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Research Paper

Restrained Convex Secure Domination in Graphs

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ABSTRACT: Let G be a connected simple graph. A set $S \subseteq V(G)$ is a restrained dominating set if every vertex not in S is adjacent to a vertex in S and to a vertex in $V(G) \setminus S$. Alternately, a subset S of $V(G)$ is a restrained dominating set if $N[S] = V(G)$ and $\langle V(G) \setminus S \rangle$ is a subgraph without isolated vertices. A set S is convex if $I_G[S] = S$. A restrained dominating set S of G is a restrained convex secure dominating set, if it is convex and for each element u in $V(G) \setminus S$ there exists an element v in S such that $uv \in E(G)$ and $(S \setminus \{v\}) \cup \{u\}$ is a dominating set. The restrained convex secure domination number of G, denoted by $\gamma_{rcs}(G)$, is the minimum cardinality of a convex secure dominating set of G. A restrained convex secure dominating set of cardinality $\gamma_{rcs}(G)$ will be called a γ_{rcs}-set. In this paper, we investigate the concept and give important results.

KEYWORDS: dominating set, restrained dominating set, convex dominating set, secure dominating set restrained convex secure dominating set.

I. INTRODUCTION

Graph Theory was born in 1736 with Euler's paper in which he solved the Konigsberg bridge problem [1]. Domination in graph was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [2]. However, it was not until following an article by Ernie Cockayne and Stephen Hedetniemi [3], that domination became an area of study by many. One type of domination parameter is the restrained domination number in a graph. This was introduced by Telle and Proskurowski [4] indirectly as a vertex partitioning problem. Restrained domination in graphs can be read in the paper of Domke et.al. [5]. Other variant of restrained domination in graphs can be read in [6-16].

A graph G is connected if there is at least one path that connects every two vertices $x, y \in V(G)$, otherwise, G is disconnected. For any two vertices u and v in a connected graph, the distance $d_G(u,v)$ between u and v is the length of a shortest path in G. A u-v path of length $d_G(u,v)$ is also referred to as u-v geodesic. The closed interval $I_G[u,v]$ consist of all those vertices lying on a u-v geodesic in G. For a subset S of vertices of G, the union of all sets $I_G[u,v]$ for $u, v \backslash in\ S$ is denoted by $I_G[S]$. Hence $x \in I_G[S]$ if and only if x lies on some u-v geodesic, where $u, v \in S$. A set S is convex if $I_G[S] = S$. Certainly, if G is connected graph, then $V(G)$ is convex. Convexity in graphs was studied in [17-21].

A complete graph of order n, denoted by K_n, is the graph in which every pair of its distinct vertices are joined by an edge. A nonempty subset S of $V(G)$ is a clique in G if the graph $\langle S \rangle$ induced by S is complete. A nonempty subset S of a vertex set $V(G)$ is a clique dominating set of G if S is a dominating set and S is a clique in G. Clique domination in a graph is found in the papers [22-26].

A dominating set S which is also convex is called a convex dominating set of G. The convex domination number $\gamma_{con}(G)$ of G is the smallest cardinality of a convex dominating set of G. A convex dominating set of cardinality $\gamma_{con}(G)$ is called a γ_{con}-set of G. A dominating set S in G is called a secure dominating set in G if for every $u \in V(G) \setminus S$, there exists $v \in S \cap N_G(u)$ such that $(S \setminus \{v\}) \cup \{u\}$ is a dominating set. The minimum cardinality of secure dominating set is called the secure domination number of G and is denoted by $\gamma_s(G)$. A secure dominating set of cardinality $\gamma_s(G)$ is called γ_s-set of G. The concept of secure domination in graphs was studied and introduced by E.J. Cockayne et.al [27,28]. Some variants of secure domination in graphs are found in [29-33].

A convex dominating set S of G is a convex secure dominating set, if for each element u in $V(G) \setminus S$ there exists an element v in S such that $uv \in E(G)$ and $(S \setminus \{v\}) \cup \{u\}$ is a dominating set. The convex secure domination number of G, denoted by $\gamma_{cs}(G)$, is the minimum cardinality of a convex secure dominating set of G.

