Quest Journals
Journal of Research in Applied Mathematics
Volume 8 ~ Issue 9 (2022) pp: 01-06
ISSN(Online) : 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

One-dimensional compressible Navier-Stokes equations: an analytical approach

Mohammad Ghorbanzadeh, AliHussein Naser,
Department of Mathematics, Imam Reza International University
Mashhad , Iran,
Department of Electric- Communication system , Imam Reza International University , Mashhad , Iran,

Abstract:
The 1D compressible Navier-Stokes equations' solution exhibits complicated behavior in half space. On the solution to the 1D compressible isentropic Navier-Stokes equations with constant viscosity coefficient on(x, t), ϵ[0,+∞) ×R ₊, we discover an intriguing phenomenon, namely the splitting in half of the solutions to the 1D compressible Navier-Stokes equations' initial boundary value problem.

Under some circumstances, the Riccati differential equation's solution can be converted into space.

R**eceived 18 August, 2022; **R**evised 31 August, 2022; **A**ccepted 02 September, 2022* ***P**ublished with open access at **www.questjournals.org

I. Introduction

We take into account the half-space 1D compressible Navier-Stokes equations in the following ways:

$\rho t + (\rho u)x = 0, (x, t) \in [0, +\infty) \times R_+$, (1)
$\rho(ut + uux) + \rho x = \mu uxx$, $(x, t) \epsilon [0, +\infty) \times R_+$, (2)
where$\rho(x,t)$, $u(x,t)$ denote the compressible flow's density and velocity .The coefficient of constant viscosity isμ. The flow's pressure is indicated by $P = P(\rho)$. We begin with the supposition that:

$(\rho, u)(x, 0) = (\rho_0, u_0)(x)$, (3)
Moreover, the restriction:
$u(0,t) = g(t)$, (4)
and allow
$\rho(x) > 0$ *and* $\rho_0(x) \epsilon C^2(0,+\infty)$, $g(t)$ ϵ $C^1|(0,+\infty), p(\odot)\ \epsilon\ C^1|(0,+\infty)$(5)

There is a good amount information material on the topics of 1D compressible Navier-Stokes solution global existence and big time behavior. Since viscosityμ is a positive number as well as the initial density being different from vacuum,Kanel [1] addressed the issues when the data were sufficiently smooth,Serre [2, 3] and Hoff [4] also looked at the issues with discontinuous starting data. Because density and viscosity are both dependent on one other permanent lower bound,[5-8] provided the system without initial vacuum with global well-posedness and long-time behavior of solutions. However, many papers on compressible fluid dynamics are related when the initial data acknowledges the presence of vacuum [9–17]. When we see how someone is properly poised

ways to fix the compressible equations of Navier-Stokes, the The presence of a vacuum is a significant challenge. Ding and associates got the widespread use of traditional techniques for 1D compressible Navier-Stokes equations on constrained domains, assuming
$\mu \epsilon C^2 [0,+\infty)$ satisfies $0 < \bar{\mu} \leq \mu() \leq C(1+P(\rho))$. Ye [19] used the limitation

$$\mu(\rho) = 1 + \rho^{\beta};\ 0 \le \beta < \gamma.$$

to find the global classical big solutions to the Cauchy problems (1) and (2).

The one-dimensional Navier-Stokes equations for viscous compressible and heat-conducting fluids in a limited region with the Robin boundary condition were the subject of a worldwide existence of classical solution by Zhang and Zhu [20]. For the Cauchy problem with the external force, Li et al. [21] calculate the uniform upper bound of density as well as the global wellposedness of strong (classical) big solutions. When the first and second viscosity coefficients areμ andλ(ρ), respectively, for the two-dimensional case, global well-posedness of classical solutions to the Cauchy issue or periodic domain problem of compressible Navier-Stokes equations with vacuum was achieved in [22–24].Global well-posedness was determined by Li and Xin [25] and published in the Hindawi Journal of Function Spaces.

