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Research Paper

Modelling the Effects of Three Natural Predators on the Aquactic and Adult Stages of Anopheles Mosquitoes in the Control of Malaria Transmission

[*1]Atanyi Yusuf Emmanuel., [2]H.K. Oduwole., [3]M.A. Umar
1. *Department of Mathematics, Federal University of Lafia, PMB 146, Lafia, Nigeria*
2&3. *Department of Mathematics, Nasarawa State University, Keffi PMB 1022 Keffi, Nigeria*

ABSTRACT
Modelling the effects of three natural predators on the aquatic and adult anopheles' mosquitoes in the control of malaria transmission was derived aimed at eradicating anopheles' larva, pupa and adult anopheles' mosquito by introduction of natural predators "copepods, tadpoles and purple martins" so that there should not be anopheles' adult mosquito for malaria transmission in our society. The new model is a control flow diagram of predator-prey interaction model in mosquito life-cycle. The population is sub-divided based on mosquito life-cycle and natural predators. Under a mosquito life-cycle, the population is divided into four compartments', Egg compartment E(t), Larva compartment L(t), Pupa compartment P(t), and Adult compartment A(t), and natural predators, it is divided into three compartments', namely; Copepods $C_P(t)$, Tadpoles $T_P(t)$ and Purple martins $P_M(t)$. From the stability analysis of steady state we observed that the model free equilibrium state is stable, implies that the equilibrium point or steady state is stable and the stability of the model (1) – (10) means, there will not be anopheles adult mosquito in our society for malaria transmission and from the idea of Beltrami's conditions and Diekmann condition, we observed that the Determinant of the Jacobian matrix is greater than zero $\{Det(j) > 0\}$*,Trace of the Jacobian matrix is less than zero* $\{Tr(j) < 0\}$ *and* $R_o < 1$ *which implies that the model disease free equilibrium state is stable. Hence the number of larva that transform to pupa is almost zero and the number of pupa that develop to adult is minimal and number of adult that escape to vector stage are inconsequential and microscopic and that means the life-cycle could be broken at the larva, pupa, and adult stages with the introduction of natural predators, with the natural implication there will not be anopheles adult mosquito for malaria transmission and we also use maple for symbolical and numerical solution and presented the results graphically.*

KEYWORDS: *Anopheles; Copepod; Tadpole; Purple martins; Malaria*

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I. Introduction

The *Anopheles* vector system in Nigeria and of course in sub-Saharan Africa is probably the strongest that exists for human *Plasmodium*. Contact with human vectors, particularly An. *gambiae* s.l., shows remarkable stability and flexibility, resulting in extremely high vaccination rates under different seasonal and geographic ecological conditions (Mokuolu et al., 2018). Malaria remains a leading cause of death and disease in most tropical regions of the world, where it is endemic in 106 countries. In 2010, out of a total of 216 million cases of malaria, around 81% occurred in Africa and 13% in Southeast Asia1. The majority (91%) of the estimated 665,000 malaria deaths occur in Africa and primarily affect children under the age of five (86%). In America in 2010 there were more than 670,000 confirmed cases of malaria with 133 deaths from malaria. The transmission is active in 21 countries and puts approximately 20% of the US population at risk. Malaria severely limits economic development and is a cause of poverty in most countries where the disease is endemic. Malaria remains an ongoing problem in sub-Saharan Africa, and while great strides have been made over the past 15 years, millions of people are still at risk of contracting the parasite (Patouillard et al., 2017).

Africa offers a stable and ecologically diverse ecosystem and hosts the world's highest vectors of malaria (Bernard et al., 2020) and is expected to remain so in the future. Climate change (Adigun et al., 2015). The main vectors of *Anopheles* malaria in sub-Saharan Africa are *Anopheles funestus* s.s. and three members of

the *Anopheles gambiae* complex: *An. Gambiae* s.s., *Anopheles coluzzii* and *Anopheles arabiensis* (Molinaro et al., 2015), which play a role in the transmission of malaria in their distribution area, e.g. the groups *Anopheles moucheti* and *Anopheles* nili (Rajeswari, 2017) and another of secondary or random vectors (Antonio-nkondjio et al., 2006). Considering that the genus *Anopheles* includes more than 500 species worldwide, of which only a few are considered important species for the transmission of malaria (Garcia Guerra et al., 2014). The morphological identification of species is crucial for allocating scarce resources solely to the fight against malaria vectors. Species groups and species complexes are common within the genus *Anopheles* (Harbach & Besansky, 2014) and this complicates vector control because not all species in a complex share similar behaviors or similar roles in transmission malaria disease (Vanelle et al., 2012a).

Mosquitoes of the family *Culicidae* are considered a nuisance and a major public health problem because their females feed on human blood and therefore transmit extremely harmful diseases such as malaria, yellow fever and *filariasis* (Tsoka-Gwegweni & Okafor, 2014). They are estimated to transmit diseases to more than 700 million people each year and are responsible for the death of around 1 in 17 people ("Malaria Policy Advisory Committee to the WHO: Conclusions and Recommendations of Eighth Biannual Meeting (September 2015)," 2016). Effective transmission of mosquito-borne diseases requires successful contact between female mosquitoes and their hosts (Vanelle et al., 2012b). Among *Anopheles*, members of the genus *Anopheles* are best known for their role in the global transmission of malaria and *filariasis* ("Malaria Policy Advisory Committee to the WHO: Conclusions and Recommendations of Fifth Biannual Meeting (March 2014)," 2014). Among these diseases, malaria, caused by the *Plasmodium* parasite, is one of the deadliest diseases in the world ("Malaria Policy Advisory Committee to the WHO: Conclusions and Recommendations of Sixth Biannual Meeting (September 2014)," 2015). ("Malaria Vaccine: WHO Position Paper, January 2016 – Recommendations," 2018) reported approximately 207 million cases of malaria in 2012, of which 200 million (80.0%) were on the affected continent. Patterns of disease spread, transmission, and intensity depend on the degree of urbanization and distance from vector breeding sites (MCNAMARA, 2005). The endemicity of malaria in each region is determined, among other things, by native *Anopheles* mosquitoes, their abundance, diet, resting behavior and *Plasmodium* infectivity (Atta & Reeder, 2014). The Federal Ministry of Health in Abuja reported that at least 50.0% of Nigerians suffer from some form of malaria, making it the most significant health problem in Nigeria (UM & AN, 2016). The high transmission rate and prevalence of malaria is the result of the various mosquito breeding sites, including convenient water reservoirs such as cans, old tires, tree holes, cisterns, open pools, drains, streams and ponds (McKenzie, 2014). Part of the fight is the official observance of April 25 each year, beginning in 2008, as World Malaria Day (CDC Weekly, 2020). Arms-only people face a variety of barriers when assessing malaria prevention, particularly with respect to knowledge of mathematical modeling and vector biology (Emmanuel et al., 2020).

II. Materials and Methods

The responsibility for the interpretation and use of the materials for larva collections lies with the target area so that larval habitants can be accessed and treated.

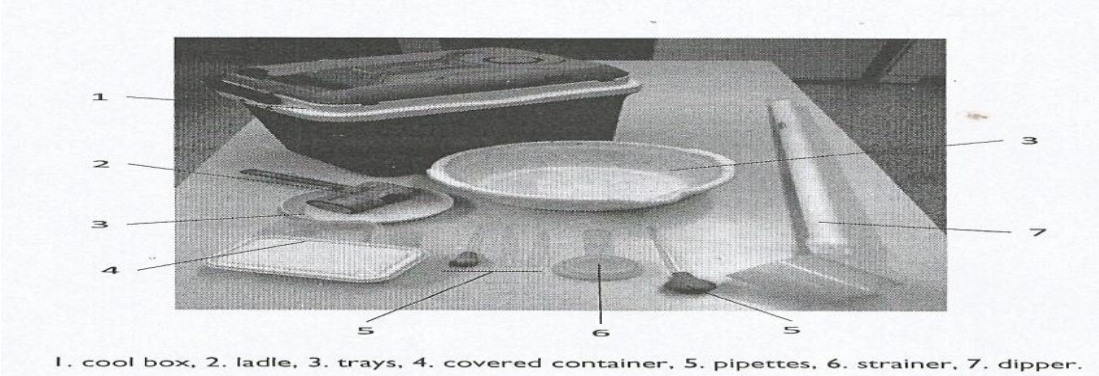


Figure 1: Main Materials for Larva Collections

There are seven (7) materials for larva collection from the above figure such as cool box, ladle, trays, covered container, pipettes, strainer and dipper.

Cool Box: Mosquitoes breed as larvae in cooler water. There are good larvicide like *diflubenzuron* of Bayer, available for this purpose.

Ladle: Ladle material are used for larvae collect sample of larvae, larva densities before treatment are taken by ladle.

Trays: The mosquito mass-rearing trays was designed to provide a large surface area to evaluate stress on larvae and pupae during the collection.

Covered Container: The container-inhabiting *Aedes* mosquitoes are the major vectors to detect larvae and pupae and associated socioeconomic surveys to collect.
Pipette: The net is an effective means of collecting *anopheline* larvae and was to use a pipette to remove all the mosquito larvae that were at.
Strainer: The purpose of this material is to provide specific and laboratory-reared mosquito larvae of known age or instars.
Dipper: Dipper for the collection of mosquito larvae and pupae is a patented telescoping water sampling dipper primarily intended for use by vector field.

Developmental Stages of *Anopheles* Mosquito
There are four stages in the life cycle of a mosquito which includes egg, larva, pupa and adult. During its life-cycle the mosquito undergoes two changes from larva to pupa and from pupa to adult (metamorphoses). The developmental stage of a mosquito is part of materials.

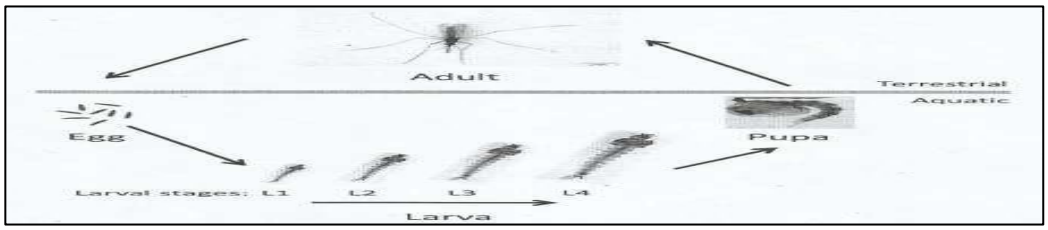


Figure 2: Stages of the life cycle of the Anopheles Mosquito

Egg Stage
a) The adult female *Anopheles* mosquito mates once and lays eggs throughout its life.
b) Females should feed on blood every 2-3 days. Blood is needed to develop eggs. Females lay a series of eggs before the next blood meal.
c) Eggs are laid in batches of 50 to 200 eggs in water (rain ponds, ponds, banks, lakes, etc.).
d) The hatching time of the eggs depends largely on the temperature.
e) At about 30°C, the eggs hatch into larvae in about 2-3 days.
f) In temperate zones (16oC), about 7-14 days

Larva Stage
a) The larva has a well-developed head with "mouth brushes" for feeding (filtering). The larva feeds on micro-agents (eg algae, bacteria) and organic matter in the water in which it reproduces.
b) The Anopheles larva does not have a respiratory siphon. It is parallel to the surface of the water to breathe.
c) There are four stages of larval development called instars (designated L1 to L4, Fig. 2).
d) Development from larva to pupa takes 5-10 days at normal tropical temperatures, depending on the species. Water temperature affects the time required for development, which is shorter in warmer waters.

Pupa Stage
a) The pupa is comma-shaped and stays on the surface of the water.
b) It has a pair of breathing trumpets through which it breathes when on the surface.
c) During this phase, there is no feeding, but the pupa is mobile and responds to stimuli.
d) It is the dormant or inactive phase during which there is a great transformation from aquatic life to surface life and extra-aquatic life.
e) The pupa stage lasts about 2 to 5 days.

