Quest Journals
Journal of Research in Applied Mathematics
Volume 9 ~ Issue 2 (2023) pp: 29-38
ISSN(Online) : 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

SK index and SK_1index of generalized transformation graphs

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ABSTRACT: *The generalized transformation graph is denoted by G^{xy}, is a graph whose vertex set is V(G) $\cup \boldsymbol{E}(\boldsymbol{G})$ and the vertices$\boldsymbol{\alpha}, \boldsymbol{\beta}$ $\in V(G^{xy})$. The vertex of G^{xy} corresponding to a vertex of G is referred to as a point vertex and the vertex e of G^{xy} corresponding to an edge e as a line vertex. There are eight distinct 3-permutations of {+,-}and corresponding to these there are eight graphical transformations of graph G. In this paper SK index and SK_1 index of $\boldsymbol{G^{xy}}$,$\overline{\boldsymbol{G^{xy}}}$,$\boldsymbol{G^{xyz}}$and $\overline{\boldsymbol{G^{xyz}}}$generalized transformation graphs are studied.*
KEYWORDS: *Complement graph,generalized transformation graph,SKindex, SK_1 index,topological index.*

Received 25 Jan., 2023; Revised 07 Feb., 2023; Accepted 09 Feb., 2023 © The author(s) 2023.
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I. INTRODUCTION

Let G = (V, E) be a graph with order |V(G)| = n and size |E(G)| = m. The degree of a vertex,denoted by d_u and defined as the number of vertices adjacent to u. Topological indices are numerical parameter obtained from graph representing a molecule. All graphs considered here are finite, undirected and simple. The complement of graph G, denoted by $\bar{G}$, is a graph having the same vertex set as G in which two vertices are adjacent if and only if they are not adjacent in G. Thus, size of $\bar{G}$ is $\binom{n}{2}$- m and $d_{\bar{G}}(v)$ = n-1-$d_G(v)$ [1].The vertex of G^{xy} corresponding to a vertex v of G is referred as a point vertex and vertex e corresponding to an edge as a line vertex. For notations and definitions refer [2-5].The transformation of G^{+++}(total graph) of G is the graph with vertex set V(G) U E(G) in which the vertices u and v are joined by an edge if and only if one of the following holds [6]:

1) Both u and v ∈ V(G) and u and v are adjacent in G;
2) Both u and v ∈ E(G) and u and v are adjacent in G;
3) One is in V(G) and the other is in E(G) and they are incident with each other in G.

The simple sufficient condition for $G^{++-}$to be Hamiltonian was obtained by L.YI. et al. in [7].Some degree based topological indices of transformation graphs are studied in [8].There are four transformations for G^{xy} and four for their complements as G^{++},G^{+-}, G^{-+}, G^{--},and for complementsG^{--},G^{-+}, G^{+-}, G^{++}.Partition the edge set of E(G^{++}) into two sets E_1 and E_2,where E_1 = {uv|u,v∈ V(G)}, E_2 = {ue| the vertex u is incident to the edge e in G}.The generalized transformation graph G^{xy} is a graph whose vertex set is V(G) $\cup E(G)$ and α, β $\in V(G^{xy})$.Then α and β are adjacent in G^{xy} if and only if (a) and (b) holds:

(a) α, β ∈V(G),$\alpha, \beta$are adjacent in G if x = + and α, β not adjacent in G if x = -.
(b) α ∈V(G) and β ∈ E(G),$\alpha, \beta$are incidentin G if y = + and α, βnot incident in G if y = -.

There are eight distinct 3-permutations of {+,-}and corresponding to those eight graphical transformations of G[9-10] as G^{+++},G^{++-},G^{+-+}, G^{-++}, and for their complementsG^{---}, G^{--+}, G^{-+-}, G^{+--}.$\overline{G^{xyz}}$ Denotes the transformation graphs of complement of a graph G.

