_5C_7[p,q]$ nanotubes by M-polynomial and exponential

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Abstract
Sombor index is newly introduced and widely studied degree based topological index defined by I.Gutman in the form of function F(x,y) = $\sqrt{x^2+y^2}$ *which was not reported in the chemical graph theory earlier. Sombor index is defined as*

SO(G) = $\sum_{uv\in E(G)}\sqrt{{d_u}^2+{d_v}^2}$*, where G is finite,simple connected graph with vertex set V(G) and edge set E(G) and* d_v *is the degree of vertex v∈V(G).In this paper different versions of Sombor index of* $VC_5C_7[p,q]$ *nanotubes are investigated by M-polynomial and exponential.*

***Keywords**: Degree, increased Sombor index, molecular graph, M-polynomial,reduced Sombor index, Sombor index,topological index,*$VC_5C_7[p,q]$ *nanotubes.*

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I. Introduction

Let G be a finite,simple,connected graph with vertex set V(G) and edge set E(G).In chemical graph theory eachvertex represents an atom of the molecule and covalent bonds between atoms are represented by the edges between the corresponding vertices.Two vertices u and v are adjacent if there is an edge e = uv between them.The degree of vertex v∈V(G),d_v is the number of edges incident with v.A topological index is a numerical parameter mathematically derived from the graph structure.M-polynomials and topological indices of $VC_5C_7[p,q]$ and $HC_5C_7[p,q]$ nanotubes are studied by Jia-B.Liu et al.[1].Topological indices of $VC_5C_7[p,q]$carbon nanotubes are reported in [2-8].Topological polynomials of molecular graphs are studied in [9-12].M-polynomials of molecular graphs are discussed in [13-17]. Different versions of novel harmonic index are discussed in [18].

Sombor coindices of graphs reported by P.Chinglensana [19].Sombor exponential,reduced Sombor exponential and average Sombor exponential of graph G is introduced by V.R.Kulli[20].The Sombor index,reduced Sombor index and average Sombor index showed good predictive potentials [21].More information on Sombor indices of graphs can be found in [22-35].

New degree based topological index called Sombor index is recently introduced and widely studied after Zagreb indices for molecular graphs in chemical graph theory, which is defined as

$$SO = SO(G) = \sum_{uv\in E(G)}\sqrt{(d_u^2+d_v^2)}\ ,$$

where d_v is degree of vertex v in graph G [36].
The first Banhatti Sombor index,first reduced Sombor index and first δ-Banhatti Sombor index of a graph defined respectively as [37]

$$BSO_1(G) = \sum_{uv\in E(G)}\sqrt{\frac{1}{d_u^2}+\frac{1}{{d_v}^2}}\ ,$$

$$RBSO_1(G) = \sum_{uv\in E(G)}\sqrt{\frac{1}{(d_u-1)^2}+\frac{1}{(d_v-1)^2}}\ ,$$

$$\text{and } \delta BSO_1(G) = \sum_{uv\in E(G)}\sqrt{\frac{1}{(d_u-\delta(G)+1)^2}+\frac{1}{(d_v-\delta(G)+1)^2}}\ , \text{ for } \delta(G)\geq 2.$$

The average Sombor index and reduced Sombor index are introduced in [38] as

$$SO_{avg}(G) = \sum_{uv\in E(G)} \sqrt{(d_u - \frac{2m}{n})^2 + (d_v - \frac{2m}{n})^2},$$

where |V(G)| = n, |E(G)| = m and

$$RSO(G) = \sum_{uv\in E(G)} \sqrt{(d_u - 1)^2 + (d_v - 1)^2}.$$

The generalized Sombor index defined as $SO_a(G) = \sum_{uv\in E(G)} \sqrt{(d_u - a)^2 + (d_v - a)^2}$.

Like reduced Banhatti Sombor index or reduced Sombor index, increased Sombor index defined by W.Ning[39]

$$SO^1(G) = \sum_{uv\in E(G)} \sqrt{(d_u + 1)^2 + (d_v + 1)^2}.$$

The Sombor index with p = $\frac{1}{2}$,

$$SO_{1/2}(G) = M_1(G) + 2RR(G),$$

where $M_1(G)$ is first Zagreb index$M_1(G) = \sum_{uv\in E(G)}(d_u + d_v)$ and RR(G) is reciprocal Randic index RR(G) = $\sum_{uv\in E(G)} \sqrt{d_u d_v}$.