Adult Stage
a) The adult animal usually emerges from the pupa at dusk.
b) After emerging from the pupa, the adult mosquito rests briefly to harden its body.
c) Both male and female mosquitoes feed on nectar for energy.
d) After mating, the female mosquito seeks blood for the development of her eggs. In some species, one diet is enough to develop the eggs. In other species, two feedings are necessary, at least for the development of the first eggs.
e) The time between the egg and the adult Anopheles can vary from 7 days at 31°C to 20 days at 20°C

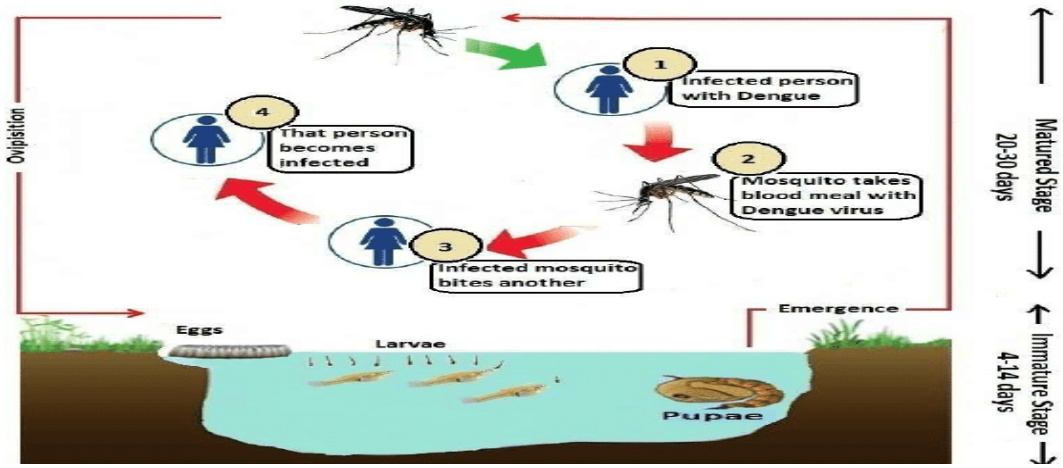


Figure 3: Mosquito Life-cycle with Predator and Disease Transmission among Human

Malaria Parasite

The malaria parasite is a vector-borne disease caused by *protozoan* parasites of the *genus Plasmodium*. There are four types of human malaria parasites: *P. falciparum*, *P. vivax*, *P.* malariae and *P. ovale*. Parasites are transmitted from person to person by female mosquitoes of the *genus Anopheles*.

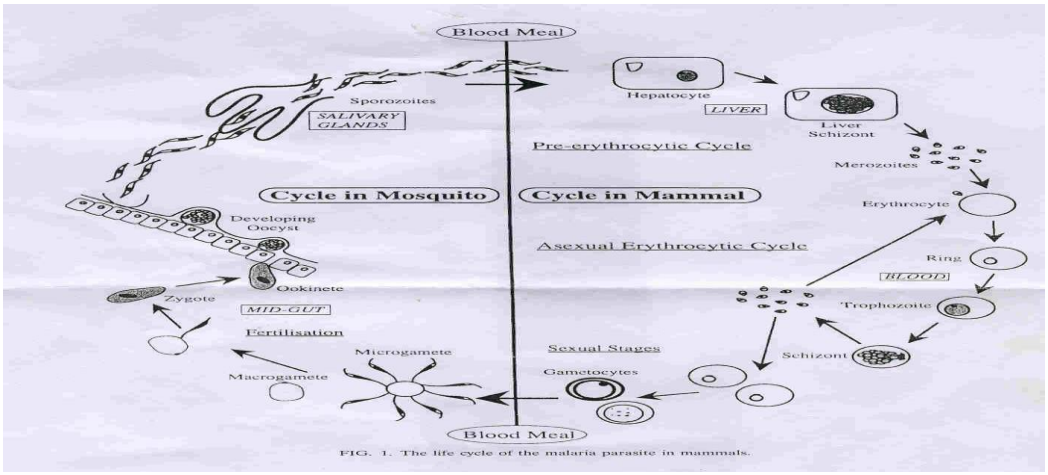


Figure 4: The life cycle of the Malaria Parasite. (Source: Phillips 2001).

Microscope

A microscope is a laboratory instrument used to examine objects too small to be seen with the naked eye. Microscopy is the science of examining small objects and structures under a microscope. Microscopic means invisible to the eye unless assisted by a microscope. In this work, we use it to observe the different stages of the life cycle of mosquitoes.

Figure 5: Viewing the Larva Stages of Mosquito Using Binocular Microscope

Methods

The modified model is used to study the uniqueness, existence, stability analysis of disease-free steady states, analyze, solve and perform numerical simulations showing a graphical representation of the results, three methods would be used. First we would use the point of equilibrium or steady state, Beltrami conditions, method of Dikeman conditions and finally we would use Maple software to display the results when three natural enemies are introduced at the same time.

Sampling Methods for Collecting Larvae

There are several methods for sampling larvae. The application of individual sampling methods depends on the type and type of hatchery and is described in the following sections. The larval collector should approach the hatchery with caution, as any disturbance will cause the larvae and pupae to descend and become inaccessible. It is important that the collector does not cast a shadow on the water. If the larvae and pupae are moving, it may be necessary to remain still until they swim again.

Diving (Dipping) procedures

1. This method is generally used to sample relatively large bodies of water, such as swamps, ditches, streams and rice fields.
2. The bucket should be gently lowered at an angle of approximately 45° to minimize disturbance and skim the surface of the water or gently lowered to allow water and nearby larvae to drain into the bucket. Be careful not to spill any water when you take the bucket out of the water.
3. The larvae should be removed from the spoon with a pipette and transferred to a properly labeled bottle or vial.
4. If vegetation appears in the hatchery, the collector should agitate the water and allow the larvae to swim to the bottom, then remove some vegetation to create a clear area for sampling and wait a few minutes before to continue sampling as previously described. . To calculate the larval density, note the number of baths in each hatchery. Also consider the time required for collection.

Figure 6: Sample Collection (Larva) by Dipping

Mode of Remuneration (Netting Method)

This method consists of using a fine-mesh net attached to a handle, with a plastic bottle or hose attached to the end. It is usually used to collect larvae and pupae in larger bodies of water such as ponds and small lakes. The net should be held at approximately a 45º angle to the water surface and pulled across the surface. The larvae and pupae are collected in the plastic bottle at the end.

Figure 7: Sample Collection (Larva) by Netting

Pipetting Method

This method is used to collect larvae from small breeding sites, such as small puddles, hoof prints, containers, plant axils, and tree cavities.

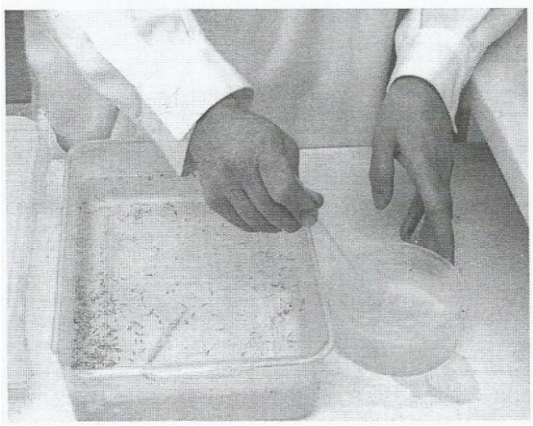


Figure: 8 Introduction of Natural Predator to the Larvae Tray

Figure 9: Showing Larva Rearing Container (left) and Larva Food (right).

Description of the Model

The new model is a control flowchart of the predator-prey interaction model in the mosquito life cycle that considers an open population of mosquitoes and predators. The population is subdivided according to the life cycle of mosquitoes and natural predators. In the life cycle of a mosquito, the population is divided into four compartments: Egg compartment E(t), Larval compartment L(t), Pupal compartment P(t), Adult compartment A(t) and natural Predator divided into three divisions. Copepods $C_P(t)$, Tadpoles $T_P(t)$, and Purple Martins $P_M(t)$.

Mathematical models provide a solid understanding of planning and risk controls in heterogeneous settings, especially when the models are based on vector population ecology and a solid understanding of entomological parameters relevant to transmission. Research conducted by (Killeen & Chitnis, 2014) that mathematical models have also played an important role in understanding the epidemiology of malaria and other infectious diseases; that mathematical models also provide an accurate quantitative description of complex nonlinear processes and a method to relate the individual infection process to the incidence of disease or infection in a population over time, yielding insights important on the introduction of natural predator to increase the interruption of the life cycle of the Anopheles mosquito at the larval, pupal and Adult stages, thereby reducing or eradicating the mosquitoes. This introduction of natural enemies reduces malaria by the biting vector. They work by reducing the intensity of malaria transmission or eradicating malaria. The classification of a natural enemy as predator or parasite largely depends on the number of prey or hosts attacked or consumed the reproductive strategy and other details of the system, in which there are many similarities in characteristics of the natural predator and in the model, to study them. Mathematical modeling of malaria is a challenging area of applied mathematics due to its peculiarities in Africa and particularly in Nigeria. Millions of people die of malaria every year. Mosquitoes are resistant to most vaccines we have today. It is important to develop preventives/methods to fight against malaria and mosquitoes in general.

Therefore, each of the two population compartments above is divided into classes below;

A(t) = Number of adult mosquitoes at time(t)

L(t) = Number of larvae at time(t)

P(t) = Number of pupae at time(t)
$C_P(t)$ = Number of natural predator for larva (Copepods)
$T_P(t)$ = Number of natural predator for pupa (Tadpoles)
$P_M(t)$ = Number of natural predator for purple martins (Purple martins)
$N_1 =$ Total population for mosquitoes at time t, $N_1 = A(t) + L(t) + P(t) + E(t)$
$N_2 =$ Total population for predator at time t, $N_2 = P_m(t) + C_P(t) + T_p(t)$
$N_2 =$ Total population at time t, $N(t) = N_1(t) + N_2(t)$

An adult female mosquito interact sexually with males or vice versa at a rate called the incidence rate, given by η. $b_1$is the natural birth rate of the adult class, β_1 is the induced mortality rate of copepods due to chemical and environmental conditions of the adult class and μ_1 is the natural mortality rate of the adult class. σ is the fraction in which the egg is harsh to larva, β_2 is the induced mortality rate of the egg due to the chemical and environmental conditions of the egg class, and μ_2 is the natural mortality rate of the compartment to eggs. λ is the fraction at which the larvae transform to pupate, β_3 is the induced mortality rate of the larvae due to the chemical and environmental conditions of the larval class, and μ_3 is the natural mortality rate of the larvae. $\boldsymbol{\pi}$ is the fraction at which the pupa transforms into an adult, β_3 is the induced death rate of the pupa due to chemical and environmental conditions of the pupal class, and μ_3 is the natural death rate of the pupal compartment. b_2 is the natural birth rate of the copepod class, β_6 is the induced mortality rate of copepods due to chemical and environmental conditions of the copepod class, μ_6 is the natural death rate of the copepod compartment and α is the probability at which mosquito larvae eaten by copepods. b_3 is the natural birth rate of the tadpole class, β_7 is the induced mortality rate of the tadpoles due to the chemical and environmental conditions of the tadpole class, μ_7 is the natural death rate of tadpoles compartment and ω is the probability at which adult mosquito are eaten up by purple martins. b_4 is the natural birth rate of purple martins class, β_5 is the induce death rate of purple martins due to chemical and environment conditions of purple martins class and μ_5 is the natural death rate of purple martins compartment and γ is the probability at which mosquito adult are eaten up by purple martins.