A convex secure dominating set of cardinality $\gamma_{cs}(G)$ will be called a γ_{cs}-set. Convex secure domination in graphs was found in the paper of Enriquez and Samper-Enriquez [34].

Motivated by the definition of restrained domination and convex secure domination in graphs, we define a new domination in a graph, the restrained convex secure domination in graphs. A restrained dominating set S of G is a restrained convex secure dominating set, if it is convex and for each element u in $V(G)\backslash S$ there exists an element v in S such that $uv \in E(G)$ and $(S\backslash\{v\}) \cup \{u\}$ is a dominating set. The restrained convex secure domination number of G, denoted by $\gamma_{rcs}(G)$, is the minimum cardinality of a convex secure dominating set of G. A restrained convex secure dominating set of cardinality $\gamma_{rcs}(G)$ will be called a γ_{rcs}-set. In this paper, we investigate the concept and give important results. For general concepts we refer the reader to [35].

II. RESULTS

Remark 2.1 A restrained convex secure dominating set of a graph G is a restrained dominating set, convex dominating set and secure dominating set of G.

From the definition of restrained convex secure dominating set, the following result is immediate.

Remark 2.2 Let G be a nontrivial connected graph. Then
(i) $\gamma(G) \leq \gamma_s(G) \leq \gamma_{rsc}(G)$; and
(ii) $\gamma_{rcs}(G) \in \{1,2,..,n-3,n-2,n\}$.

We need the following theorems for our next results.

Theorem 2.3 \cite{Castillano} Let G be a graph of order $n \geq 1$. Then $\gamma_s(G) = 1$ if and only if $G = K_n$.

The following result is the characterization of dominating sets with restrained convex secure domination number of one.

Theorem 2.4 Let G be a graph of order $n \geq 3$. Then $\gamma_{rcs}(G) = 1$ if and only if G is a complete graph.

Proof: Suppose that $\gamma_{rcs}(G) = 1$. Let $S = \{v\}$ be a γ_{rcs}-set in G. Then by Remark 2.1, S is a secure dominating set of G. Hence, G is a complete graph by Theorem 2.3.

For the converse, suppose that G is a complete graph. Then $\gamma_s(G) = 1$ by Theorem 2.1. Let $S = \{x\}$ be a minimum secure dominating set of G. Since S is convex set, it follows that S is a convex secure dominating set of G. Further, since the order of the complete graph G is $n \geq 3$, every vertex $u \in V(G) \setminus S$ is adjacent to a vertex in S and a vertex not in S. Hence, S is a restrained dominating set of G. Accordingly, S is a restrained convex secure dominating set of G. Thus, $\gamma_{rcs}(G) = 1$. ■

It is worth mentioning that the upper bound in Remark 2.2(ii) is sharp. For example, $\gamma_{rcs}(C_n) = n$ for all $n \geq 6$ (Note: C_n is not convex for all $n \geq 6$, it follows that). The lower bound is also attainable as the following result shows.

Theorem 2.5 Given positive integers k and n such that $k \in \{1,2,..,n-3,n-2,n\}$, there exists a connected nontrivial graph G with $|V(G)| = n$ and $\gamma_{rcs}(G) = k$.

Proof: Consider the following cases:

Case 1. Suppose $k = 1$. Let $G = K_n$. Clearly, $|V(G)| = n$ and $\gamma_{rcs}(G) = 1$ by Theorem 2.4.

Case 2. Suppose $2 \leq k \leq n-2$. Let $H = K_r$ $(r \geq 2)$ and $\overline{K}_m = [a_1, a_2, \dots, a_m]$ $(m \geq 1)$. Let $n = r + m$ and $k = m + 1$. Consider the graph G obtained from H by adding the edges $va_1, va_2, \dots, va_{m-1}, va_m$ (see Figure 1).