Volume 2021, Article 6 pages, ID 6339203 https://doi.org/10.1155/2021/6339203

asymptotic behavior of strong and classical systems, and large-time

Navier-Stokes equations for viscous compressible barotropic flows provide solutions to the Cauchy problem in two or utilizing the vacuum's far field density in three spatial dimensions,
if the smooth beginning data have a low total energy, and the viscosity coefficients consist of two constants.
This work examines an intriguing occurrence involving the solutions of the 1D compressible Navier-Stokes equations (1) and (2) with constant viscosity coefficient.
Half-space may be used to modify the issue (1) and (2) to In particular circumstances, the Riccati differential equation's resolutionfavorable circumstances Then, we first state the major findingsdenote

$U(t)$ =1 $+ g(t)$
A1(x,t)=μ($\frac{\rho_0''(x+t)}{\rho_0^2(x+t)} - \frac{2(\rho_0'(x+t))^2}{\rho_0^3(x+t)}$),

B1(x,t) =$\frac{\rho_0'(x+t)}{\rho_0^2(x+t)}$
c1(x,t) =P$'(\rho_0(x+t))\rho_0'(x+t)$
A(x,t)=$\frac{\rho_0'(t)}{\rho_0(t)}$ + A1(x,t)
B(x,t)=$-\rho_0(t)$B1(x,t)
c1(x,t)= $-\frac{\text{c1}(x,t)}{\rho_0(t)}$(6)

First Theorem : The compressible Navier-Stokes equations (1) and (2) have a solution represented by the function,$(\rho,u)(x,t) = (\rho_0(x+t),(1+g(t))(\rho_0(t))/\ (\rho_0(x+t))) - 1)$.using the preliminary data (3) and boundary condition (4), provided that$U(t) - 1\ =\ g(t)$
at all times.
The differential equation of Riccati is satisfied by $U(t)$:
$U'(t)$ +$Q(t)U(t)$ + $P(t)U^2(t) = R(t)$ (7)
Where :
$\rho(x) > 0$ *and* $\rho_0(x) \epsilon C^2(0,+\infty)$, $g(t)\ \epsilon\ C'|(0,+\infty)$, $p(\odot)\ \epsilon\ C'|(0,+\infty)$ (8)

And :
Three functions$Q(t)$, $P(t)$, and $R(t)$ exist that solely rely on t in the sense that :
$Q(t) = \frac{\rho_0'(t)}{\rho_0(t)} + A1(x,t)$, $P(t) =\ -\rho_0(t)\ B1(x,t)$,
$R(t) = -\frac{\text{c1}(x,t)}{\rho_0(t)}$, (9)
If $(x,t)\epsilon[0,+\infty)\times[0,+\infty)$
Second theorem : When
$2Q' + Q^2 - 2Q\frac{P'}{P} - 2\frac{P''}{P} + 3(\frac{P'}{P})^2 + 4R(t) > 0$, (10)
then there is the presence of (7)
Remark 3. Because the existence of a generic solution to the Ricatti equation is an unsolved issue, we do not know if the solution$U(t)$ of First Theorem exists universally. The existence of (7) on a global scale may be shown if we add the conditions$P(t)$,$Q(t)$, and $R(t)$such as (25) and (26).achieved, as Second theorem illustrates.

Remark 4. In Theorem 1, the starting value$\rho_0(x), u_0(x)$, can be limited in Environment that allows space or not, depending on the initial value's unique function, which is provided.

- 1. Boundary value case, Additionally, assuming that

$\|\rho_0(x)\| \leq C$, $\rho_0(0)=0$, $L^1([0,+\infty)$,and $g(t)=0$,

After that we have the starting energy$\rho_0(x)u_0{}^2(x)\epsilon L^1([0,+\infty)$. In actuality

$\int_0^\infty \rho_0(x)u_0{}^2(x)dx = \int_0^\infty \rho_0(x)(\frac{(1+g(0))\rho_0(0)}{\rho_0(x)} - 1)^2 x dx$

$= \int_0^\infty \rho_0(x)\, dx \leq$ C(11)

Using a simple energy estimation, we can quickly determine

$\int_0^\infty \rho(x,t)u(x,t)^2 dx \leq$ C (13)

- 2. Boundless case. The boundary condition of velocity $g(t) \neq 0$ is apparent

2.The main result's evidence

- 1st Theorem Proof ,If the analytical function is available:

$\rho(x,t) = \rho_0(x+t)$ (14)

We will obtain

$u(x,t) = (1+u(0,t))exp\left\{-\int_0^x \frac{\rho_0{}'(y+t)}{\rho_0(y+t)}dy\right\} - 1$

$= (1+g(t))\frac{\rho_0(t)}{\rho_0(x,t)} - 1$, (15)

even with the equation

$\rho_0{}'(x+t) + u\rho_0{}'(x+t) + \rho_0(x+t)u\,x$(16)

, as well as the border condition (4)

Therefore, $u(x,t)$derivatives are

$ut(x,t) = g'(t)\frac{\rho_0(t)}{\rho_0(x+t)} + (1+g(t))\frac{\rho_0{}'(t)\rho_0(x+t)}{\rho_0{}^2(x+t)}$, (17)