Model Variables and Parameters Defined
In table below, variables and parameters used in the new model are defined below

Table 1: Variables and Parameters Defined

Variables	**Description**
A(t)	Number of adult mosquitoes at time(t)
E (t)	Number of eggs at time(t)
L(t)	Number of larvae at time(t)
P(t)	Number of pupae at time(t)
N(t)	Total population
$\boldsymbol{C_P}$(t)	Number of natural Predator for larva at time(t) (Copepods)
T_p(t)	Number of natural Predator for pupa at time(t)(Tadpoles)
P_m(t)	Number of natural Predator for Adult at time(t) (Purple Martins)
Parameters	**Description**
b_1	Natural birth rate of adult class
b_2	Natural birth rate of copepods class
b_3	Natural birth rate of tadpoles' class
b_4	Natural birth rate of purple martins' class
μ_1	Natural death rate of adult class
μ_2	Natural death rate of egg class
μ_3	Natural death rate of larva class
μ_4	Natural death rate of pupa class
μ_5	Natural death rate of purple martins' class
μ_6	Natural death rate of copepods class
μ_7	Natural death rate of tadpoles' class
β_1	Induce death rate of adult due to chemical and environmental conditions
β_2	Induce death rate of egg due to chemical and environmental conditions
β_3	Induce death rate of larva due to chemical and environmental conditions
β_4	Induce death rate of pupa due to chemical and environmental conditions
β_5	Induce death rate of purple martins' due to chemical and environmental conditions

β_6	Induce death rate of copepods due to chemical and environmental conditions
β_7	Induce death rate of tadpoles' due to chemical and environmental conditions
η	The incidence rate (the rate at which adult mosquitoes oviposit)
σ	The proportion at which egg harsh to larva
λ	The proportion of larva that transform to pupa
π	The proportion of pupa that transform to adult
α	The probability at which mosquito larva are eaten up by copepods
ω	The probability at which mosquito pupa are eaten up by tadpoles
γ	The probability at which mosquito adult are eaten up by purple matins
C	The average temperature of the water culture
N_L	Number of larva been eaten up by copepods at time(t)
N_P	Number of pupa been eaten up by tadpoles at time(t)
N_A	Number of adult been eaten up by purple martins at time(t)

Model Assumptions

When formulating the model, the following assumptions were made

1) The total population of Anopheles mosquitoes consists of four populations such as egg, larva, pupa and adult.
2) The total population of natural predators consists of three populations such as copepods, tadpoles and purple martins.
3) The parasite of a mosquito, transmitted from one mosquito to another, is transmitted only through the host, this is called horizontal transmission.
4) Predators can consume infinite amounts of prey.
5) Emigration and immigration of the Anopheles mosquito population does not occur in this population; however, the population increases only by the natural birth rate and decreases only by the natural death rate and also due to environmental factors.
6) The prey population grows exponentially when the predator is absent.
7) The Anopheles mosquito is thought to transmit malaria only through direct contact.
8) The predator population will starve in the absence of the prey population

The following diagram describes the flux control of the predator-prey interaction; It will be useful in formulating models**.**

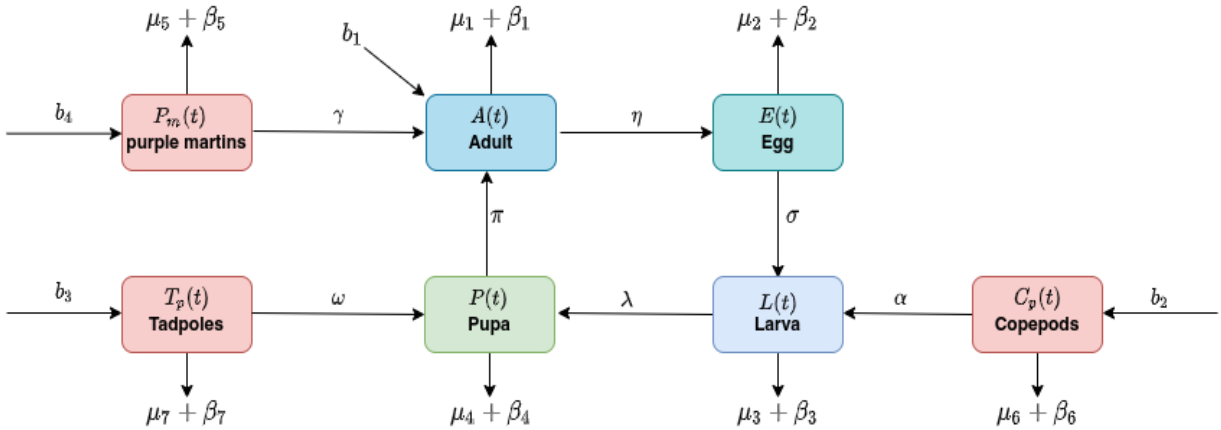


Figure 10: Flow Control Diagram of Predator-prey Interaction Model in Mosquito Life-Cycle

The Model Equations

From the above assumptions and flowchart, the following equations were derived

Model Equations for Mosquito Life-Cycle

$$\frac{dA}{dt} = b_1 + \gamma P_m(t) + \pi P(t) - (\mu_1 + \beta_1 + \eta)A(t) \quad \ldots \quad (1)$$

$$\frac{dE}{dt} = \eta A(t) - (\mu_2 + \beta_2 + \sigma)E(t) \quad \ldots \quad (2)$$

$$\frac{dL}{dt} = \sigma E(t) + \alpha C_P(t) - (\mu_3 + \beta_3 + \lambda)L(t) \quad \ldots \quad (3)$$

$$\frac{dP}{dt} = \lambda L(t) + \omega T_p(t) - (\mu_4 + \beta_4 + \pi)P(t) \quad \ldots \quad (4)$$

Model Equations for Natural Predators

$$\frac{dC_P}{dt} = b_2 - (\mu_6 + \beta_6 + \alpha)C_P(t) \quad \ldots (5)$$

$$\frac{dT_p}{dt} = b_3 - (\mu_7 + \beta_7 + \omega)T_p(t) \quad \ldots (6)$$

$$\frac{dP_m}{dt} = b_4 - (\mu_5 + \beta_5 + \gamma)P_m(t) \quad \ldots (7)$$

Model Equations for Total Population

$N_1 = A(t) + L(t) + P(t) + E(t)$

$N_2 = P_m(t) + C_P(t) + T_p(t)$

$N(t) = N_1(t) + N_2(t))$

$N(t) = A(t) + L(t) + P(t) + E(t) + P_m(t) + C_P(t) + T_p(t)$

$\frac{dN}{dt} = b_1 + \gamma P_m(t) + \pi P(t) - (\mu_1 + \beta_1 + \eta)A(t) + \eta A(t) - (\mu_2 + \beta_2 + \sigma)E(t) + \sigma E(t) + \alpha C_P(t) -$
$(\mu_3 + \beta_3 + \lambda)L(t) + \lambda L(t) + \omega T_p(t) - (\mu_4 + \beta_4 + \pi)P(t) + b_2 - (\mu_6 + \beta_6 + \alpha)C_P(t) + b_3 - (\mu_7 + \beta_7 +$
$\omega Tpt + b4 - \mu5 + \beta5 + \gamma Pmt \quad \ldots (8)$

Mosquito-Free Steady-State Stability Analysis Using Beltrami Conditions

The Beltrami conditions state that if the determinants of the Jacobian are greater than zero and the trace elements of the Jacobian are less than zero, then the mosquito-free equilibrium stability analysis model is stable; otherwise it is unstable. Before using the result to construct the mosquito-free equilibrium stability analysis E_P using Beltrami's conditions, the following theorems are given without proof.

Theorem 4.1

Let $\mathcal{R}$ be a commutative sub ring of ${}^{n}F^{n}$, where F is a field (or a commutative ring) and $M \in {}^{n}F^{n}$.

$LET\ M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$, where A, B, C, D are $n \times n$ block matrices over F, so that $M \in {}^{2n}F^{2n}$. Suppose that C = 0, the $n \times n$ zero matrices, then

(a). $det_F M = det_F \begin{pmatrix} A & B \\ O & D \end{pmatrix} = det_F AD = det_F A.det_F D$

(b). $Trace_F M = Trace_F \begin{pmatrix} A & B \\ 0 & D \end{pmatrix} = Trace_F(A + D) = Trace_F(A) + Trace_F(D)$

Proof: (See Silvester, 2000).

Theorem 4.2

The eigenvalues $\lambda_{1,2}$ of a 2 by 2 matrix satisfy J satisfy $Re\lambda_{1,2} < 0$ if and only if $Det(J) > 0 and\ Trace(J) < 0$. they are pure imaginary if and only if $Trace(J) = 0$. moreover the eigenvalues fulfills the following conditions $\lambda_1 < 0 < \lambda_2\ or\ \lambda_2 < 0 < \lambda_1\ if\ and\ only\ if\ Det(J) < 0$

Proof: (See page 255 of Geland, 2012).

Theorem 4.3

Let A be an $n \times n$ matrix with n distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$, and J(A) the Jacobian matrix of A evaluated at equilibrium state E_P

(i). If the eigenvalues of the Jacobian Matrix J(A) all have real parts less than zero, then the equilibrium state is stable or predictable

(ii). If at least one of the eigenvalues of the Jacobian Matrix J(A) has real parts less than zero, then the equilibrium state is unstable or uncertain

Proof: (See Thomas, 2008).

Equations of the Model Associated with Equations (1) – (8)

To study the behavior of the system (1)-(8) around the mosquitoes free equilibrium state $Let\ E_P = (A_0, E_0, L_0, P_0, C_{PO}, T_{PO}, P_{MO}\ and\ N_0)$ be the equilibrium points of the model, we resort to the linearized stability approach.

Let

$$f_1 = b_1 + \gamma P_m(t) + \pi P(t) - (\mu_1 + \beta_1 + \eta)A(t) \quad \ldots (9)$$

$$f_2 = \eta A(t) - (\mu_2 + \beta_2 + \sigma)E(t) \quad \ldots (10)$$

$$f_3 = \sigma E(t) + \alpha C_P(t) - (\mu_3 + \beta_3 + \lambda)L(t) \quad \ldots (11)$$

$$f_4 = \lambda L(t) + \omega T_p(t) - (\mu_4 + \beta_4 + \pi)P(t) \quad \ldots (12)$$

$$f_5 = b_2 - (\mu_6 + \beta_6 + \alpha)C_P(t) \quad \ldots (13)$$

$$f_6 = b_3 - (\mu_7 + \beta_7 + \omega)T_p(t) \quad \ldots (14)$$

$$f_7 = b_4 - (\mu_5 + \beta_5 + \gamma)P_m(t) \quad \ldots \quad (15)$$

$$f_8 = b_1 + \gamma P_m(t) + \pi P(t) - (\mu_1 + \beta_1 + \eta)A(t) + \eta A(t) - (\mu_2 + \beta_2 + \sigma)E(t) + \sigma E(t) + \alpha C_P(t) - (\mu_3 + \beta_3 + \lambda)L(t) + \lambda L(t) + \omega T_p(t) - (\mu_4 + \beta_4 + \pi)P(t) + b_2 - (\mu_6 + \beta_6 + \alpha)C_P(t) + b_3 - (\mu_7 + \beta_7 + \omega)T_p(t) + b_4 - (\mu_5 + \beta_5 + \gamma)P_m(t) \quad \ldots \quad (16)$$

Then

$$f_1 = b_1 + \gamma P_m(t) + \pi P(t) - (\mu_1 + \beta_1 + \eta)A(t)$$

$$\frac{\partial f_1}{\partial A} = -(\mu_1 + \beta_1 + \eta), \frac{\partial f_1}{\partial E} = 0, \frac{\partial f_1}{\partial L} = 0, \frac{\partial f_1}{\partial P} = \pi, \frac{\partial f_1}{\partial C_P} = 0, \frac{\partial f_1}{\partial T_P} = 0, \frac{\partial f_1}{\partial P_M} = \gamma, \frac{\partial f_1}{\partial N} = 0 \quad \ldots \quad (17)$$

$$f_2 = \eta A(t) - (\mu_2 + \beta_2 + \sigma)E(t)$$

$$\frac{\partial f_2}{\partial A} = \eta, \frac{\partial f_2}{\partial E} = -(\mu_2 + \beta_2 + \sigma), \frac{\partial f_2}{\partial L} = 0, \frac{\partial f_2}{\partial P} = 0, \frac{\partial f_2}{\partial C_P} = 0, \frac{\partial f_2}{\partial T_P} = 0, \frac{\partial f_2}{\partial P_M} = 0, \frac{\partial f_2}{\partial N} = 0 \quad \ldots \quad (18)$$

$$f_3 = \sigma E(t) + \alpha C_P(t) - (\mu_3 + \beta_3 + \lambda)L(t)$$

$$\frac{\partial f_3}{\partial A} = 0, \frac{\partial f_3}{\partial E} = \sigma, \frac{\partial f_3}{\partial L} = -(\mu_3 + \beta_3 + \lambda), \frac{\partial f_3}{\partial P} = 0, \frac{\partial f_3}{\partial C_P} = \alpha, \frac{\partial f_3}{\partial T_P} = 0, \frac{\partial f_3}{\partial P_M} = 0, \frac{\partial f_3}{\partial N} = 0 \quad \ldots \quad (19)$$

$$f_4 = \lambda L(t) + \omega T_p(t) - (\mu_4 + \beta_4 + \pi)P(t)$$

$$\frac{\partial f_4}{\partial A} = 0, \quad \frac{\partial f_4}{\partial E} = 0, \frac{\partial f_4}{\partial L} = \lambda, \quad \frac{\partial f_4}{\partial P} = -(\mu_4 + \beta_4 + \pi), \frac{\partial f_4}{\partial C_P} = 0, \frac{\partial f_4}{\partial T_P} = \omega, \frac{\partial f_4}{\partial P_M} = 0, \frac{\partial f_4}{\partial N} = 0 \quad \ldots \quad (20)$$