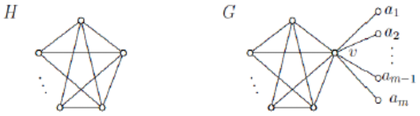

Figure 1: A graph G with $\gamma_{rcs}(G) = k$

Subcase 1. Suppose that $k = 2$. Let $m = 1$. Then the set $S = \{v, a_1\}$ is a γ_{rcs}-set of G. Thus, $|V(G)| = r + 1 = n$ and $\gamma_{rcs}(G) = 2$.

Subcase 2. Suppose that $3 \leq k \leq n - 2$. Let $m \geq 2$. Then the set $S = \{v, a_1, a_2, \ldots, a_m\}$ is a γ_{rcs}-set of G. Thus, $|V(G)| = r + m = n$ and $\gamma_{rcs}(G) = m + 1 = k$. In particular, if $m = 2$, then $k = 3$. Further, if $r = 3$ then $k = m + 1 = m + 3 - 2 = m + r - 2 = n - 2$.

Case 3. Suppose $k = n$. Let $G = C_n$ for all $n \backslash geq$ 6. Then $|V(G)| = n$ and $\gamma_{rcs}(G) = n$.

This proves the assertion. ■

Corollary 2.6 The difference $\gamma_{rcs}(G) - \gamma(G)$ can be made arbitrarily large.

Proof: Let $n \geq 2$ be a positive integer. By Theorem 2.5, there exists a connected graph G such that $\gamma_{rcs}(G) = n + 1$ and $\gamma(G) = 1$. Thus, $\gamma_{rcs}(G) - \gamma(G) = n$, showing that $\gamma_{rcs}(G) - \gamma(G)$ can be made arbitrarily large. ■

The following remark is useful for our next results, the join of two graphs.

Remark 2.7 Every clique dominating set is a convex dominating set.

The converse of Remark 2.7 need not be true. For example the minimum convex dominating set of P_n for all $n \geq 5$ where $V(P_n) = \{x_1, x_2, \ldots, x_{n-1}, x_n\}$ is $S = \{x_2, x_3, \ldots, x_{n-1}\}$. But S is not a clique dominating set of P_n for all $n \geq 5$.

The join of two graphs G and H is the graph $G + H$ with vertex-set $V(G + H) = V(G) \cup V(H)$ and edge-set $E(G + H) = E(G) \cup E(H) \cup \{uv : u \in V(G), v \in V(H)\}$.

The next result is the characterization of a restrained convex secure dominating set in the join of two graphs.

Theorem 2.8 Let G and H be connected non-complete graphs. Then a nonempty proper subset S of $V(G + H)$ is a restrained convex secure dominating set in $G + H$ if and only if one of the following statement is satisfied.

(i) S is a clique secure dominating set of G and $|S| \geq 2$.
(ii) S is a clique secure dominating set of H and $|S| \geq 2$.
(iii) $S = S_G \cup S_H$ where $S_G = \{v\} \subset V(G)$ and $S_H = \{w\} \subset V(H)$ and
  $a)$ S_G is a dominating set of G and S_H is a dominating set of H; or
  $b)$ S_G is a dominating set of G and $V(H) \setminus N_H[S_H]$ is clique in H; or
  $c)$ S_H is a dominating set of H and $V(G) \setminus N_G[S_G]$ is clique in G; or
  $d)$ $V(G) \setminus N_G[S_G]$ is clique in G and $V(H) \setminus N_H[S_H]$ is clique in H.
(iv) $S = S_G \cup S_H$ where S_G is a clique in G ($|S_G| \geq 2$) and $S_H = \{w\} \subset V(H)$ and
  $a)$ S_G is a dominating set of G; or
  $b)$ $V(G) \setminus N_G[S_G]$ is clique in G.
(v) $S = S_G \cup S_H$ where $S_G = \{v\} \subset V(G)$ and S_H is a clique in H ($|S_H| \geq 2$) and
  $a)$ S_H is a dominating set of G; or
  $b)$ $V(H) \setminus N_H[S_H]$ is clique in H.
(vi) $S = S_G \cup S_H$ where S_G is a clique in G ($|S_G| \geq 2$) and S_H is a clique in H ($|S_H| \geq 2$).