For p = -1 [40] the Sombor index defined as $SO_{-1}(G) = \sum_{uv\in E(G)} (\frac{d_u d_v}{d_u + d_v})$ which is inverse sum index.

Modified Sombor index and modified reduced Sombor index are proposed by V.R.Kullias

$$^{m}SO(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_u^2 + d_v^2}},$$

and modified reduced Sombor index

$$^{m}RSO(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{(d_u - 1)^2 + (d_v - 1)^2}}.$$

The M-polynomial of graph G is defined as

$$M(G;x,y) = \sum_{\delta\le i\le j\le\Delta} m_{ij}(G)\, x^i y^j,$$

where δ = min{d_v|v ∈ V(G)}, Δ = max{d_v|v∈ V(G)}, and $m_{ij}(G)$ is the edge vu ∈ E(G) such that i ≤ j , with $D_x = x\frac{\partial f(x,y)}{\partial x}$, $D_y = y\frac{\partial f(x,y)}{\partial y}$, $S_x = \int_0^x \frac{f(t,y)}{t}dt$, $S_y = \int_0^y \frac{f(x,t)}{t}dt$, J(f(x,y)) = f(x,x), $Q_\alpha(f(x,y)) = x^\alpha f(x,y)$.

By using the first derivative of the Schultz,modified Schultz polynomials of Jahangir graph $J_{3,m}$(evaluated at x = 1)one can compute the Schultz, modified Schultz indices [41] as

$$Sc(J_{3,m}) = \frac{\partial Sc(J_{3,m},x,y)}{\partial x}|_{x=1}.$$

The first and second Zagreb indices computed from Zagreb polynomialsin [42] for $HC_5C_7[p,q]$ by using first derivative of the polynomials

$Zg_1(HC_5C_7[p,q]) = \frac{\partial Zg_1(G,x)}{\partial x}|_{x=1}$ and $Zg_2(HC_5C_7[p,q]) = \frac{\partial Zg_2(G,x)}{\partial x}|_{x=1}$.

The terms and notations used in this paper are standard and mainly taken from books of graph theory [43-47].

In this paper Sombor index (SO(G)),first Banhatti index ($BSO_1(G)$),first reduced Banhatti Sombor index ($RBSO_1(G)$),firstδ-Banhatti Sombor index ($\delta BSO_1(G)$),increased Sombor index ($SO^1(G)$),reduced Sombor index (RSO(G)),generalized Sombor index($SO_a(G)$),p-Sombor index(with p = -1) ($SO_{-1}(G)$) and p-Sombor index (with p =$\frac{1}{2}$)($SO_{1/2}$ (G)), are investigated by M-polynomial and average Sombor index ($SO_{avg}(G)$),modified Sombor index ($^{m}SO(G)$) andmodified reduced Sombor index ($^{m}RSO(G)$) by Sombor polynomial.

II. Materials and Methods

A molecular graph is a simple and connected graph.The two dimensional graph of $VC_5C_7[p,q]$ with p =3 and q = 4 is shown in figure 1.Let the graph of$VC_5C_7[p,q]$ nanotube be denoted by G.

There are three edges in G given by E_1= p,E_2 =10p,E_3 = 24pq-14p as E_1 = {uv∈E(G)|d_u=d_v=2}, E_2 = {uv∈E(G)|d_u= 2 and d_v= 3}and E_3 = {uv∈E(G)|d_u=d_v= 3}.It is observed from figure that the vertex set = 16pq+3p and edge set = 24pq-3p.

The edge partion of $VC_5C_7[p,q]$nanotube is used todetermine different versions of Sombor index by M-polynomial and exponential.The derivational formulas for M-polynomialof different Sombor indices are presented in table2.The Sombor indices SO(G),$BSO_1(G)$,$RBSO_1(G)$,$\delta BSO_1(G)$,$SO^1(G)$,$SO_{red}(G)$,$SO_a(G)$,$SO_{-1}(G)$ and $SO_{1/2}(G)$ are determined by M-polynomial and $SO_{avg}(G)$,$^{m}SO(G)$ and $^{m}RSO(G)$by Sombor polynomial.