$$f_5 = b_2 - (\mu_6 + \beta_6 + \alpha)C_P(t)$$

$$\frac{\partial f_5}{\partial A} = 0, \frac{\partial f_5}{\partial E} = 0, \frac{\partial f_5}{\partial L} = 0, \frac{\partial f_5}{\partial P} = 0, \frac{\partial f_5}{\partial C_P} = -(\mu_6 + \beta_6 + \alpha), \frac{\partial f_5}{\partial T_P} = 0, \frac{\partial f_5}{\partial P_M} = 0, \frac{\partial f_5}{\partial N} = 0 \quad \ldots \quad (21)$$

$$f_6 = b_3 - (\mu_7 + \beta_7 + \omega)T_p(t)$$

$$\frac{\partial f_6}{\partial A} = 0, \frac{\partial f_6}{\partial E} = 0, \frac{\partial f_6}{\partial L} = 0, \frac{\partial f_6}{\partial P} = 0, \frac{\partial f_6}{\partial C_P} = 0, \frac{\partial f_6}{\partial T_P} = -(\mu_7 + \beta_7 + \omega), \frac{\partial f_6}{\partial P_M} = 0, \frac{\partial f_6}{\partial N} = 0 \quad \ldots \quad (22)$$

$$f_7 = b_4 - (\mu_5 + \beta_5 + \gamma)P_m(t)$$

$$\frac{\partial f_7}{\partial A} = 0, \frac{\partial f_7}{\partial E} = 0, \frac{\partial f_7}{\partial L} = 0, \frac{\partial f_7}{\partial P} = 0, \frac{\partial f_7}{\partial C_P} = 0, \frac{\partial f_7}{\partial T_P} = 0, \frac{\partial f_7}{\partial P_M} = -(\mu_5 + \beta_5 + \gamma), \frac{\partial f_7}{\partial N} = 0 \quad \ldots \quad (23)$$

$$f_8 = b_1 - (\mu_1 + \beta_1 + \eta)A(t) + \eta A(t) - (\mu_2 + \beta_2 + \sigma)E(t) + \sigma E(t) - (\mu_3 + \beta_3 + \lambda)L(t) + \lambda L(t) + \pi P(t) - (\mu_4 + \beta_4 + \pi)P(t) + b_2 - (\mu_6 + \beta_6 + \alpha)C_P(t) + \alpha C_P(t) + b_3 - (\mu_7 + \beta_7 + \omega)T_p(t) + \omega T_p(t) + b_4 - (\mu_5 + \beta_5 + \gamma)P_m(t) + \gamma P_m(t)$$

$$\frac{\partial f_8}{\partial A} = -(\mu_1 + \beta_1), \frac{\partial f_8}{\partial E} = -(\mu_2 + \beta_2), \frac{\partial f_8}{\partial L} = -(\mu_3 + \beta_3), \frac{\partial f_8}{\partial P} = -(\mu_4 + \beta_4), \frac{\partial f_8}{\partial C_P} = -(\mu_6 + \beta_6), \frac{\partial f_8}{\partial T_P} = -(\mu_7 + \beta_7), \frac{\partial f_7}{\partial P_M} = -(\mu_5 + \beta_5), \frac{\partial f_8}{\partial N} = 0 \quad \ldots \quad (24)$$

Jacobian Matrix (J) Associated with Model Equations (9) – (16)

Theorem 4.4

The mosquito's free equilibrium state of the model (9) – (16) is locally asymptotically stable if $R_o < 1$ and the following threshold conditions hold $(i).R_1 < 1\ (ii).R_2 < 1\ (i).R_3 < 1$, otherwise E_P is unstable.

Proof: The Jacobian matrix of the system is given below

$$J = \begin{pmatrix} -(\mu_1+\beta_1+\eta) & 0 & 0 & \pi & 0 & 0 & \gamma & 0 \\ 0 & -(\mu_2+\beta_2+\sigma) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma & -(\mu_3+\beta_3+\lambda) & \alpha & 0 & 0 & 0 \\ 0 & 0 & \lambda & -(\mu_4+\beta_4+\pi) & 0 & \omega & 0 & 0 \\ 0 & 0 & 0 & 0 & -(\mu_6+\beta_6+\alpha) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -(\mu_7+\beta_7+\omega) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -(\mu_5+\beta_5+\gamma) & 0 \\ -(\mu_1+\beta_1) & -(\mu_2+\beta_2) & -(\mu_3+\beta_3) & -(\mu_4+\beta_4) & -(\mu_6+\beta_6) & -(\mu_7+\beta_7) & -(\mu_5+\beta_5) & 0 \end{pmatrix} \quad \ldots \quad (4.4.1)$$

If the Jacobian is evaluated in the free equilibrium state of the mosquito, then the criterion required for a stable equilibrium (by Theorem 4.2 and Theorem 4.3) is that the determinant of the Jacobian be positive and the trace of the Jacobian be negative .

$$Det(J)$$

$$= \begin{pmatrix} -(\mu_1+\beta_1+\eta) & 0 & 0 & \pi & 0 & 0 & \gamma & 0 \\ 0 & -(\mu_2+\beta_2+\sigma) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma & -(\mu_3+\beta_3+\lambda) & \alpha & 0 & 0 & 0 \\ 0 & 0 & \lambda & -(\mu_4+\beta_4+\pi) & 0 & \omega & 0 & 0 \\ 0 & 0 & 0 & 0 & -(\mu_6+\beta_6+\alpha) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -(\mu_7+\beta_7+\omega) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -(\mu_5+\beta_5+\gamma) & 0 \\ -(\mu_1+\beta_1) & -(\mu_2+\beta_2) & -(\mu_3+\beta_3) & -(\mu_4+\beta_4) & -(\mu_6+\beta_6) & -(\mu_7+\beta_7) & -(\mu_5+\beta_5) & 0 \end{pmatrix} \quad \ldots \quad (4.4.2)$$

Using the result of Theorem 4.1 above, we partition the matrix (Z) represented in equation (4.4.2) above as follows:

$$Z = \begin{pmatrix} A & \cdots & B \\ \vdots & \ddots & \vdots \\ C & \cdots & D \end{pmatrix} \quad \ldots \ (4.4.3)$$

where A, B, C and D are block matrices defined as follows

$$\boldsymbol{Det(J)} = \begin{pmatrix} -(\mu_1+\beta_1+\eta) & 0 & 0 & \pi & 0 & 0 & \gamma & 0 \\ 0 & -(\mu_2+\beta_2+\sigma) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma & -(\mu_3+\beta_3+\lambda) & \alpha & 0 & 0 & 0 \\ 0 & 0 & \lambda & -(\mu_4+\beta_4+\pi) & 0 & \omega & 0 & 0 \\ 0 & 0 & 0 & 0 & -(\mu_6+\beta_6+\alpha) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -(\mu_7+\beta_7+\omega) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -(\mu_5+\beta_5+\gamma) & 0 \\ -(\mu_1+\beta_1) & -(\mu_2+\beta_2) & -(\mu_3+\beta_3) & -(\mu_4+\beta_4) & -(\mu_6+\beta_6) & -(\mu_7+\beta_7) & -(\mu_5+\beta_5) & 0 \end{pmatrix} \quad \ldots \ (4.4.4)$$

$$\boldsymbol{A} = \begin{bmatrix} -(\mu_1+\beta_1+\eta) & 0 & 0 & \pi \\ 0 & -(\mu_2+\beta_2+\sigma) & 0 & 0 \\ 0 & 0 & \sigma & -(\mu_3+\beta_3+\lambda) \\ 0 & 0 & \lambda & -(\mu_4+\beta_4+\pi) \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & 0 \\ \sigma & 0 & 0 & 0 \\ 0 & \omega & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{C} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -(\mu_1+\beta_1) & -(\mu_2+\beta_2) & -(\mu_3+\beta_3) & -(\mu_4+\beta_4) \end{bmatrix} \boldsymbol{and\ D}$$

$$= \begin{bmatrix} -(\mu_6+\beta_6+\alpha) & 0 & 0 & 0 \\ 0 & -(\mu_7+\beta_7+\omega) & 0 & 0 \\ 0 & 0 & -(\mu_5+\beta_5+\gamma) & 0 \\ -(\mu_6+\beta_6) & -(\mu_7+\beta_7) & -(\mu_5+\beta_5) & 0 \end{bmatrix} \quad \ldots (4.4.5)$$

The determinant of a matrix is a scalar or numeric value associated with any square matrix, which can be a real or complex number, positive, negative or zero. The determinant is usually denoted by det(A) or |A|.

We have that the determinant of the Jacobian matrix (J) is given by

$$\boldsymbol{A} = \begin{bmatrix} -(\mu_1+\beta_1+\eta) & 0 & 0 & \pi \\ 0 & -(\mu_2+\beta_2+\sigma) & 0 & 0 \\ 0 & 0 & \sigma & -(\mu_3+\beta_3+\lambda) \\ 0 & 0 & \lambda & -(\mu_4+\beta_4+\pi) \end{bmatrix} \quad \ldots \ (4.4.6)$$

$$\boldsymbol{Det(A)} = -(\mu_1+\beta_1+\eta)\begin{bmatrix} -(\mu_2+\beta_2+\sigma) & 0 & 0 \\ 0 & \sigma & -(\mu_3+\beta_3+\lambda) \\ 0 & \lambda & -(\mu_4+\beta_4+\pi) \end{bmatrix} - 0\begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma & -(\mu_3+\beta_3+\lambda) \\ 0 & 0 & -(\mu_4+\beta_4+\pi) \end{bmatrix}$$

$$+ 0\begin{bmatrix} 0 & -(\mu_2+\beta_2+\sigma) & 0 \\ 0 & 0 & -(\mu_3+\beta_3+\lambda) \\ 0 & 0 & -(\mu_4+\beta_4+\pi) \end{bmatrix} - 0\begin{bmatrix} 0 & -(\mu_2+\beta_2+\sigma) & 0 \\ 0 & 0 & \sigma \\ 0 & 0 & \lambda \end{bmatrix}$$

$$\boldsymbol{Det(A)} = -(\mu_1+\beta_1+\eta)\begin{bmatrix} -(\mu_2+\beta_2+\sigma) & 0 & 0 \\ 0 & \sigma & -(\mu_3+\beta_3+\lambda) \\ 0 & \lambda & -(\mu_4+\beta_4+\pi) \end{bmatrix}$$

$$\boldsymbol{Det(A)} = -(\mu_1+\beta_1+\eta)\left\{-(\mu_2+\beta_2+\sigma)\begin{bmatrix} \sigma & -(\mu_3+\beta_3+\lambda) \\ \lambda & -(\mu_4+\beta_4+\pi) \end{bmatrix} - 0\begin{bmatrix} 0 & -(\mu_3+\beta_3+\lambda) \\ 0 & -(\mu_4+\beta_4+\pi) \end{bmatrix} + 0\begin{bmatrix} 0 & \sigma \\ 0 & \lambda \end{bmatrix}\right\}$$

$$\boldsymbol{Det(A)} = -(\mu_1+\beta_1+\eta)\left\{-(\mu_2+\beta_2+\sigma)\begin{bmatrix} \sigma & -(\mu_3+\beta_3+\lambda) \\ \lambda & -(\mu_4+\beta_4+\pi) \end{bmatrix}\right\}$$

$$\boldsymbol{Det(A)} = -(\mu_1+\beta_1+\eta)\{-(\mu_2+\beta_2+\sigma)[-(\mu_4+\beta_4+\pi)\sigma + (\mu_3+\beta_3+\lambda)\lambda]\}$$

$$\boldsymbol{Det(A)} = -(\mu_1+\beta_1+\eta)\{(\mu_2+\beta_2+\sigma)(\mu_4+\beta_4+\pi)\sigma - (\mu_2+\beta_2+\sigma)(\mu_3+\beta_3+\lambda)\lambda\}$$

$$\boldsymbol{Det(A)} = \{-(\mu_1+\beta_1+\eta)(\mu_2+\beta_2+\sigma)(\mu_4+\beta_4+\pi)\sigma - (\mu_1+\beta_1+\eta)(\mu_2+\beta_2+\sigma)(\mu_3+\beta_3+\lambda)\lambda\}$$