Proof: Suppose that a nonempty proper subset S of $V(G + H)$ is a restrained convex secure dominating set of $G + H$. Consider the following cases:

Case 1. Suppose that $S \cap V(H) = \emptyset$. Then $S \subseteq V(G)$. This implies that S is a restrained convex secure dominating set of G and hence a secure dominating set. Now suppose that $|S| = 1$, say $S = \{a\}$. Since S is also a secure dominating set of $G + H$, $(S \setminus \{a\}) \cup \{z\} = \{z\}$ is a dominating set of $G + H$ and hence of H for every $z \in V(H)$. This implies that H is a complete graph, contrary to our assumption. Thus, $|S| \geq 2$. Let $x, y \in S$. Since S is also a convex dominating set of $G + H$, it follows that $I_{G+H}[x, y] \subseteq S$. This implies that $xy \in E(G)$ for all $x, y \in S$, otherwise there exists $z \in V(H)$ such that $I_{G+H}[x, y] = \{x, z, y\} \nsubseteq S$. Thus, S must be a clique dominating set of G, that is, S is a clique secure dominating set of G. This proves statement (i).

Case 2. Suppose that $S \cap V(G) = \emptyset$. Then $S \subseteq V(H)$. Using similar arguments in Case 1, S is a clique secure dominating set of H and $|S| \geq 2$ proving statement (ii).

Case 3. Suppose that $S \cap V(G) \neq \emptyset$ and $S \cap V(H) \neq \emptyset$. Let $S_G = S \cap V(G)$ and $S_H = S \cap V(H)$. Then $S = S_G \cup S_H$. Consider the following subcases.

Subcase 1. Suppose that $S_G = \{v\} \subset V(G)$ and $S_H = \{w\} \subset V(H)$. If S_G is a dominating set of G and S_H is a dominating set of H, then statement $(iii)(a)$ is proved. Suppose that S_G is a dominating set of G and S_H is not a dominating set of H. Let $x \in V(H) \setminus N_H[S_H]$. Since S is a secure dominating set of $G + H$, $(S \setminus \{v\}) \cup \{x\} = \{w, x\}$ is a dominating set of $G + H$ (and hence in H). Since $x \in V(H) \setminus N_H[S_H]$, $wx \notin E(H)$. Suppose there exists $y \notin N_H[S_H]$ such that $xy \notin E(H)$. Then $(S \setminus \{v\}) \cup \{y\} = \{w, y\}$ is not a dominating set of H and hence of $G + H$. This is contrary to our assumption that S is a secure dominating set. Thus, for every $y \notin N_H[S_H]$, $xy \in E(H)$. Since x was arbitrarily chosen, it follows that the subgraph induced by $V(H) \setminus N_H[S_H]$ is complete. Hence, $V(H) \setminus N_H[S_H]$ is clique in H proving statement $(iii)(b)$. Similarly, if S_H is dominating set of H and S_G is not a dominating set of G, then $V(G) \setminus N_G[S_G]$ is clique in G proving statement $(iii)(c)$. If S_G is not a dominating set of G and S_H is not a dominating set of H, then by following similar arguments in $(iii)(b)$ and $(iii)(c)$, $V(G) \setminus N_G[S_G]$ is clique in G and $V(H) \setminus N_H[S_H]$ is clique in H showing statement $(iii)(d)$.