III. Results and Discussion

3.1 Sombor indices of $VC_5C_7[p,q]$ nanotubes where p,q>1 by M-polynomial

The molecular graph of $VC_5C_7[p,q]$ nanotube with p=3 and q=4 is shown in figure 1.Let the graph of$VC_5C_7[p,q]$ nanotube be denoted by G. The structure of nanotube consists of cycles C_5 and C_7 in trivalent decoration which covers either a cylinder or torus.

The 2-D graph of $VC_5C_7[p,q]$ nanutubes has 24pq-3p edges and 16pq+3p vertices and degrees of vertices are 2 and 3.The edge partition of $VC_5C_7[p,q]$ nanotube for p,q> 1 is given in table 1.

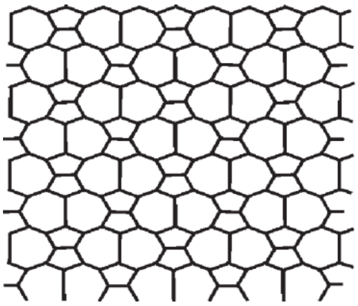

Figure 1. 2-D graph of $VC_5C_7[p,q]$ nanotube with p=3,q=4.

(d_u,d_v)	(2,2)	(2,3)	(3,3)
Number of edges	p	10p	24pq-14p

Table 1. Edge partition of $VC_5C_7[p,q]$ nanotube for p,q> 1.

Theorem 1. The Sombor index of $VC_5C_7[p,q]$ nanotube is 102pq -21p.

Proof. Consider a molecular graph of $VC_5C_7[p,q]$ nanotube as shown in figure 1.By using the definition of Sombor index $SO(G) = \sum_{uv\in E(G)} \sqrt{(d_u^2+d_v^2)}$ and table 1and 2 we get SO(G).

Let M-polynomial of $VC_5C_7[p,q]$ is

$M(G;x,y) = \sum_{\delta\le i\le j\le\Delta} m_{ij}(G)\, x^i y^j$

$=\sum_{2\le 2} m_{22}(G)\, x^2y^2 + \sum_{2\le 3} m_{23}(G)\, x^2y^3 + \sum_{3\le 3} m_{33}(G)\, x^3y^3$

$=|E_{(2,2)}|x^2y^2+|E_{(2,3)}|x^2y^3+|E_{(3,3)}|x^3y^3$

$=px^2y^2+10px^2y^3+(24pq-14p)x^3y^3$.

In order tofind SO(G) we need the following,

$M(G;x,y) = px^2y^2+10px^2y^3+(24pq-14p)x^3y^3$.

$D_x^2M(G;x,y) = 4px^2y^2+4*10px^2y^3+9(24pq-14p)x^3y^3$.

$D_y^2M(G;x,y) = 4px^2y^2+9*10px^2y^3+9(24pq-14p)x^3y^3$.

$(D_x^2+D_y^2)^{\frac{1}{2}}(M(G;x,y)) = \sqrt{8}\, px^2y^2 + \sqrt{13}\, 10px^2y^3 + \sqrt{18}\,(24pq-14p)x^3y^3$.

$SO(G) = (D_x^2+D_y^2)^{\frac{1}{2}}(M(G;x,y))|_{x=y=1}$

= 102pq -21p.

Theorem 2. The first Banhatti-Sombor index of $VC_5C_7[p,q]$ nanotube is 11pq - p.

Proof. Consider a molecular graph of $VC_5C_7[p,q]$ nanotube as shown in figure 1.By using the definition of first Banhatti-Sombor index $BSO_1(G) = \sum_{uv\in E(G)} \sqrt{\frac{1}{d_u^2}+\frac{1}{d_v^2}}$

and table 1and 2 we get $BSO_1(G)$.