$$\boldsymbol{Det(A)} = -\{(\mu_1+\beta_1+\eta)(\mu_2+\beta_2+\sigma)(\mu_3+\beta_3+\lambda)\lambda + (\mu_1+\beta_1+\eta)(\mu_2+\beta_2+\sigma)(\mu_4+\beta_4+\pi)\sigma\}$$

$$\boldsymbol{Det(A)} = -(\mu_1+\beta_1+\eta)(\mu_2+\beta_2+\sigma)\{(\mu_3+\beta_3+\lambda)\lambda + (\mu_4+\beta_4+\pi)\sigma\}$$

$$\boldsymbol{B} = \begin{bmatrix} 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & 0 \\ \sigma & 0 & 0 & 0 \\ 0 & \omega & 0 & 0 \end{bmatrix} \quad \ldots \ (4.4.7)$$

$$\boldsymbol{Det(B)} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \boldsymbol{\gamma} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{\sigma} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\omega} & \mathbf{0} & \mathbf{0} \end{bmatrix} = \mathbf{0}$$

$$\boldsymbol{C} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -(\boldsymbol{\mu_1} + \boldsymbol{\beta_1}) & -(\boldsymbol{\mu_2} + \boldsymbol{\beta_2}) & -(\boldsymbol{\mu_3} + \boldsymbol{\beta_3}) & -(\boldsymbol{\mu_4} + \boldsymbol{\beta_4}) \end{bmatrix} \quad \ldots (\mathbf{4.4.8})$$

$$\boldsymbol{Det(C)} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -(\boldsymbol{\mu_1} + \boldsymbol{\beta_1}) & -(\boldsymbol{\mu_2} + \boldsymbol{\beta_2}) & -(\boldsymbol{\mu_3} + \boldsymbol{\beta_3}) & -(\boldsymbol{\mu_4} + \boldsymbol{\beta_4}) \end{bmatrix} = \mathbf{0}$$

$$\boldsymbol{D} = \begin{bmatrix} -(\boldsymbol{\mu_6} + \boldsymbol{\beta_6} + \boldsymbol{\alpha}) & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -(\boldsymbol{\mu_7} + \boldsymbol{\beta_7} + \boldsymbol{\omega}) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -(\boldsymbol{\mu_5} + \boldsymbol{\beta_5} + \boldsymbol{\gamma}) & \mathbf{0} \\ -(\boldsymbol{\mu_6} + \boldsymbol{\beta_6}) & -(\boldsymbol{\mu_7} + \boldsymbol{\beta_7}) & -(\boldsymbol{\mu_5} + \boldsymbol{\beta_5}) & \mathbf{0} \end{bmatrix} \quad \ldots (\mathbf{4.4.9})$$

$$\boldsymbol{Det(D)} = -(\boldsymbol{\mu_6} + \boldsymbol{\beta_6} + \boldsymbol{\alpha}) \begin{bmatrix} -(\boldsymbol{\mu_7} + \boldsymbol{\beta_7} + \boldsymbol{\omega}) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -(\boldsymbol{\mu_5} + \boldsymbol{\beta_5} + \boldsymbol{\gamma}) & \mathbf{0} \\ -(\boldsymbol{\mu_7} + \boldsymbol{\beta_7}) & -(\boldsymbol{\mu_5} + \boldsymbol{\beta_5}) & \mathbf{0} \end{bmatrix}$$
$$- \mathbf{0} \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -(\boldsymbol{\mu_5} + \boldsymbol{\beta_5} + \boldsymbol{\gamma}) & \mathbf{0} \\ -(\boldsymbol{\mu_6} + \boldsymbol{\beta_6}) & -(\boldsymbol{\mu_5} + \boldsymbol{\beta_5}) & \mathbf{0} \end{bmatrix} + \mathbf{0} \begin{bmatrix} \mathbf{0} & -(\boldsymbol{\mu_7} + \boldsymbol{\beta_7} + \boldsymbol{\omega}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -(\boldsymbol{\mu_6} + \boldsymbol{\beta_6}) & -(\boldsymbol{\mu_7} + \boldsymbol{\beta_7}) & \mathbf{0} \end{bmatrix}$$
$$- \mathbf{0} \begin{bmatrix} \mathbf{0} & -(\boldsymbol{\mu_7} + \boldsymbol{\beta_7} + \boldsymbol{\omega}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -(\boldsymbol{\mu_5} + \boldsymbol{\beta_5} + \boldsymbol{\gamma}) \\ -(\boldsymbol{\mu_6} + \boldsymbol{\beta_6}) & -(\boldsymbol{\mu_7} + \boldsymbol{\beta_7}) & -(\boldsymbol{\mu_5} + \boldsymbol{\beta_5}) \end{bmatrix}$$

$$\boldsymbol{Det(D)} = -(\boldsymbol{\mu_6} + \boldsymbol{\beta_6} + \boldsymbol{\alpha}) \begin{bmatrix} -(\boldsymbol{\mu_7} + \boldsymbol{\beta_7} + \boldsymbol{\omega}) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -(\boldsymbol{\mu_5} + \boldsymbol{\beta_5} + \boldsymbol{\gamma}) & \mathbf{0} \\ -(\boldsymbol{\mu_7} + \boldsymbol{\beta_7}) & -(\boldsymbol{\mu_5} + \boldsymbol{\beta_5}) & \mathbf{0} \end{bmatrix}$$

$$\boldsymbol{Det(D)} = -(\boldsymbol{\mu_6} + \boldsymbol{\beta_6} + \boldsymbol{\alpha}) \left\{ -(\boldsymbol{\mu_7} + \boldsymbol{\beta_7} + \boldsymbol{\omega}) \begin{bmatrix} -(\boldsymbol{\mu_5} + \boldsymbol{\beta_5} + \boldsymbol{\gamma}) & \mathbf{0} \\ -(\boldsymbol{\mu_5} + \boldsymbol{\beta_5}) & \boldsymbol{\lambda} \end{bmatrix} - \mathbf{0} \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ -(\boldsymbol{\mu_7} + \boldsymbol{\beta_7}) & \mathbf{0} \end{bmatrix} \right.$$
$$\left. - \mathbf{0} \begin{bmatrix} \mathbf{0} & -(\boldsymbol{\mu_5} + \boldsymbol{\beta_5} + \boldsymbol{\gamma}) \\ -(\boldsymbol{\mu_7} + \boldsymbol{\beta_7}) & -(\boldsymbol{\mu_5} + \boldsymbol{\beta_5}) \end{bmatrix} \right\}$$

$$\boldsymbol{Det(D)} = -(\boldsymbol{\mu_6} + \boldsymbol{\beta_6} + \boldsymbol{\alpha}) \left\{ -(\boldsymbol{\mu_7} + \boldsymbol{\beta_7} + \boldsymbol{\omega}) \begin{bmatrix} -(\boldsymbol{\mu_5} + \boldsymbol{\beta_5} + \boldsymbol{\gamma}) & \mathbf{0} \\ -(\boldsymbol{\mu_5} + \boldsymbol{\beta_5}) & \boldsymbol{\lambda} \end{bmatrix} \right\}$$

$$\boldsymbol{Det(D)} = -(\boldsymbol{\mu_6} + \boldsymbol{\beta_6} + \boldsymbol{\alpha}) \{ (\boldsymbol{\mu_7} + \boldsymbol{\beta_7} + \boldsymbol{\omega})(\boldsymbol{\mu_5} + \boldsymbol{\beta_5} + \boldsymbol{\gamma})\boldsymbol{\lambda} \}$$

$$\boldsymbol{Det(D)} = -(\boldsymbol{\mu_6} + \boldsymbol{\beta_6} + \boldsymbol{\alpha})(\boldsymbol{\mu_7} + \boldsymbol{\beta_7} + \boldsymbol{\omega})(\boldsymbol{\mu_5} + \boldsymbol{\beta_5} + \boldsymbol{\gamma})\boldsymbol{\lambda}$$

$$\boldsymbol{Det(J)} = \boldsymbol{detA.detD}$$
$$= -\{(\boldsymbol{\mu_1} + \boldsymbol{\beta_1} + \eta)(\boldsymbol{\mu_2} + \boldsymbol{\beta_2} + \boldsymbol{\sigma})(\boldsymbol{\mu_3} + \boldsymbol{\beta_3} + \boldsymbol{\lambda})\boldsymbol{\lambda}$$
$$+ (\boldsymbol{\mu_1} + \boldsymbol{\beta_1} + \eta)(\boldsymbol{\mu_2} + \boldsymbol{\beta_2} + \boldsymbol{\sigma})(\boldsymbol{\mu_4} + \boldsymbol{\beta_4} + \boldsymbol{\pi})\boldsymbol{\sigma}\}\{-(\boldsymbol{\mu_6} + \boldsymbol{\beta_6} + \boldsymbol{\alpha})(\boldsymbol{\mu_7} + \boldsymbol{\beta_7} + \boldsymbol{\omega})(\boldsymbol{\mu_5} + \boldsymbol{\beta_5}$$
$$+ \boldsymbol{\gamma})\boldsymbol{\lambda}\}$$

$$\boldsymbol{Det(J)} = \boldsymbol{detA.detD}$$
$$= \{(\boldsymbol{\mu_1} + \boldsymbol{\beta_1} + \eta)(\boldsymbol{\mu_2} + \boldsymbol{\beta_2} + \boldsymbol{\sigma})(\boldsymbol{\mu_3} + \boldsymbol{\beta_3} + \boldsymbol{\lambda})\boldsymbol{\lambda}$$
$$+ (\boldsymbol{\mu_1} + \boldsymbol{\beta_1} + \eta)(\boldsymbol{\mu_2} + \boldsymbol{\beta_2} + \boldsymbol{\sigma})(\boldsymbol{\mu_4} + \boldsymbol{\beta_4} + \boldsymbol{\pi})\boldsymbol{\sigma}\}\{(\boldsymbol{\mu_6} + \boldsymbol{\beta_6} + \boldsymbol{\alpha})(\boldsymbol{\mu_7} + \boldsymbol{\beta_7} + \boldsymbol{\omega})(\boldsymbol{\mu_5} + \boldsymbol{\beta_5}$$
$$+ \boldsymbol{\gamma})\boldsymbol{\lambda}\} > 0$$

Similarly, the Trace of the Jacobian Matrix (J) is given by

$$Trace(J) = Trace \begin{pmatrix} A & 0 \\ 0 & D \end{pmatrix} = Trace(A + D) = Trace(A) + Trace(D)$$

$$\boldsymbol{A} = \begin{bmatrix} -(\boldsymbol{\mu_1} + \boldsymbol{\beta_1} + \eta) & \mathbf{0} & \mathbf{0} & \boldsymbol{\pi} \\ \mathbf{0} & -(\boldsymbol{\mu_2} + \boldsymbol{\beta_2} + \boldsymbol{\sigma}) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\sigma} & -(\boldsymbol{\mu_3} + \boldsymbol{\beta_3} + \boldsymbol{\lambda}) \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\lambda} & -(\boldsymbol{\mu_4} + \boldsymbol{\beta_4} + \boldsymbol{\pi}) \end{bmatrix}$$

$$\boldsymbol{Trace(A)} = -(\boldsymbol{\mu_1} + \boldsymbol{\beta_1} + \eta) - (\boldsymbol{\mu_2} + \boldsymbol{\beta_2} + \boldsymbol{\sigma}) - (\boldsymbol{\mu_4} + \boldsymbol{\beta_4} + \boldsymbol{\pi}) + \boldsymbol{\sigma}$$

$$\boldsymbol{Trace(A)} = -\{(\boldsymbol{\mu_1} + \boldsymbol{\beta_1} + \eta) + (\boldsymbol{\mu_2} + \boldsymbol{\beta_2}) + (\boldsymbol{\mu_4} + \boldsymbol{\beta_4} + \boldsymbol{\pi})\} < 0$$

$$\boldsymbol{D} = \begin{bmatrix} -(\boldsymbol{\mu_6}+\boldsymbol{\beta_6}+\boldsymbol{\alpha}) & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -(\boldsymbol{\mu_7}+\boldsymbol{\beta_7}+\boldsymbol{\omega}) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -(\boldsymbol{\mu_5}+\boldsymbol{\beta_5}+\boldsymbol{\gamma}) & \mathbf{0} \\ -(\boldsymbol{\mu_6}+\boldsymbol{\beta_6}) & -(\boldsymbol{\mu_7}+\boldsymbol{\beta_7}) & -(\boldsymbol{\mu_5}+\boldsymbol{\beta_5}) & \mathbf{0} \end{bmatrix}$$

$$Trace(D) = -(\mu_6+\beta_6+\alpha)-(\mu_7+\beta_7+\omega)-(\mu_5+\beta_5+\gamma)$$

$$Trace(D) = -\{(\mu_6+\beta_6+\alpha)+(\mu_7+\beta_7+\omega)+(\mu_5+\beta_5+\gamma)\}$$

$$Trace(J) = Trace(A)+Trace(D)$$
$$= -\{(\mu_2+\beta_2)+(\mu_4+\beta_4+\pi)\}-\{(\mu_6+\beta_6+\alpha)+(\mu_7+\beta_7+\omega)+(\mu_5+\beta_5+\gamma)\}$$

$$Trace(J) = Trace(A)+Trace(D)$$
$$= -\{(\mu_2+\beta_2)+(\mu_4+\beta_4+\pi)+(\mu_6+\beta_6+\alpha)+(\mu_7+\beta_7+\omega)+(\mu_5+\beta_5+\gamma)\} < 0$$

Result for Diekmann Conditions

Since $R_0 < 1$ under Diekmann's conditions, the stability analysis of the free equilibrium state is stable. Since $\alpha = \frac{N_L(t)}{L}, \omega = \frac{N_P(t)}{P}\ and\ \gamma = \frac{N_A(t)}{A}$, With natural implication, it means that the rate at which the proportion of mosquito larvae turns into pupae and pupae into adults is low, almost equal to zero, there will be no adult Anopheles mosquito for malaria transmission in our society when more natural predators are employed to feed on larvae, pupae and adult mosquito.