$ux(x,t) = -(1+g(t))\frac{\rho_0{}'(t)\rho_0(x+t)}{\rho_0{}^2(x+t)}$, (18)

$uxx(x,t) = -(1+g(t))\frac{\rho_0(t)\,\rho_0{}^2(x+t)\,\rho_0{}''(x+t) - 2\rho_0(t)\rho_0(x+t)\rho_0{}'(x+t))^2}{\rho_0{}^3(x,t)}$

, (19)

When (18) and (19) are substituted into the present equation, we get

$$\rho_0(x+t)\left(g'(t)\frac{\rho_0(t)}{\rho_0(x+t)} + (1+g(t))\frac{\rho_0{}'(t)\rho_0(x+t) - \rho_0(t)\rho_0{}'(x+t)}{\rho_0{}^2(x+t)} - \left\{(1+g(t))\frac{\rho_0(t)}{\rho_0(x+t)} - 1\right\}(1+g(t))\frac{\rho_0(t)\rho_0{}'(x+t)}{\rho_0{}^2(x+t)}\right) + P'(\rho_0(x+t)\rho_0{}'(x+t) = -\mu(1+g(t))\frac{\rho_0(t)\,\rho_0{}^2(x+t)\,\rho_0{}''(x+t) - 2\rho_0(t)\rho_0(x+t)\rho_0{}'(x+t))^2}{\rho_0{}^3(x,t)} \quad ,(20)$$

i.e.,

$g'(t) + (1+g(t))\left[\frac{\rho_0{}'(t)}{\rho_0(t)} + \mu\left(\frac{\rho_0{}''(x+t)}{\rho_0{}^2(x+t)} - \frac{2\left(\rho_0{}'(x+t)\right)^2}{\rho_0{}^3(x,t)}\right)\right] - (1+g(t))^2\frac{\rho_0(t)}{\rho_0(x+t)}\frac{\rho_0{}'(x+t)}{\rho_0(x+t)} = \frac{P'(\rho_0(x+t))}{\rho_0(t)}\rho_0{}'(x+t)$

, (21)

$U(t)$ is satisfied if $U(t) = 1 + g(t)$

$U'(t) + U(t)\left[\frac{\rho_0{}'(t)}{\rho_0(t)} + \mu\left(\frac{\rho_0{}''(x+t)}{\rho_0{}^2(x+t)} - \frac{2\left(\rho_0{}'(x+t)\right)^2}{\rho_0{}^3(x,t)}\right)\right] - (U(t))^2\frac{\rho_0(t)}{\rho_0(x+t)}\frac{\rho_0{}'(x+t)}{\rho_0(x+t)} = \frac{P'(\rho_0(x+t))}{\rho_0(t)}\rho_0{}'(x+t)$, (22)

also$\left(\frac{\rho_0{}''(x+t)}{\rho_0{}^2(x+t)} - \frac{2\left(\rho_0{}'(x+t)\right)^2}{\rho_0{}^3(x,t)}\right)$, $\left(\frac{\rho_0{}'(x+t)}{\rho_0{}^2(x+t)}\right)$ and

$$P'(\rho_0(x+t))\,\rho_0{}'(x+t)$$

are independent of the positional variable x .As a result, $U(t)$ is the answer to the Riccati differential equation (7) under the assumptions that (9).

$U(t)$ meets condition (7) if $g(t)$ = $U(t)$ −1, and the other requirements are met. It is simple to obtain in (8) and (9)

$[\rho_0(x+t)]\mathrm{t} + \left(\rho_0(x+t)\left[(1+g(t))\frac{\rho_0(t)}{\rho_0(x+t)} - 1\right]\right)x = 0$,

$\rho_0(x+t)\left(\left[1+g(t)\frac{\rho_0(t)}{\rho_0(x+t)} - 1\right] + \left[(1+g(t))\frac{\rho_0(t)}{\rho_0(x+t)} - 1\right] \times \left[(1+g(t))\frac{\rho_0(t)}{\rho_0(x+t)} - 1\right]x\right) P'(\rho_0(x+$ $t)\rho_0'x+t=\mu(1+g(t))\rho_0\ t\rho_0 x+txx$(23)

So that, $(x,t)(\rho,u) = \rho_0(x+t), (1+g(t))\frac{\rho_0(t)}{\rho_0(x+t)} - 1)$is the result of compressible Navier-Stokes equations (1) and (2) given starting information (3) and operating conditions (4).