Let M-polynomial of $VC_5C_7[p,q]$ is

$M(G;x,y) = \sum_{\delta\le i\le j\le\Delta} m_{ij}(G)\, x^i y^j$

$=\sum_{2\le 2} m_{22}(G)\, x^2y^2 + \sum_{2\le 3} m_{23}(G)\, x^2y^3 + \sum_{3\le 3} m_{33}(G)\, x^3y^3$

$=|E_{(2,2)}|x^2y^2+|E_{(2,3)}|x^2y^3+|E_{(3,3)}|x^3y^3$

$=px^2y^2+10px^2y^3+(24pq-14p)x^3y^3$.

In order tofind $BSO_1(G)$ we need the following,

$M(G;x,y) = px^2y^2+10px^2y^3+(24pq-14p)x^3y^3$.

$S_x^{1/2}M(G;x,y) = \frac{p}{4}x^2y^2+\frac{10}{4}px^2y^3+\frac{1}{9}(24pq-14p)x^3y^3$.

$S_y^{1/2}M(G;x,y) = \frac{p}{4}x^2y^2+\frac{10}{9}px^2y^3+\frac{1}{9}(24pq-14p)x^3y^3$.

$BSO_1(G) = (S_x^2+S_y^2)^{1/2}(M(G;x,y))|_{x=y=1} = 11pq-p$.

Theorem 3. The first reduced Banhatti-Sombor index of $VC_5C_7[p,q]$ nanotube is 17pq +2p.

Proof. Consider a molecular graph of $VC_5C_7[p,q]$ nanotube.By using the definition of first reduced Banhatti-Sombor index

$RBSO_1(G) = \sum_{uv\in E(G)} \sqrt{\frac{1}{(d_u-1)^2}+\frac{1}{(d_v-1)^2}}$ and table 1and 2 we get $RBSO_1(G)$.

Let M-polynomial of $VC_5C_7[p,q]$ is

$M(G;x,y) = px^2y^2+10px^2y^3+(24pq-14p)x^3y^3$.

In order tofind $RBSO_1(G)$ we need the following,

$M(G;x,y) = px^2y^2 + 10px^2y^3 + (24pq-14p)x^3y^3$.
$Q_{x(-1)}Q_{y(-1)}M(G;x,y) = pxy + 10\,pxy^2 + (24pq-14p)x^2y^2$.
$S_x^2M(G;x,y) = pxy + 10\,pxy^2 + \frac{1}{4}(24pq-14p)x^2y^2$.
$S_y^2M(G;x,y) = pxy + \frac{10}{4}pxy^2 + \frac{1}{4}(24pq-14p)x^2y^2$.
$RBSO_1(G) = (S_x^2 + S_y^2)^{1/2}Q_{x(-1)}Q_{y(-1)}(M(G;x,y))|_{x=y=1} = 17pq+2p$.

Theorem 4. The first δ-Banhatti Sombor index of $VC_5C_7[p,q]$ nanotube is 17pq –2.516p.

Proof. Consider a molecular graph of $VC_5C_7[p,q]$ nanotube as shown in figure 1.By using the definition of first δ-Banhatti Sombor index

$\delta BSO_1(G) = \sum_{uv\in E(G)}\sqrt{\frac{1}{(d_u-\delta(G)+1)^2}+\frac{1}{(d_v-\delta(G)+1)^2}}$ and table 1and 2 we get $\delta BSO_1(G)$.For graph G we have minimum degree among the vertices of G as $\delta(G) = 2$.
Let M-polynomial of $VC_5C_7[p,q]$ is
$M(G;x,y) = px^2y^2 + 10px^2y^3 + (24pq-14p)x^3y^3$.
In order tofind $\delta BSO_1(G)$ we need the following,
$M(G;x,y) = px^2y^2 + 10px^2y^3 + (24pq-14p)x^3y^3$.
$Q_{x(-1)}Q_{y(-1)}M(G;x,y) = pxy + 10\,pxy^2 + (24pq-14p)x^2y^2$.
$S_x^2M(G;x,y) = pxy + 10\,pxy^2 + \frac{1}{4}(24pq-14p)x^2y^2$.
$S_y^2M(G;x,y) = pxy + \frac{10}{4}pxy^2 + \frac{1}{4}(24pq-14p)x^2y^2$.
$\delta BSO_1(G) = (S_x^2 + S_y^2)^{1/2}Q_{x(-1)}Q_{y(-1)}(M(G;x,y))|_{x=y=1} = 17pq-2.516p$.