Using degree of a line vertex and a point vertex in generalized transformation graphs G^{xy} and $\overline{G^{xy}}$ the SK index and SK_1 index can be obtained [11].The SK index of a graph G = (V, E) is defined as SK(G) = $\sum_{uv\in E(G)} \frac{d_u+d_v}{2}$ and SK_1 index as SK_1(G) = $\sum_{uv\in E(G)} \frac{d_u d_v}{2}$,where d_u and d_v are the degrees of the vertices u and v in G[12-13].There are many papers on the topological indices of generalized transformation graphs as [14-27].In this paper SK index and SK_1index of G^{xy} ,$\overline{G^{xy}}$,G^{xyz} and $\overline{G^{xyz}}$ generalized transformation graphs are studied.

II.MATERIALS AND METHODS

For a simple, connected graph with vertex set V(G) and edge set E(G) and complement graph with $V(\overline{G})$and edge set $E(\overline{G})$are related by the equations:$|V(G)|=|V(\overline{G})|$ and $|E(G)|+|E(\overline{G})| = \frac{n(n-1)}{2}$.The procedure of obtaining a new graph from a given graph using adjacency(or non adjacency) and incidence(no incidence) relationship between elements of a graph is known as transformation graph. There are four transformations of a graph for G^{xy}, and four for their complements $\overline{G^{xy}}$.For three variables x,y,z there are eight distinct 3-permutations of {+,-} so eight corresponding graph transformations. Using degree of a line vertex and point vertex inG^{xy},$\overline{G^{xy}}$,G^{xyz} and $\overline{G^{xyz}}$: the SK index and SK_1 index can be obtained for generalized transformation graphs.

III.RESULTS AND DISCUSSION

For a graph G(V,E) of order n≥3,let the variables x,y,z takes the values + or -. The transformation graph G^{xyz} is a graph having $V(G) \cup E(G)$ as a vertex set and for $\alpha, \beta \in V(G) \cup E(G)$,$\alpha$ and β are adjacent in G^{xyz} if and only if

1. $\alpha, \beta \in V(G)$, x = + ifα is adjacent to β in Gotherwise x = -.
2. $\alpha, \beta \in E(G)$,y = +if α is adjacent to β in G otherwise y = -.
3. $\alpha \in V(G)$ and$\beta \in E(G)$,z = +ifα and β are incident to each other in Gotherwise z = -.

The edge set of G^{xyz} can be partitioned in E_x,E_y and E_z where E_x = {uv|u,v∈ V(G)},E_y = {st | s,t∈ E(G)},and E_z = {ue|u∈ V(G),e∈ E(G)}.In this section SK index and SK_1 index of $G^{xy}, \overline{G^{xy}}, G^{xyz}$ and $\overline{G^{xyz}}$ generalized transformation graphs are obtained.

SK index of transformation graphs G^{xy}

Theorem 1.1: Let G be a graph with n vertices and m edges.

Then $SK(G^{++})= 2SK(G^{++}) + \sum_{ue\in E_2}[d_{(G)}(u) + 1]$.

Proof. Partition the edge set $E(G^{++})$ in two sets E_1 and E_2, where E_1 = {uv | u,v∈E(G)},E_2 = {ue |the vertex u is incident to the edge e in G} and so $|E_1|$=m and $|E_2|$= 2m.By using the proposition if u∈ V(G) then $d_{(G^{++})}(u)=2d_G(u)$ and if e∈ E(G) then $d_{(G^{++})}(e) = 2$.

Therefore $SK(G^{++})=\sum_{uv\in E(G^{++})}\frac{d_{(G^{++})}(u)+d_{(G^{++})}(v)}{2}$

$=\sum_{uv\in E_1}\frac{d_{(G^{++})}(u)+d_{(G^{++})}(v)}{2}+\sum_{ue\in E_2}\frac{d_{(G^{++})}(u)+d_{(G^{++})}(e)}{2}$

$=\sum_{uv\in E_1}\frac{2d_{(G)}(u)+2d_{(G)}(v)}{2}+\sum_{ue\in E_2}\frac{2d_{(G)}(u)+2}{2}$

$= 2SK(G^{++}) +\sum_{ue\in E_2}[d_{(G)}(u) + 1]$.