The Riccati equation has not had a universal solution since Riccati proposed it in the seventeenth century.

300 years or more. Despite the fact that there are several special solutions, none of them can inherently resolve this problem.

Here, we state the existence of the Riccati equation globally (7)

Given a certain set of $P(t)$, $Q(t)$, and $R(t)$ conditions,

driven bythe outcomes shown in the source [26, 27].

Verification of Second theorem. Equation (7) changes when $W(t)$ = $P(t)U(t)$, is used.

$W'(t) = -W^2(t) - f(t)\,W + R(t)$ (24)

Where $f(t) = Q(t) - (P'(t)/P(t))$ Because $\rho_0(x) \epsilon C^2(0,+\infty)$and (10)There are

$f(t)\epsilon\ C^1|(0,+\infty)$ (25)

$\frac{1}{2}f'(t) + \frac{1}{4}f^2(t) + R(t) > 0$ (26)

According to [26, 27], we may determine the Riccati equation's global existence using (25) and (26).

3. Example

Here, we provide a few samples. It is simple to verify that first.

Example 1. Assuming that the starting values ρ_0 and $u0$ are both constants and that the pressure is $P(\rho) = \rho^\gamma$ for anyγ> 0, we can determine that the solutions to the compressible Navier-Stokes equations (1) and (2) satisfy first Theorem 's conclusion.

Particularly, if a certain nonphysical condition exists, we can infer the following fascinating example.

Example 2. Assuming that $P(\rho) = -\rho^{-\gamma}$, and consider that

$\rho_0(x) = \frac{1}{x+1}$, $u_0(x) = \frac{2c_0+1}{2c_0-1}x + \frac{2}{2c_0-1}(c_0 \neq 0)$, (27)

Following that, we may obtain the analytically nontrivial answer to the compressible Navier-Stokes equations (1) and (2)

$\rho_0(x+t) = \frac{1}{x+t+1}, u(x+t) = \frac{2c_0e^{2t}+1}{2c_0e^{2t}-1}(x+t+1) - 1$ (28)

Additionally, we may obtain the compressible flow's particle path.

$x(t) = \frac{x(0)+1}{|2c_0-1|}\ e^{-t}|2c_0e^{2t} - 1| - t - 1$ (29)

where $x(0)$ denotes the particle's original location

Verification. We have the earliest data from (27)

$\rho_0(x,0) = \rho_0(x) = \frac{1}{x+1}$, $u_0(x) = \frac{2c_0+1}{2c_0-1}x + \frac{2}{2c_0-1}$, (30)

and the requirement for compatibility

$g(t) = u(0,t)|_{t=0} = u(0,0) = u_0(0) = \frac{2}{2c_0-1}$,(31)

Using the preliminary data, we have

$\mathrm{A1}(x,t) = \mu\left(\frac{\rho_0''(x+t)}{\rho_0^2(x+t)} - \frac{2(\rho_0'(x+t))^2}{\rho_0^3(x+t)}\right) = 0$

$\mathrm{B1}(x,t) = \frac{\rho_0'(x+t)}{\rho_0^2(x+t)} = -1$

$\mathrm{c1}(x,t) = \mathrm{P}'(\rho_0(x+t))\rho_0'(x+t) = -1$

$Q(t) = -\frac{1}{1+t},\ P(t) = \frac{1}{1+t}, R(t) = 1+t$ (33)

Because of the variable replacement

$U(t) = 1 + g(t)$ (34)

We get

$U'(t) = -\frac{1}{1+t}\ U(t) + \frac{1}{1+t}U^2(t) = 1+t$ (35)

$V(t) = U(t) - (t+1)$ (36)

Afterwards, we have that V t satisfies.

$V'(t) = \left(\frac{1}{1+t} - 2\right)\ V(t) - \frac{1}{1+t}V^2(t)$ (37)

When we divide the equation (37) by $\left(-\frac{1}{V^2}\right)$ we obtain

$\left(\frac{1}{V^2}\right)' = \left(2-\frac{1}{1+t}\right)\left(\frac{1}{V^2}\right)+\frac{1}{1+t}\left(\frac{1}{V^2}\right)^2$ (38)

Using the approach of constant variation and the compatibility requirement $g(0) = u_0(0) = \frac{2}{2c_0-1}$

Then

$V(t) = \frac{2(1+t)}{2c_0e^{2t}-1}$ (39)

The result is obtained by combining the variant replacements (34) and (36).