Theorem 5. The increased Sombor index of $VC_5C_7[p,q]$ nanotube is 136pq -25p.

Proof. Consider a molecular graph of $VC_5C_7[p,q]$ nanotube.By using the definition of increased Sombor index $SO^1(G) = \sum_{uv\in E(G)}\sqrt{(d_u+1)^2+(d_v+1)^2}$ and table 1and 2 we get $SO^1(G)$.
Let M-polynomial of $VC_5C_7[p,q]$ is
$M(G;x,y) = px^2y^2 + 10px^2y^3 + (24pq-14p)x^3y^3$.
In order tofind $SO^1(G)$ we need the following,
$M(G;x,y) = px^2y^2 + 10px^2y^3 + (24pq-14p)x^3y^3$.
$Q_{x(+1)}Q_{y(+1)}M(G;x,y) = px^3y^3 + 10px^3y^4 + (24pq-14p)x^4y^4$.
$D_x^2Q_{x(+1)}Q_{y(+1)}M(G;x,y) = 9px^3y^3 + 9*10px^3y^4 + 16(24pq-14p)x^4y^4$.
$D_y^2Q_{x(+1)}Q_{y(+1)}M(G;x,y) = 9px^3y^3 + 16*10px^3y^4 + 16(24pq-14p)x^4y^4$.
$(D_x^2 + D_y^2)Q_{x(+1)}Q_{y(+1)}M(G;x,y) = 18\,px^3y^3 + 25*10px^3y^4 + 32(24pq-14p)x^4y^4$.
$SO^1(G) = (D_x^2 + D_y^2)^{\frac{1}{2}}(M(G;x,y))|_{x=y=1}$
= 136pq -25p.

Theorem 6. The reduced Sombor index of $VC_5C_7[p,q]$ nanotube is 67.87pq -15.8p.

Proof. Consider a molecular graph of $VC_5C_7[p,q]$ nanotube.By using the definition of reduced Sombor index $RSO(G) = \sum_{uv\in E(G)}\sqrt{(d_u-1)^2+(d_v-1)^2}$ and table 1and 2 we get RSO(G).
Let M-polynomial of $VC_5C_7[p,q]$ is
$M(G;x,y) = px^2y^2 + 10px^2y^3 + (24pq-14p)x^3y^3$.
In order tofind RSO(G) we need the following,
$M(G;x,y) = px^2y^2 + 10px^2y^3 + (24pq-14p)x^3y^3$.
$Q_{x(-1)}Q_{y(-1)}M(G;x,y) = pxy + 10pxy^2 + (24pq-14p)x^2y^2$.
$D_x^2Q_{x(-1)}Q_{y(-1)}M(G;x,y) = pxy + 10pxy^2 + 4(24pq-14p)x^2y^2$.
$D_y^2Q_{x(-1)}Q_{y(-1)}M(G;x,y) = pxy + 4*10pxy^2 + 4(24pq-14p)x^2y^2$.
$RSO(G) = (D_x^2 + D_y^2)^{1/2}Q_{x(-1)}Q_{y(-1)}(M(G;x,y))|_{x=y=1}$
= 67.87pq-15.8p.

Theorem 7. The generalized Sombor index of $VC_5C_7[p,q]$ nanotube is
$(8-8a+2a^2)^{\frac{1}{2}}p + (13-10a+2a^2)^{\frac{1}{2}}10p + (18-12a+2a^2)^{\frac{1}{2}}(24pq-14p)$.

Proof. Consider a molecular graph of $VC_5C_7[p,q]$ nanotube.By using the definition of generalized Sombor index $SO_a(G) = \sum_{uv\in E(G)}\sqrt{(d_u-a)^2+(d_v-a)^2}$ and table 1and 2 we get $SO_a(G)$.
Let M-polynomial of $VC_5C_7[p,q]$ is
$M(G;x,y) = px^2y^2 + 10px^2y^3 + (24pq-14p)x^3y^3$.
In order tofind $SO_a(G)$ we need the following,
$M(G;x,y) = px^2y^2 + 10px^2y^3 + (24pq-14p)x^3y^3$.
$Q_{x(-a)}Q_{y(-a)}M(G;x,y) = px^{2-a}y^{2-a} + 10px^{2-a}y^{3-a} + (24pq-14p)x^{3-a}y^{3-a}$.