Theorem 1.2: Let G be a graph with n vertices and m edges. Then $SK(G^{+-})= m^2+m(n-2)\frac{m+(n-2)}{2}$.

Proof. Partition the edge set $E(G^{+-})$ in two sets E_1 and E_2, where E_1 = {uv | u,v∈E(G)},E_2 = {ue |the vertex u is not incident to the edge e in G} and so $|E_1|$=m,$|E_2|$=m(n-2).By using the proposition if u∈ V(G) then $d_{(G^{+-})}(u) = m$ and if e∈ E(G) then$d_{(G^{+-})}(e) = n-2$.

Therefore $SK(G^{+-})=\sum_{uv\in E(G^{+-})}\frac{d_{(G^{+-})}(u)+d_{(G^{+-})}(v)}{2}$

$=\sum_{uv\in E_1}\frac{d_{(G^{+-})}(u)+d_{(G^{+-})}(v)}{2}+\sum_{ue\in E_2}\frac{d_{(G^{+-})}(u)+d_{(G^{+-})}(e)}{2}$

$=\sum_{uv\in E_1}\frac{m+m}{2}+\sum_{ue\in E_2}\frac{m+(n-2)}{2}$

$= m^2+m(n-2)\frac{m+(n-2)}{2}$.

Theorem 1.3: Let G be a graph with n vertices and m edges.

Then $SK(G^{-+})= (\frac{n(n-1)-2m}{2})(n-1)+m(n+1)$.

Proof. Partition the edge set $E(G^{-+})$ in two sets E_1 and E_2, where E_1 = {uv | u,v∉ E(G)},E_2 = {ue |the vertex u is incident to the edge e in G}and so $|E_1|=\binom{n}{2} - m$ and $|E_2|$=2m.By using the proposition if u∈ V(G) then $d_{(G^{-+})}(u) = n-1$ and if e∈ E(G) then $d_{(G^{-+})}(e) = 2$.

Therefore $SK(G^{-+})=\sum_{uv\in E(G^{-+})}\frac{d_u+d_v}{2}$

$=\sum_{uv\in E_1}\frac{d_{(G^{-+})}(u)+d_{(G^{-+})}(v)}{2}+\sum_{ue\in E_2}\frac{d_{(G^{-+})}(u)+d_{(G^{-+})}(e)}{2}$

$=\sum_{u,v\notin E_1}\frac{(n-1)+(n-1)}{2}+\sum_{u,e\in E_2}\frac{2+(n-1)}{2}$

$= [\binom{n}{2} - m](n-1)+2m[\frac{(n-1)+2}{2}]$

=$(\frac{n(n-1)-2m}{2})$(n-1)+m(n+1).

SK index of transformation graphs $\overline{G^{xy}}$

Theorem 1.4: Let G be a graph withn vertices and m edges. Then SK($\overline{G^{++}}$)=$\sum_{uv\notin E(G)}(m+n-1-d_G(u)-d_G(v))+\sum_{ue\in E_2}(m+n-2-d_G(u))+\sum_{ef\in E_3}(m+n-3)$.

Proof. Partition the edge set E($\overline{G^{++}}$)in three sets E_1,E_2 and E_3, where E_1 = {uv | u,v∉ E(G)},E_2 = {ue |the vertex u is not incident to the edge e in G}and E_3 = {ef|e,f∈ $E(G)$} and so |E_1|=$\binom{n}{2}$-m,|E_2| = m(n-2)and |E_3|=$\binom{m}{2}$.By using the proposition if u∈ V(G) $d_{(\overline{G^{++}})}$(u) = m+n-1-2d_G(u) and if e∈ E(G) then $d_{(\overline{G^{++}})}$ (e) = m+n-3.