$g(t) = \frac{2(1+t)}{2c_0e^{2t}-1}+t$ (40)

Finally, we derive from first theorem (28).

Since the particle route meets the condition

$x'(t) = u(x,t)$, we obtain, from (28),

$x'(t) = \frac{2c_0e^{2t}+1}{2c_0e^{2t}-1}(x+t+1)-1$ (41)

$x(t) = (x(0)+1)exp\left\{t+\int_0^{2t}\frac{1}{2c_0e^{s}-1}ds\right\}-t-1,$ (42)

Those are the results (29)

Remark 5. We are able to obtain $\forall P>1$ in The first theorem .

$$\|\rho_0(x)\|L^P([0,+\infty)) = \int_0^\infty \frac{1}{(x+1)^p}dx \le 1,$$

$\|\rho_0(x)\|L^P([0,+\infty)) = \int_0^\infty \frac{1}{(x+t+1)^p}dx \le \frac{1}{(1+t)^{\frac{p-1}{p}}}$(43)

However, $\|\rho_0(x)\|L^1([0,+\infty))$ is unbounded, and $\|\rho_0{u_0}^2\|L^1([0,+\infty))$and $\|\rho u^2\|L^1([0,+\infty))$boundedness are difficult to get.

for the specified starting data circumstance (27)

The initial velocity data for $u_0(x) = -1$, if$c_0 = -(\frac{1}{2})$.

The answer to the compressible Navier-Stokes equations (1) and (2) under the pressure $P(\rho) = -\rho^{-1}$ is thus given by

$\rho(x+t) = \frac{1}{x+t+1}$, $u(x+t) = \frac{e^{2t}+1}{e^{2t}-1}(x+t+1)-1$, (44)

Additionally, the compressible flow's particle path is

$x(t) = \frac{x(0)+1}{2}e^{-t}(e^{2t}+1)-t-1$ (45)

Remark 6. The solutions to the next Euler equations are the functions $\rho(x+t) = \frac{1}{x+t+1}$ and $u(x+t) = \frac{e^{2t}+1}{e^{2t}-1}(x+t+1)-1$

$\rho t + (\rho u)x = 0, (x,t) \in [0,+\infty) \times R_+$,

$\rho(ut+uux)+\rho x = \mu uxx$, $(x,t) \epsilon [0,+\infty) \times R_+$, (46)

assuming the pressure and starting information

$P(\rho)=-\rho^{-\gamma}$,

$\rho_0(x) = \frac{1}{x+1}$, $u_0(x) = \frac{2c_0+1}{2c_0-1}x+\frac{2}{2c_0-1}(c_0 \neq 0)$ (47)

With respect to the spatial variable x, the second order derivative of $u(x+t)$ is equal to zero .

Availability of Data

This publication basically gets some fascinating theorems that need serious verification; there is no analysis of data in this study, and all of the references can be located upon that Web of Science.

References

[1]. I. Kanel, "A model system of equations for the onedimensional motion of a gas (in Russian)," Differencial'nyeUravnenija, vol. 4, pp. 721–734, 1968.

[2]. D. Serre, "Solutions faiblesglobales des quations de NavierStokes pour un fluide compressible," ComptesRendus de l'Académie des Sciences - Series I - Mathematics, vol. 303, pp. 639–642, 1986.

[3]. D. Serre, "On the one-dimensional equation of a viscous compressible heat-conducting fluid," ComptesRendus de l'Académie des Sciences - Series I - Mathematics, vol. 303, pp. 703–706, 1986.

[4]. D. Hoff, "Global existence for 1D compressible isentropic Navier-Stokes equations with large initial data," Transactions of the American Mathematical Society, vol. 303, pp. 169–181, 1987.

[5]. A. A. Amosov and A. A. Zlotnik, "Global generalized solutions of the equations of the one-dimensional motion of a viscous heat-conducting gas," Soviet Mathematics-Doklady, vol. 38, pp. 1–5, 1989.

[6]. H. Beirao da Veiga, "Long time behavior for one-dimensional motion of a general barotropic viscous fluid," Archive for Rational Mechanics and Analysis, vol. 108, no. 2, pp. 141– 160, 1989.