Subcase 2. Suppose that S_G is a clique in G ($|S_G| \geq 2$) and $S_H = \{w\} \subset V(H)$. If S_G is a dominating set of G then statement $(iv)(a)$ follows. Suppose that S_G is not a dominating set of G. Let $x \in V(G) \setminus N_G[S_G]$. Since S is a secure dominating set of $G + H$, $S_x = (S \setminus \{w\}) \cup \{x\}$ is a dominating set of $G + H$ (and hence of G). Since $x \in V(G) \setminus N_G[S_G]$, $vx \notin E(G)$ for every $v \in S_G$. Suppose there exists $y \notin N_G[S_G]$ such that $xy \notin E(G)$. Then $(S \setminus \{w\}) \cup \{y\} = S_G \cup \{y\}$ is not a dominating set of G and hence of $G + H$. This is contrary to our assumption that S is a secure dominating set. Thus, for every $y \notin N_G[S_G]$, $xy \in E(G)$. This implies that for every $x, y \in V(G) \setminus N_G[S_G]$, $xy \in E(G)$, that is, the subgraph induced by $V(G) \setminus N_G[S_G]$ is complete. Hence $V(G) \setminus N_G[S_G]$ is a clique in G. This proves statement $(iv)(b)$.

Subcase 3. Suppose that S_H is a clique in H ($|S_H| \geq 2$) and $S_G = \{v\} \subset V(G)$. If S_H is a dominating set of H then statement $(v)(a)$ follows. Suppose that S_H is not a dominating set of H. Let $x \in V(H) \setminus N_H[S_H]$. Since S is a secure dominating set of $G + H$, $S_x = (S \setminus \{v\}) \cup \{x\}$ is a dominating set of $G + H$ (and hence of H). Since $x \in V(H) \setminus N_H[S_H]$, $wx \notin E(H)$ for every $w \in S_H$. Suppose there exists $y \notin N_H[S_H]$ such that $xy \notin E(H)$. Then $(S \setminus \{v\}) \cup \{y\} = S_H \cup \{y\}$ is not a dominating set of H and hence of $G + H$. This is contrary to our assumption that S is a secure dominating set. Thus, for every $y \notin N_H[S_H]$, $xy \in E(H)$. This implies that for every $x, y \in V(H) \setminus N_H[S_H]$, $xy \in E(H)$, that is, the subgraph induced by $V(H) \setminus N_H[S_H]$ is complete. Hence $V(H) \setminus N_H[S_H]$ is a clique in H. This proves statement $(v)(b)$.

Subcase 4. Suppose that S_G is a clique in G where $|S_G| \geq 2$ and $|S_H| \geq 2$. Since S is convex dominating set of $G + H$, S_H must be a clique in H. This proves statement (vi).

For the converse, suppose that statement (i) is satisfied, that is, S is a clique secure dominating set in G and $|S| \geq 2$. Then S is a clique dominating set and a secure dominating set in G and hence in $G + H$. By Remark 2.7, every clique dominating set is a convex dominating, that is, S is a convex dominating set and secure dominating set of $G + H$. Thus, S is a convex secure dominating set of a graph $G + H$. If $S \subseteq V(G)$, then $N[S] = V(G + H)$ and $\langle V(G + H) \setminus S \rangle$ is a subgraph without isolated vertices because $V(H) \subset \langle V(G + H) \setminus S \rangle$ where H is a connected non-complete graph. Thus, S is a restrained convex secure dominating set of a graph $G + H$. Similarly, if $S \subseteq V(H)$, then S is a restrained convex secure dominating set of a graph $G + H$. If $S \nsubseteq V(G)$ and $S \nsubseteq V(H)$, then $S = S_G \cup S_H$ where $S_G \subset V(G)$ and $S_H \subset V(H)$. Let $x \in V(G) \setminus S_G$ and $y \in V(H) \setminus S_H$. Then $xy \in E(G + H)$, that is, every vertex not in S is adjacent to a vertex in S and to another vertex in $V(G + H) \setminus S$. Hence, then S is a restrained convex secure dominating set of a graph $G + H$. Similarly, if statement (ii) is satisfied, then S is a convex secure dominating set of a graph $G + H$.