$(D_x^2+D_x^2)^{\frac{1}{2}}Q_{x(-a)}Q_{y(-a)}M(G;x,y)= (8-8a+2a^2)^{\frac{1}{2}}px^{2-a}y^{2-a}+(13-10a+2a^2)^{\frac{1}{2}}10px^{2-a}y^{3-a}+(18-12a+2a^2)^{\frac{1}{2}}(24pq-14p)x^{3-a}y^{3-a}$.

$SO_a(G) = (D_x^2 + D_y^2)^{1/2}Q_{x(-a)}\, Q_{y(-a)}(M(G;x,y))|_{x=y=1}$

$= (8-8a+2a^2)^{\frac{1}{2}}p+(13-10a+2a^2)^{\frac{1}{2}}10p+(18-12a+2a^2)^{\frac{1}{2}}(24pq-14p)$.

Theorem 8. The p-Sombor index (with p = -1) of $VC_5C_7[p,q]$ nanotube is 36pq -8p.

Proof. Consider a molecular graph of $VC_5C_7[p,q]$ nanotube. By using the definition of p-Sombor index(with p=-1) $SO_p(G) = \sum_{uv\in E(G)}(\frac{d_u d_v}{d_u+d_v})$ and table 1and 2 we get $SO_p(G)$(with p=-1).

Let M-polynomial of $VC_5C_7[p,q]$ is

$M(G;x,y) = px^2y^2+ 10px^2y^3+ (24pq-14p)x^3y^3$.

In order tofind $SO_p(G)$(p=-1)with we need the following,

$M(G;x,y) = px^2y^2+ 10px^2y^3+ (24pq-14p)x^3y^3$.

$D_yM(G;x,y) = 2px^2y^2+3*10px^2y^3+3(24pq-14p)x^3y^3$.

$D_xD_yM(G;x,y) = 4px^2y^2+6*10px^2y^3+9(24pq-14p)x^3y^3$.

$JD_xD_yM(G;x,y) = 4px^4+6*10px^5+9(24pq-14p)x^6$.

$S_xJD_xD_yM(G;x,y) = px^4+12px^5+1.5(24pq-14p)x^6$.

$SO_p(G) = S_xJD_xD_y(M(G;x,y))|_{x=1}= 36pq -8p$.

Theorem 9. The p-Sombor index (with p= $\frac{1}{2}$) of $VC_5C_7[p,q]$ nanotube is 288pq-61p.

Proof. Consider a molecular graph of $VC_5C_7[p,q]$ nanotube. By using the definition of p-Sombor index(with p=$\frac{1}{2}$)

$SO_{1/2}(G) = \sum_{uv\in E(G)}(d_u + d_u)+2\sum_{uv\in E(G)}\sqrt{(d_u d_v)}$ and table 1and 2 we get $SO_{1/2}(G)$.

Let M-polynomial of $VC_5C_7[p,q]$ is

$M(G;x,y) =px^2y^2+ 10px^2y^3+ (24pq-14p)x^3y^3$.

In order tofind $SO_{1/2}(G)$ we need the following,

$M(G;x,y) = px^2y^2+ 10px^2y^3+ (24pq-14p)x^3y^3$.

$D_xM(G;x,y) = 2px^2y^2+2*10px^2y^3+3(24pq-14p)x^3y^3$.

$D_yM(G;x,y) = 2px^2y^2+3*10px^2y^3+3(24pq-14p)x^3y^3$.

$(D_x+D_y)M(G;x,y)= 4px^2y^2+5*10px^2y^3+6(24pq-14p)x^3y^3$.

$D_y^{\frac{1}{2}}M(G;x,y) = 2^{\frac{1}{2}}px^2y^2+3^{\frac{1}{2}}\,10px^2y^3+3^{\frac{1}{2}}(24pq-14p)x^3y^3$.