[7]. A. V. Kazhikhov, "Cauchy problem for viscous gas equations," Siberian Mathematical Journal, vol. 23, no. 1, pp. 44–49, 1982.
[8]. I. Straškraba and A. Zlotnik, "On a decay rate for 1D-viscous compressible barotropic fluid equations," Journal of Evolution Equations, vol. 2, no. 1, pp. 69–96, 2002.
[9]. Y. Cho and H. Kim, "On classical solutions of the compressible Navier-Stokes equations with nonnegative initial densities," Manuscripta Mathematica, vol. 120, no. 1, pp. 91–129, 2006.
[10]. E. Feireisl, Dynamics of Viscous Compressible Fluids, Oxford University Press, New York, NY, USA, 2004.
[11]. F. M. Huang, J. Li, and Z. P. Xin, "Convergence to equilibria and blowup behavior of global strong solutions to the Stokes approximation equations for two-dimensional compressible flows with large data," Journal de MathématiquesPures et Appliquées, vol. 86, no. 6, pp. 471–491, 2006.
[12]. X. D. Huang and J. Li, "Serrin-type blowup criterion for viscous compressible and heat conducting Navier Stokes and magnetohydrodynamic flows," Communications in Mathematical Physics, vol. 324, no. 1, pp. 147–171, 2013.
[13]. X. D. Huang, J. Li, and Z. P. Xin, "Global well-posedness of classical solutions with large oscillations and vacuum to the three-dimensional isentropic compressible Navier-Stokes equations," Communications on Pure and Applied Mathematics, vol. 65, no. 4, pp. 549–585, 2012.
[14]. J. Li, J. W. Zhang, and J. N. Zhao, "On the global motion of viscous compressible barotropic flows subject to large external potential forces and vacuum," SIAM Journal on Mathematical Analysis, vol. 47, no. 2, pp. 1121–1153, 2015.
[15]. P.-L. Lions, Mathematical Topics in Fluid Mechanics, vol. 2 of Compressible Models, Oxford University Press, New York, NY, USA, 1998.
[16]. A. Matsumura and T. Nishida, "The initial value problem for the equations of motion of viscous and heat conductive gases," Journal of Mathematics of Kyoto University, vol. 20, pp. 67– 104, 1980.
[17]. Z. P. Xin, "Blowup of smooth solutions to the compressible Navier-Stokes equation with compact density," Communications on Pure and Applied Mathematics, vol. 51, no. 3, pp. 229 –240, 1998.
[18]. S. J. Ding, H. Y. Wen, and C. J. Zhu, "Global classical large solutions to 1D compressible Navier-Stokes equations with density-dependent viscosity and vacuum, " Journal of Differential Equations, vol. 251, no. 6, pp,1969-1725,2011
[19]. Y. L. Ye, "Global classical solution to 1D compressible NavierStokes equations with no vacuum at infinity," Mathematical Methods in the Applied Sciences, vol. 39, no. 4, pp. 776 –795, 2016.
[20]. P. X. Zhang and C. J. Zhu, "Global classical solutions to 1D full compressible Navier-Stokes equations with the Robin boundary condition on temperature," Nonlinear Analysis: Real World Applications, vol. 47, pp. 306 –323, 2019.
[21]. K. Li, B. Lv, and Y. Wang, "Global well-posedness and largetime behavior of 1D compressible Navier Stokes equations with density-depending viscosity and vacuum in unbounded domains, " Science China Mathematics, vol. 62, pp. 1 –14, 2019.
[22]. Q. S. Jiu, Y. Wang, and Z. P. Xin, "Global well-posedness of the Cauchy problem of two-dimensional compressible NavierStokes equations in weighted spaces," Journal of Differential Equations, vol. 255, no. 3, pp. 351 –404, 2013.
[23]. Q. S. Jiu, Y. Wang, and Z. P. Xin, "Global well-posedness of 2D compressible Navier-Stokes equations with large data and vacuum," Journal of Mathematical Fluid Mechanics, vol. 16, no. 3, pp. 483 –521, 2014.
[24]. J. Li and Z. L. Liang, "On local classical solutions to the Cauchy problem of the two-dimensional barotropic compressible Navier-Stokes equations with vacuum, " Journal de MathématiquesPures et Appliquées, vol. 102, no. 4, pp. 640–671, 2014
[25]. J. Li and Z. P. Xin, "Global well-posedness and large time asymptotic behavior of classical solutions to the compressible Navier-Stokes equations with vacuum, " Annals PDE, vol. 5, no. 1, p. 7, 2019.
[26]. J. C. Kegley, "A global existence and uniqueness theorem for a Riccati equation, " SIAM Journal on Mathematical Analysis , vol. 14, no. 1, pp. 47 –59, 1983.
[27]. W. Zhang, "Global existence and blow-up for Riccati equation," Dynamic Systems and Applications, vol. 12, no. 3-4, pp. 251 –258, 2003.