List of Numerical Experiments of the Model

The following experiments are carried out

Experiment 1: Effect of introducing one natural predator, copepod on mosquitoes' larva ($C_p = 500\ T_p = 0, and\ \ P_m = 0$).

Experiment 2: Effect of introducing two natural predators, copepod and tadpole on mosquitoes' larva and pupa respectively ($C_p = 500, T_p = 500\ and P_m = 0$).

Experiment 3: Effect of introducing three natural predators, copepod, tadpole and purple martins on mosquitoes' larva, pupa and adult respectively ($C_p = 500, T_p = 500\ and\ P_m = 130$).

Experiment 4: Comparison of the effect of introducing one, two and three natural predator on larva.

Experiment 5: Comparison of the effect of introducing two and three natural predator on pupa.

Experiment 6: Effect of introducing one natural predator, tadpole on mosquitoes' pupa($T_p = 500$).

Experiment 7: Effect of introducing two natural predators, tadpole and purple martins on mosquitoes' pupa and adult respectively ($C_p = 0, T_p = 500\ and\ P_m = 130$).

Experiment 8: Comparison of the effect of introducing one, two and three natural predator on pupa.

Experiment 9: Comparison of the effect of introducing two and three natural predator on adult.

Experiment 10: Effect of introducing one natural predator, purple martins on mosquitoes' adult ($P_m = 130, C_p = 0, and\ T_p = 0$).

Table 2: Numerical values of the variables and parameters

Variables/Parameters	Values	Source
A(t)	500	Assumed
E (t)	100000	Guerra, (2014)
L(t)	90000	Assumed
P(t)	80000	Assumed
N(t)	270000	Assumed
$\mathbf{C_P}$(t)	500	Practical
T_p(t)	500	Practical
P_m(t)	130	Assumed
b_1	0.02	Olivier, (202)
b_2	0.21	Gearty, (2021)
b_3	0.9	Calef, (1973)
b_4	0.5	Joshua,(1971)
μ_1	0.4	Mathews, (2020)
μ_2	0.3	Clements, (1981)
μ_3	0.2	Couret, (2014)
μ_4	0.1	Mondragon, (2020)

μ_5	0.5	Jervis, (2019)
μ_6	0.02	Charyl, (2011)
μ_7	0.01	Szekely, (2022)
β_1	40^oC(0.3)	Beck-Johnson,, (2013)
β_2	37^oC(0.57)	Sukiato, (2019)
β_3	28^oC(0.0110)	Adam, (2014)
β_4	28^oC(0.0110)	Adam, (2014)
β_5	25^oC (0.13)	Fred, (2014)
β_6	40^oC(0.01)	Jiang, (2014)
β_7	35^oC(0.02)	Halsbank-Lenk,(2014)
η	0.002	Practical
σ	0.00004	Practical
λ	0.00005	Practical
π π	0.01	Practical
α	0.5	Practical
ω	0.5	Practical
γ	0.9	Practical

Experiment 1: Effect of introducing one natural predator, copepod on mosquitoes' larva.

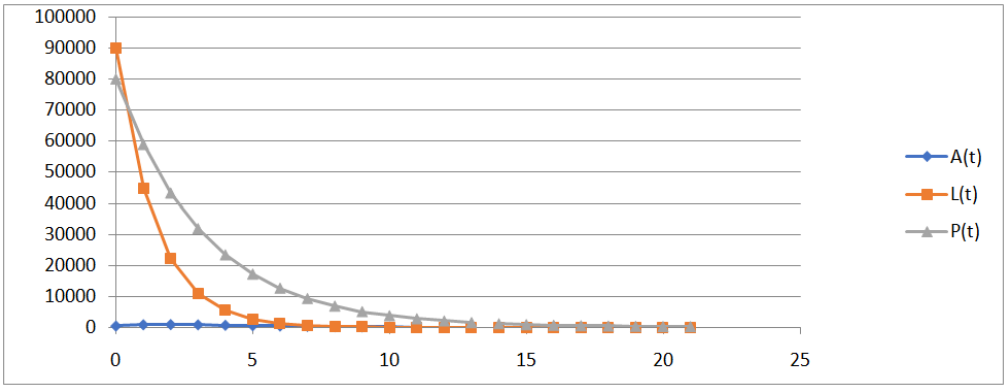


Figure 11: Number of mosquitoes' larva when one natural predator, copepod was introduced ($C_p = 500, T_p = 0, P_m = 0, \alpha = 0.5,\ \mu_6 = 0.02,\ \beta_6 = 0.01\ and\ b_2 = 0.21$).

Experiment 2: Effect of introducing two natural predators, copepod and tadpole on mosquitoes' larva and pupa respectively.

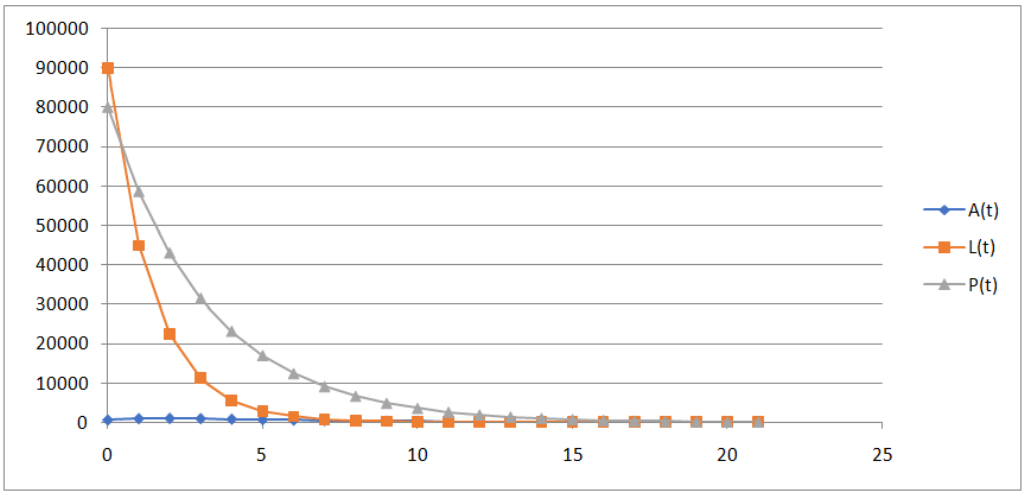


Figure 12: Number of mosquitoes' larva and pupa when two natural predators, copepod and tadpole are introduced respectively($C_p = 500, T_p = 500,\ P_m = 0,\ \alpha = 0.5,\ \mu_6 = 0.02,\ \beta_6 = 0.01,\ b_2 = 0.21, \omega = 0.5, \mu_7 = 0.01,\ \beta_7 = 0.02\ and\ b_3 = 0.9$)

Experiment 3: Effect of introducing three natural predators, copepod, tadpole and purple martins on mosquitoes' larva, pupa and adult.

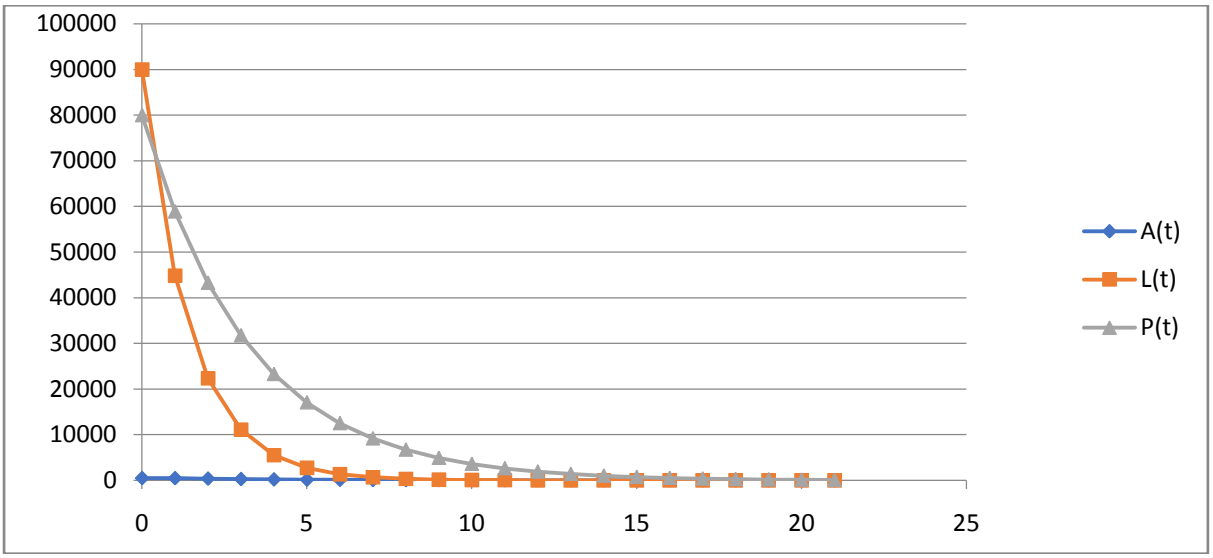


Figure 13: Number of mosquitoes' larva, pupa and adult, when three natural predators, copepod, tadpole and purple martins are introduced respectively ($C_p = 500, T_p = 500,\ P_m = 130, \alpha = 0.5,\ \mu_6 = 0.02,\ \beta_6 = 0.01,\ b_2 = 0.21,\ \omega = 0.5,\ \mu_7 = 0.01,\ \beta_7 = 0.02, b_3 = 0.9,\ \gamma = 5,\ \mu_5 = 0.5,\ \beta_5 = 0.13\ and\ b_4 = 0.5$).

Experiment 4: Comparison of the effect of introducing one, two and three natural predator, copepod on mosquitoes' larva.