Now, suppose statement that $(iii)a$) is satisfied, that is, $S = S_G \cup S_H$ where $S_G = \{v\} \subset V(G)$ and $S_H = \{w\} \subset V(H)$ and S_G is a dominating set of G and S_H is a dominating set of H. Clearly, S is a convex dominating set and a secure dominating set of $G + H$, that is, S is a convex secure dominating set of $G + H$. Let $x \in V(G) \setminus \{v\}$ and $y \in V(H) \setminus \{w\}$. Then $xv \in E(G + H)$ and $xy \in E(G + H)$, that is, every vertex not in S is adjacent to a vertex in S and to a vertex in $V(G + H) \setminus S$. Hence, then S is a restrained convex secure dominating set of a graph $G + H$.

Next, suppose that statement that $(iii)b)$ is satisfied, that is, $S = S_G \cup S_H$ where $S_G = \{v\} \subset V(G)$ and $S_H = \{w\} \subset V(H)$ and S_G is dominating set of G and $V(H) \setminus N_H[S_H]$ is clique in H. Clearly, S is a convex dominating set of $G + H$. Let $x \in V(H) \setminus S_H$. If $xw \in E(H)$, then $S' = (S \setminus \{w\}) \cup \{x\} = \{v, x\}$. Since $v \in V(G)$ and $x \in V(H)$, S' is a dominating set of $G + H$. Thus, S is a convex secure dominating set of $G + H$. If $xw \notin E(H)$, then $x \in V(H) \setminus N_H[S_H]$. This means that $S' = (S \setminus \{v\}) \cup \{x\} = \{w, x\}$. Since w dominate $N_H[S_H]$ and x dominate $V(H) \setminus N_H[S_H]$, it follows that S' is a dominating set of H and hence of $G + H$. Thus, S is a convex secure dominating set of $G + H$. Since S is a restrained dominating set of $G + H$ (by the same argument in $(iii)a)$), it follows that S is a restrained convex secure dominating set of a graph $G + H$. Similarly, if statement $(iii)c)$ or statement $(iii)d)$ is satisfied, then S is a restrained convex secure dominating set of a graph $G + H$.

By following the same argumentations above, if statement (iv), (v) or (vi) is satisfied, then S is a restrained convex secure dominating set of a graph $G + H$. This proves the Theorem. ■

The following result is an immediate consequence of Theorem 2.8.

Corollary 2.9 Let G and H be nontrivial connected graphs.

$$\gamma_{rcs}(G+H) = \begin{cases} 1 & if\ G\ and\ H\ are\ complete \\ 2 & if\ \gamma_{cl}(G) = 2\ or\ \gamma_{cl}(H) = 2 \\ 3 & if\ S_G\ or\ order\ m \geq 2\ and\ V(G) \setminus N_G[S_G]\ are\ cliques\ in\ G\ or \\ & S_H\ or\ order\ n \geq 2\ and\ V(H) \setminus N_H[S_H]\ are\ cliques\ in\ H \\ 4 & if\ S_G\ or\ order\ m \geq 2\ is\ cliques\ in\ G\ and\ S_H\ of\ oder\ n \geq 2 \\ & n \geq 2\ is\ clique\ in\ H \end{cases}$$

Let G and H be graphs of order m and n, respectively. The corona of two graphs G and H is the graph $G \circ H$ obtained by taking one copy of G and m copies of H, and then joining the ith vertex of G to every vertex of the ith copy of H. The join of vertex v of G and a copy H^v of H in the corona of G and H is denoted by $v + H^v$.

Remark 2.10 For any connected graph G, $V(G)$ is a minimum dominating set in $G \circ H$.

III. CONCLUSION

In this paper, we introduced a new parameter of domination in graphs G - the restrained convex secure domination number of G. Some realization problem of the restrained convex secure dominating set was investigated and the restrained convex secure domination number is computed. Further, we give the characterization of the restrained convex secure dominating set in the join of two nontrivial connected graphs and compute its restrained convex secure domination number. This study will motivate new research such as bounds and some binary operations of two graphs such as the corona, lexicographic, and Cartesian product of two graphs. Other parameters involving restrained convex secure domination in graphs may also be explored. Finally, the characterization of restrained domination in graphs and its bounds is a promising extension of this study.

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