$D_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M(G;x,y) =2^{\frac{1}{2}}2^{\frac{1}{2}}\,px^2y^2+2^{\frac{1}{2}}3^{\frac{1}{2}}\,10px^2y^3+3^{\frac{1}{2}}3^{\frac{1}{2}}(24pq-14p)x^3y^3$.

$SO_{1/2}(G) = [(D_x+D_y)+2(D_x^{1/2}D_y^{1/2})](M(G;x,y))|_{x=y=1}$

=288 pq-61p.

3.2 Sombor indicesby exponential

In the following section average Sombor index,modified Sombor index andreduced modified Sombor index are determined by Sombor exponential.

Theorem 10. The average Sombor index of $VC_5C_7[p,q]$ nanotube is

$[(\frac{-32pq}{16pq+3p})^2]^{\frac{1}{2}}p+[(\frac{-16pq}{16pq+3p})^2+(\frac{3p}{16pq+3p})^2]^{\frac{1}{2}}10p+[(\frac{6p}{16pq+3p})^2]^{\frac{1}{2}}(24pq-14p)$.

Proof. Consider a molecular graph of $VC_5C_7[p,q]$ nanotube as shown in figure 1. By using the definition of Sombor index $SO_{avg}(G) =\sum_{uv\in E(G)}\sqrt{(d_u-\frac{2m}{n})^2+(d_v-\frac{2m}{n})^2}$ and table 1and 2 we get $SO_{avg}(G)$.

Let $SO_{avg}(G,x)$ be the average Sombor exponential of $VC_5C_7[p,q]$.In this case

$E(G) = 24pq-3p$ and $V(G) =16pq+3p$,here $\bar{d} =\frac{2m}{n} = 2\frac{24pq-3p}{16pq+3p} = \frac{48pq-6p}{16pq+3p}$.

$SO_{avg}(G,x) =\sum_{uv\epsilon E(G)} x^{[(d_u-\bar{d})^2+(d_v-\bar{d})^2]^{\frac{1}{2}}}$.

$SO_{avg}(G,x) =px^{[(2-\frac{48pq-6p}{16pq+3p})^2+(2-\frac{48pq-6p}{16pq+3p})^2]^{\frac{1}{2}}}+10px^{[(2-\frac{48pq-6p}{16pq+3p})^2+(3-\frac{48pq-6p}{16pq+3p})^2]^{\frac{1}{2}}}+(24pq-14p)x^{[(3-\frac{48pq-6p}{16pq+3p})^2+(3-\frac{48pq-6p}{16pq+3p})^2]^{\frac{1}{2}}}$

$=px^{[(\frac{-32pq}{16pq+3p})^2]^{\frac{1}{2}}}+10px^{[(\frac{-16pq}{16pq+3p})^2+(\frac{3p}{16pq+3p})^2]^{\frac{1}{2}}}+(24pq-14p)x^{[(\frac{6p}{16pq+3p})^2]^{\frac{1}{2}}}$.

$SO_{avg}(G,x) = \frac{\partial(G,x)}{\partial x}|_{x=1}$

$=[(\frac{-32pq}{16pq+3p})^2]^{\frac{1}{2}}p+[(\frac{-16pq}{16pq+3p})^2+(\frac{3p}{16pq+3p})^2]^{\frac{1}{2}}10p+[(\frac{6p}{16pq+3p})^2]^{\frac{1}{2}}(24pq-14p).$

Theorem 11. The modified Sombor index of $VC_5C_7[p,q]$ nanotube is 6pq-0.1732p.

Proof. Consider a molecular graph of $VC_5C_7[p,q]$ nanotube. By using the definition of modified Sombor index $^mSO(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_u^2+d_v^2}}$ and table 1and 2 we get $^mSO(G)$.