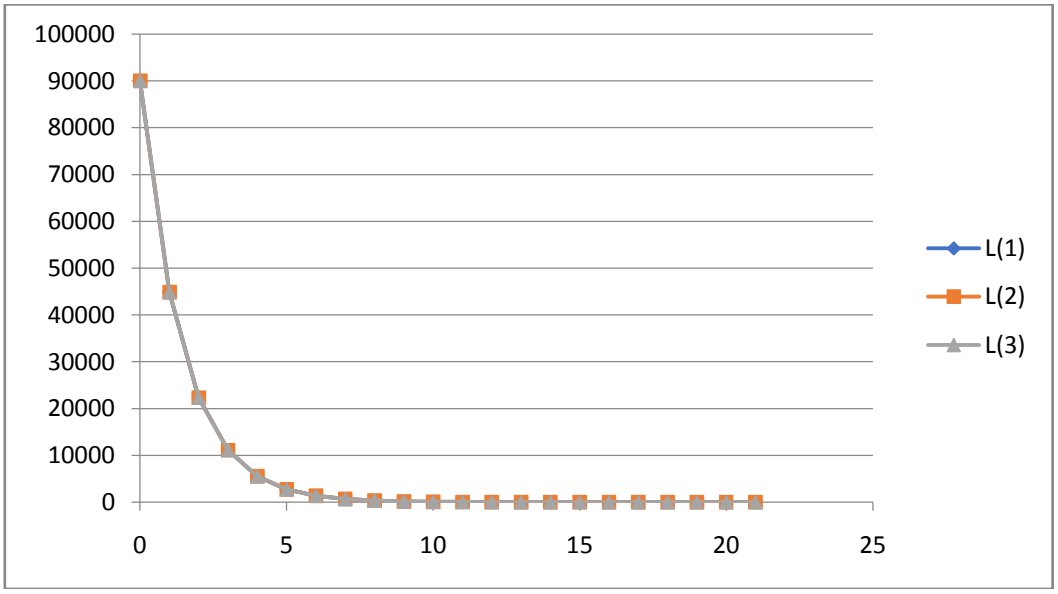


Figure 14: Number of mosquitoes' larva when one, two and three natural predators tadpoles are compared on larva respectively ($T_{1,2,\&3} = 500,\ \omega = 0.5,\ \mu_7 = 0.01,\ \beta_7 = 0.02,\ and\ b_3 = 0.9$).

Experiment 5: Comparison of the effect of introducing two and three natural predator, purple martins on mosquitoes' adult.

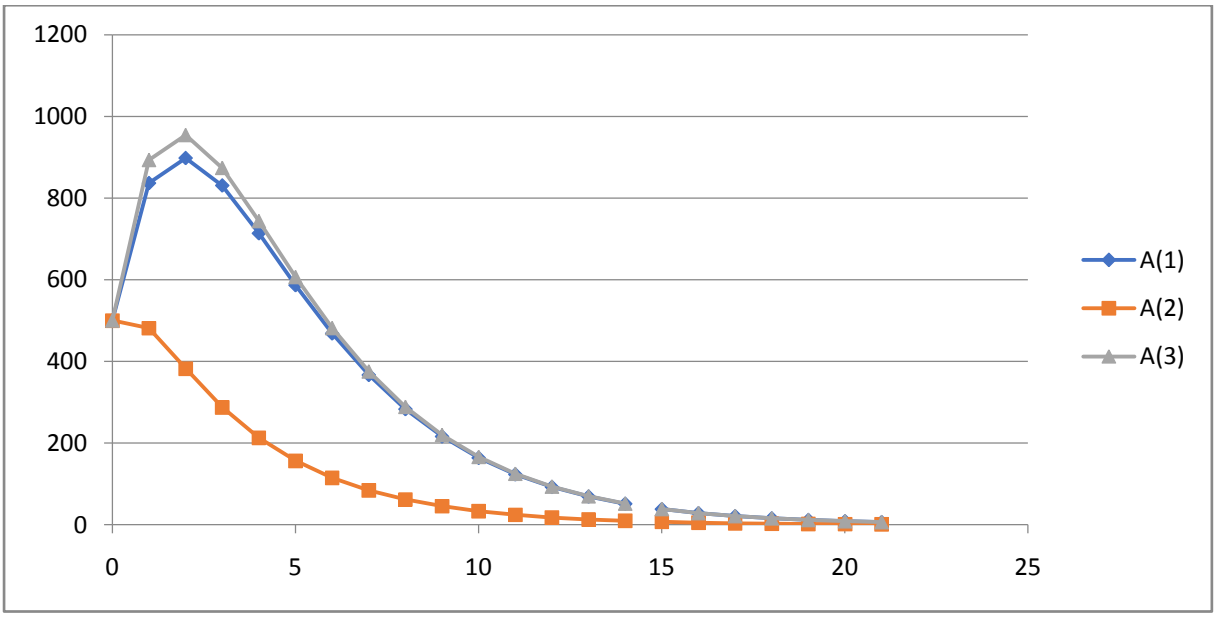


Figure 15: Number of mosquitoes' adult when two and three natural predator, purple martins are compared respectively ($P_{2\&3} = 500, \gamma = 5,\ \mu_5 = 0.5,\ \beta_5 = 0.13, and\ \ b_4 = 0.5$).

Experiment 6: Effect of introducing one natural predator, tadpole on mosquitoes' pupa.

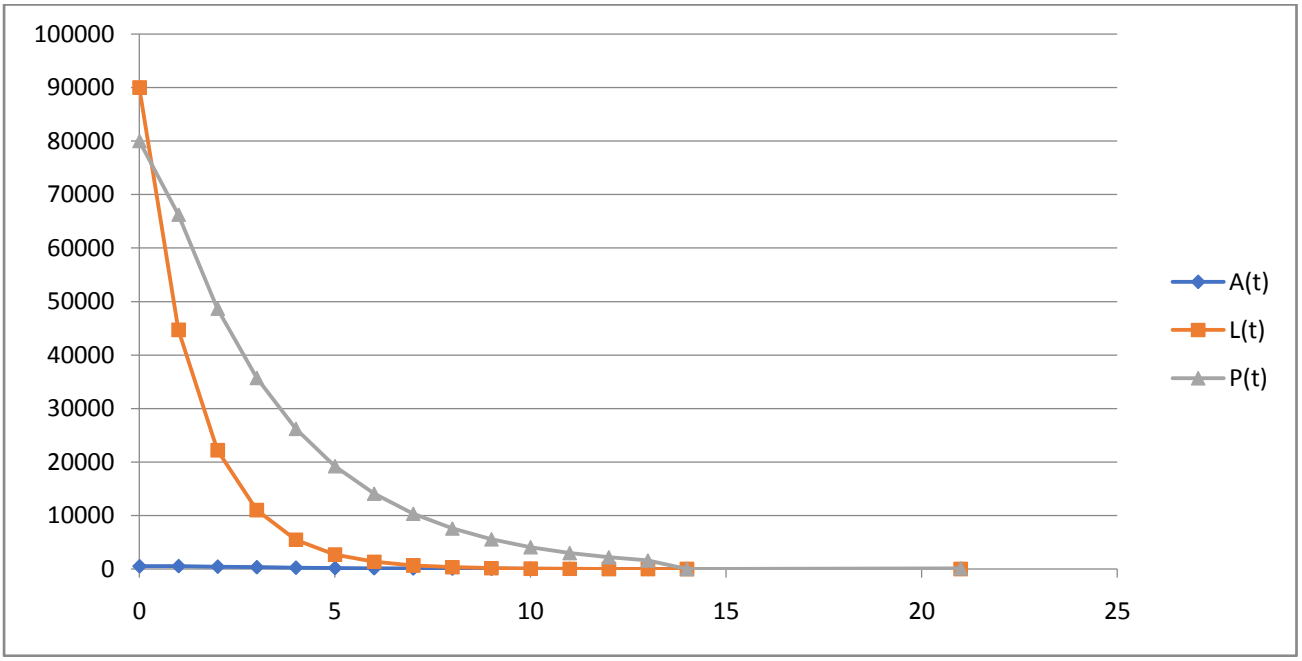


Figure 16: Number of mosquitoes' pupa when one natural predator, tadpole was introduced to mosquito pupa ($T_m = 500\ C_p = 0,\ P_m = 0,\ \omega = 0.5,\ \mu_7 = 0.01,\ \beta_7 = 0.02,\ and\ \ b_3 = 0.9$)

Experiment 7: Effect of introducing two natural predators, tadpole and purple martins on mosquitoes' pupa and adult respectively.

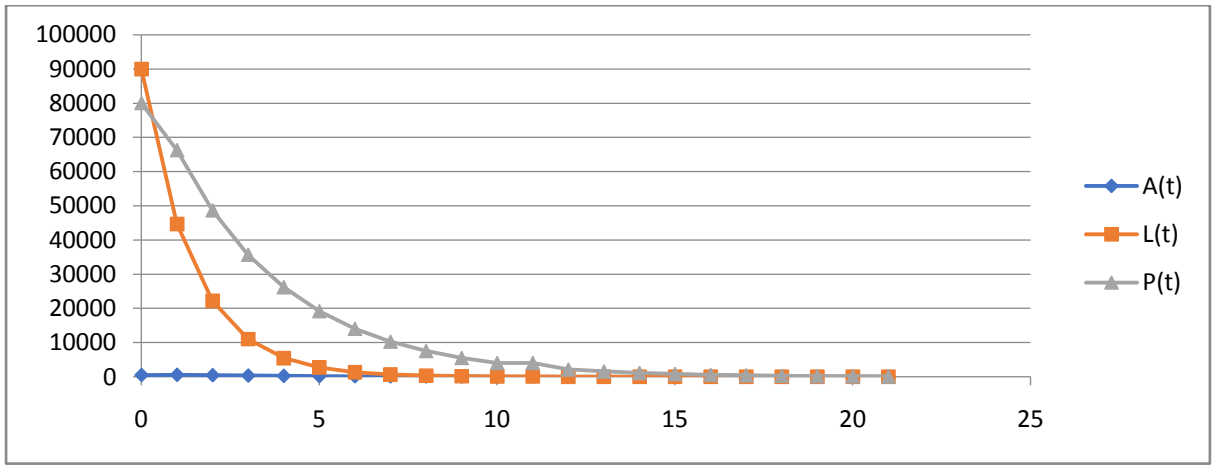
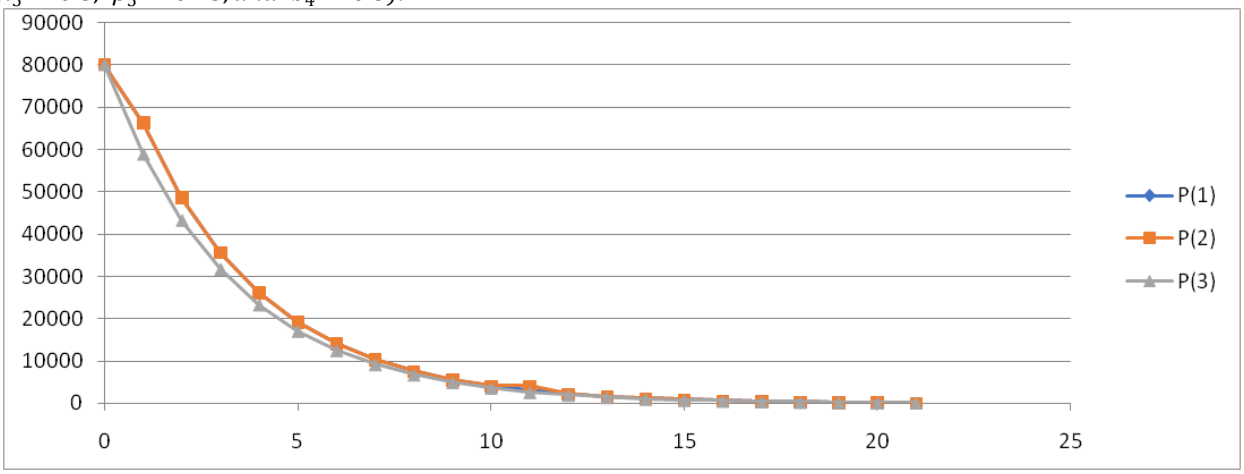


Figure 17: Number of mosquitoes' pupa and adult when two natural predators, tadpole and purple martins are introduced respectively ($T_p = 500, P_m = 130,\ C_p = 0, \omega = 0.5,\ \mu_7 = 0.01,\ \beta_7 = 0.02,\ b_3 = 0.9,\ \gamma = 5,\ \mu_5 = 0.5,\ \beta_5 = 0.13, and\ \ b_4 = 0.5$).



Experiment 8: Comparison of the effect of introducing one, two and three natural predator on pupa.

Figure 18: Number of mosquitoes' pupa when one, two and three natural predators are compared respectively ($T_{1,2\ \&3} = 500,\ \omega = 0.5,\ \mu_7 = 0.01,\ \beta_7 = 0.02,\ and\ b_3 = 0.9$).

Experiment 9: Comparison of the effect of introducing two and three natural predator purple martins on adult.