Let $^mSO(G,x)$ be the modified Sombor exponential of $VC_5C_7[p,q]$

$^mSO(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{d_u^2+d_v^2}}}$

$=p\,x^{\frac{1}{\sqrt{4+4}}}+10p\,x^{\frac{1}{\sqrt{4+9}}}+(24pq-14p)x^{\frac{1}{\sqrt{9+9}}}.$

Modified Sombor index is $^mSO(G)=\frac{\partial(G,x)}{\partial x}|_{x=1}$

$=\frac{1}{\sqrt{8}}p+\frac{1}{\sqrt{13}}10p+\frac{1}{\sqrt{18}}(24pq-14p)=6pq-0.1732p.$

Theorem 12. The reduced modified Sombor index of $VC_5C_7[p,q]$ nanotube is 8.485pq-0.23p.

Proof. Consider a molecular graph of $VC_5C_7[p,q]$ nanotube. By using the definition of reduced modified Sombor index $^mRSO(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{(d_u-1)^2+(d_v-1)^2}}$ and table 1and 2 we get $^mRSO(G)$.

Let $^mRSO(G,x)$ bethe reduced modified Sombor exponential of $VC_5C_7[p,q]$

$^mRSO(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{(d_u-1)^2+(d_v-1)^2}}}$

$=px^{\frac{1}{\sqrt{(2-1)^2+(2-1)^2}}}+10p\,x^{\frac{1}{\sqrt{(2-1)^2+(3-1)^2}}}+(24pq-14p)x^{\frac{1}{\sqrt{(3-1)^2+(3-1)^2}}}$

$=p\,x^{\frac{1}{\sqrt{2}}}+10p\,x^{\frac{1}{\sqrt{5}}}+(24pq-14p)x^{\frac{1}{\sqrt{8}}}.$

Reduced modified Sombor index is $^mRSO(G)=\frac{\partial(G,x)}{\partial x}|_{x=1}$

$=\frac{1}{\sqrt{2}}p+\frac{1}{\sqrt{5}}10p+\frac{1}{\sqrt{8}}(24pq-14p)$

$=8.485pq-0.23p.$

Topological index	Derivation from M(G;x,y)
Sombor index (SO(G))	$(D_x^2+D_y^2)^{1/2}(M(G;x,y))|_{x=y=1}$
First Banhatti Sombor index ($BSO_1(G)$)	$(S_x^2+S_y^2)^{1/2}(M(G;x,y))|_{x=y=1}$
First reduced Banhatti Sombor index ($RBSO_1(G)$)	$(S_x^2+S_y^2)^{1/2}Q_{x(-1)}Q_{y(-1)}(M(G;x,y))|_{x=y=1}$
First δ-Banhatti Sombor index ($\delta BSO_1(G)$)	$(S_x^2+S_y^2)^{1/2}Q_{x(-1)}Q_{y(-1)}(M(G;x,y))|_{x=y=1}$
Increased Sombor index ($SO^1(G)$)	$(D_x^2+D_y^2)^{1/2}Q_{x(+1)}Q_{y(+1)}(M(G;x,y))|_{x=y=1}$
Reduced Sombor index (RSO(G))	$(D_x^2+D_y^2)^{1/2}Q_{x(-1)}Q_{y(-1)}(M(G;x,y))|_{x=y=1}$
Generalized Sombor index ($SO_a(G)$)	$(D_x^2+D_y^2)^{1/2}Q_{x(-a)}Q_{y(-a)}(M(G;x,y))|_{x=y=1}$
p-Sombor index (p =-1)($SO_{-1}(G)$)	$S_xJD_xD_y(M(G;x,y))|_{x=1}$
p-Sombor index(p = $\frac{1}{2}$) ($SO_{1/2}(G)$)	$[(D_x+D_y)+2(D_x^{1/2}D_y^{1/2})](M(G;x,y))|_{x=y=1}$

Table 2.Derivational formulas for Sombor indices by M-polynomial.

IV. Conclusion

In this paper Sombor index, first Banhatti Sombor index, first reduced Banhatti Sombor index, first δ-Banhatti Sombor index, increased Sombor index, reduced Sombor index, generalized Sombor index, p-Sombor index(with p=-1), p-Sombor index (with p=$\frac{1}{2}$) are determined by M-polynomial and average Sombor index, modified Sombor index and reduced modified Sombor index by Sombor exponential for $VC_5C_7[p,q]$ nanotubes.

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