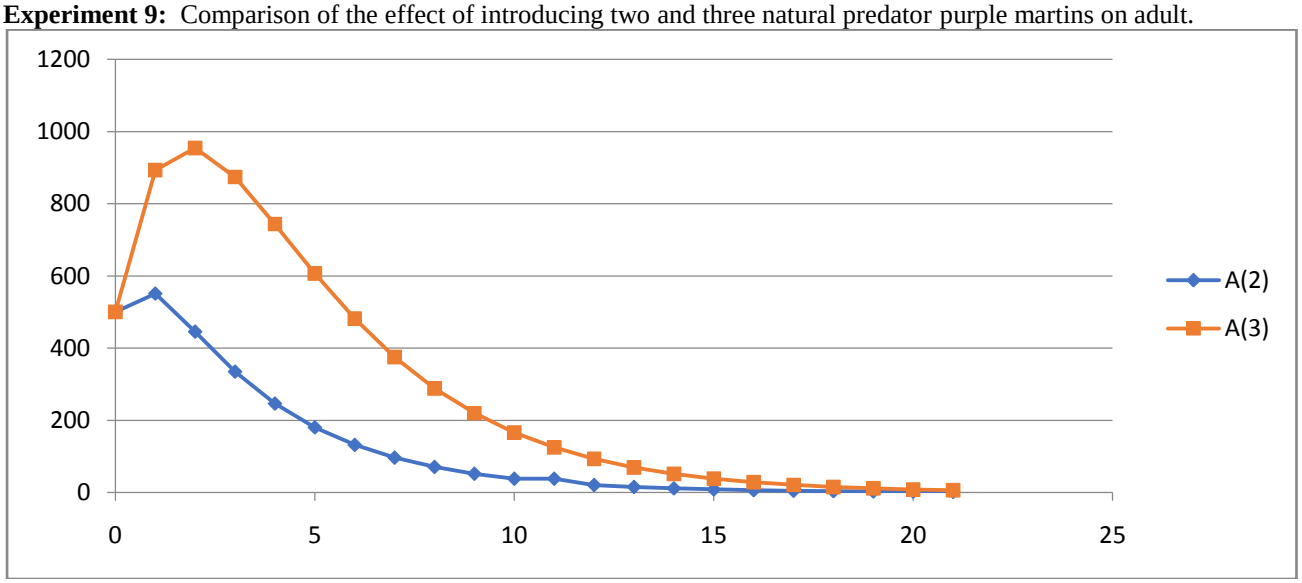


Figure 19: Number of mosquitoes' adult when two and three natural predators, purple martins are compared respectively ($P_{2\&3} = 130, \gamma = 5,\ \mu_5 = 0.5,\ \beta_5 = 0.13\ and\ b_4 = 0.5$).

Experiment 10: Effect of introducing one natural predator, purple martins on mosquitoes' adult.

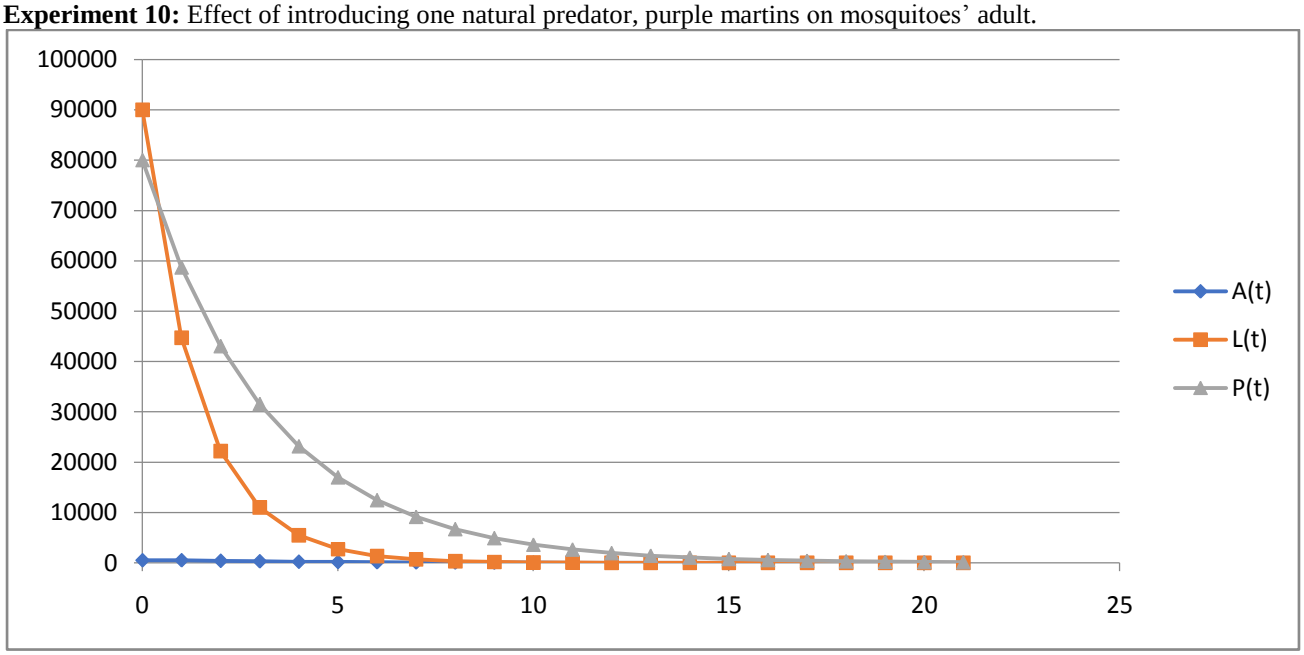


Figure 20: Number of mosquitoes' adult when one natural predator, purple martins was introduced to mosquito adult ($P_m = 130, C_p = 0, T_p = 0, \gamma = 5,\ \mu_5 = 0.5,\ \beta_5 = 0.13\ and\ b_4 = 0.5$).

III. Discussion of Results

A model of the effects of three natural predators on the aquatic and adult stages of Anopheles mosquitoes to control malaria transmission is presented. In introduction, we discuss the prevalence of mosquitoes in our society, where two million deaths are due to malaria parasites in sub-Saharan Africa in general and Nigeria in particular, one third of which are children.

In results, we performed the disease free steady state stability analysis of the model using equilibrium point idea, Beltrami conditions, Diekman conditions and also used Maple software for symbolic solution, numerically and plotted the results showing the effects of the introduction of three natural predators (copepods, tadpoles and purple martins) at the larval, pupal and adult stages. From the result, we see that the stability analysis of the free equilibrium state is stable. With the natural implication, there will be no adult female Anopheles mosquito for malaria transmission in our society.

The new model used the parameters shown in Table 1. These parameters are chosen with the thresholds obtained in the steady-state disease-free stability analysis of the model. In the analytic output, model analysis showed the existence of a unique disease-free steady state (E_0) that is locally and asymptotically stable when $R_0 < 1$. We also identified the basic reproduction number R_0 in terms of model parameters. These threshold parameters mentioned in Table 2 above should be considered when implementing the above model to provide control measures to reduce the prevalence of malaria parasites in our society and consequently eradicate the disease of mosquito in Nigeria. In the numerical results, numerical experiments performed using the variables and parameter values in Table 2 and applying disease-free steady state stability conditions (E_0) yield the following result:

In Experiment 1, the effect of introducing a natural predator, copepod, on mosquito larvae and the numerical values of variables and parameters were investigated as shown in Table 2 were analyze, solve and run numerical simulation with graphical presentation of result as in figure 11 predator was introduced, this indicate that, the number of larvae decreased significantly, and they pupated.

In Experiment 2, the effect of the introducing two natural predators, copepods and tadpoles, on mosquito larvae and pupae was studied and the numerical values of the variables and parameters were those given in table 2 which were analyze, solve and run numerical simulation with graphical representation of result as in Figure 12 When two natural predators were introduced, by implication the number of larvae to pupae was greatly reduced and the transformation of pupae to adults was minimal.

In Experiment 3, the effect of the introducing three natural predators (copepods, tadpoles and purple swallows) on mosquito larvae, pupae and adults was studied, and the numerical values of the variables and parameters were those shown in Table 2, were analyze, solve and run numerical simulation with graphical presentation of result, shown in Figure 13 when three natural enemies are introduced simultaneously. Infection in the adult Anopheles mosquito population is significantly slowed down and thus eradicated, and the probability of transmission from the adult Anopheles mosquito to the human population is very low.

In experiment 4, comparing the effect of introducing one, two and three natural predators on the larvae, the numerical values of variables and parameters are shown in Table 2, was analyze, solve and run numerical simulation with graphical representation of result which shown in Figure 14 when one, two and three predators were examined. This result shows that the infection rate in Figure 14 decreases significantly to prevent reinjection with malaria, which is the prevention strategy in the fight against malaria.

In Experiment 5, comparison of the effect of introducing two and three natural predators, purple swallows, on adult mosquitoes and the numerical values of variables and parameters were examined in Table 2 and was analyze, solve and run numerical simulation with graphical representation result as shown in Figure 15 when two and three naturals predators were introduced respectively. The result shows that the infection rate in Figure 15 decreases significantly. To prevent a new malaria infection, the transmission rate must be close to zero.

In Experiment 6, the effect of introducing a natural predator, tadpoles, on the mosquito pupa, and the numerical values of the variables of variables and parameter were investigated as shown in Table 2 were analyze, solve and run numerical simulation with graphical representation of the result and the graphical result in Figure 16, shown that when a natural predator, the tadpole, was introduced Infection in the adult Anopheles mosquito population slowed down and the probability of transmission from the pupa to the adult Anopheles mosquito population is very low.

In Experiment 7, the effect of the introducing two natural predators, tadpoles and purple swallows, on mosquito pupae and adults was studied, and the numerical values of the variables and parameters were those given in Table 2 were analyze, solve run numerical simulation with graphical representation of the result is in Figure 17, which vindicated that Infection in the adult Anopheles mosquito population is substantially slowed down and the probability of transforming to adult Anopheles mosquito is very deep.

In Experiment 8, comparing the effect of introducing one, two and three natural enemies into mosquito pupae and the numerical values shown in Table 2 and the graphical result shown in Figure 18 when one, two and three natural enemies are present respectively are entered and verified. The result shows that the infection rate in Figure 18 decreases significantly. To prevent reinjection with malaria, the transmission rate must be close to zero.

In Experiment 9, comparing the effect of introducing two and three natural predators on adult mosquitoes and the numerical values of variables and parameters shown in Table 2, was analyze, solve and run numerical simulation with graphical representation of the result in figure 19, shows that the infection rate decreases appropriately to prevent malaria infection.

In Experiment 10, the effect of introducing a natural predator, purple swallow, on adult mosquitoes was studied, and the numerical values of variables and parameters were as shown in Table 2 were analyze, solve and run numerical simulation with graphical representation of result plotted in Figure 20 when a natural predator, the Crimson Swallow, was introduced, Infection in adult mosquitoes of the Anopheles family is fairly stagnant and the percentage of transmission is very small.

Considering the total population, the effect of the introducing three natural predators, one, two and three, on the larva, larva and pupa and larva, pupa and adult (copepods, tadpoles and martens) respectively. (compare Figure 11, 12, 13 with Figure 14 and 15). The infectious agent content is greatly reduced and the infection of the egg, larva and pupa is eradicated, but persists at a low level in the adult Anopheles mosquito.

Evaluating the total population, the effect of the introducing two natural predators one and two on pupae, pupae and adults (tadpoles and purple martins) respectively, was examined (compare Figure 16, 17, 18 with Figure 19 and 20). The infectious agent content is greatly reduced and the infection of the egg, larva and pupa is eradicated, but persists at a low level in the adult Anopheles mosquito.

Finally, to understand the effects of introducing three natural enemies (copepods, tadpoles and purple martins) on the larva, pupa and adult when three natural predators are introduced in each, Figures 11, 12, 13, 14, 15… 20 graphically specify the representations to be provided. It could be clearly observed that the transmission speed was reduced to the indispensable minimum. This could be achieved since research should focus on formulating models that capture preventive strategies based on stability analysis to prevent the onset of the disease and thus eradicate it.

IV. Conclusion

We find out that based on the conditions of the Beltrami condition, when the determinants of the Jacobian matrix are greater than zero and the trace is less than zero, the disease-free steady-state stability analysis is stable and Diekmann's conditions which indicate when $R0 < 1$, the steady state stability analysis without disease is stable. We conclude that if the natural predators introduced are large, the number of larvae leading to pupae will be almost zero and the number of pupae developing into adults will be zero, which will prolong the life cycle of the interrupted Anopheles mosquito. Therefore, in our society, there will be no adult Anopheles mosquitoes for the transmission of malaria parasites.

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