Therefore SK(($\overline{G^{++}}$)=$\sum_{uv\in E_1(\overline{G^{++}})}\frac{d_{(\overline{G^{++}})}(u)+d_{(\overline{G^{++}})}(v)}{2}$

=$\sum_{uv\in E_1}\frac{d_{(\overline{G^{++}})}(u)+d_{(\overline{G^{++}})}(v)}{2}+\sum_{ue\in E_2}\frac{d_{(\overline{G^{++}})}(u)+d_{(\overline{G^{++}})}(e)}{2}+\sum_{ef\in E_3}\frac{d_{(\overline{G^{++}})}(e)+d_{(\overline{G^{++}})}(f)}{2}$

=$\sum_{uv\notin E(G)}\frac{m+n-1-2d_G(u)+m+n-1-2d_G(v)}{2}+\sum_{ue\in E_2}\frac{m+n-1-2d_G(u)+m+n-3}{2}+\sum_{ef\in E_3}\frac{(m+n-3)+(m+n-3)}{2}$

=$\sum_{uv\notin E(G)}(m+n-1-d_G(u)-d_G(v))+\sum_{ue\in E_2}(m+n-2-d_G(u))+\sum_{ef\in E_3}(m+n-3)$.

Theorem 1.5: Let G be a graph with n vertices and m edges.

Then SK($\overline{G^{+-}}$) =$\frac{[n(n-1)-2m](n-1)}{2}$+m(n+m)+$\frac{m(m-1)(m+1)}{2}$.

Proof. Partition the edge set E($\overline{G^{+-}}$)in three sets E_1,E_2 and E_3, where E_1={uv | u,v∉ E(G)},

E_2={ue |the vertex u is incident to the edge e in G}and E_3 = {ef|e,f∈ $E(G)$} and so |E_1|=$\binom{n}{2}$-m,|E_2|=2m and |E_3|=$\binom{m}{2}$.By using the proposition if u∈ V(G) then $d_{(\overline{G^{+-}})}$(u) = n-1 and if e∈ E(G) then $d_{(\overline{G^{+-}})}$ (e) = m+1.

Therefore SK(($\overline{G^{+-}}$)=$\sum_{uv\in E_1(\overline{G^{+-}})}\frac{d_{(\overline{G^{+-}})}(u)+d_{(\overline{G^{+-}})}(v)}{2}$

=$\sum_{uv\in E_1}\frac{d_{(\overline{G^{+-}})}(u)+d_{(\overline{G^{+-}})}(v)}{2}+\sum_{ue\in E_2}\frac{d_{(\overline{G^{+-}})}(u)+d_{(\overline{G^{+-}})}(e)}{2}+\sum_{ef\in E_3}\frac{d_{(\overline{G^{+-}})}(e)+d_{(\overline{G^{+-}})}(f)}{2}$

=$\sum_{uv\notin E(G)}\frac{(n-1)+(n-1)}{2}+\sum_{ue\in E_2}\frac{(n-1)+(m+1)}{2}+\sum_{ef\in E_3}\frac{(m+1)+(m+1)}{2}$

= $\frac{[n(n-1)-2m](n-1)}{2}$+m(n+m)+$\frac{m(m-1)(m+1)}{2}$.

Theorem 1.6: Let G be a graph with n vertices and m edges.

Then SK($\overline{G^{-+}}$)= m^2+m(n-2)($\frac{2m+n-3)}{2}$)+$\frac{m(m-1)}{2}(m+n-3)$.

Proof. Partition the edge set E($\overline{G^{-+}}$)in three sets E_1,E_2 and E_3, where E_1 = {uv | u,v∈E(G)},E_2 = {ue |the vertex u is not incident to the edge e in G}and E_3 = {ef|e,f∈ $E(G)$} and so |E_1|=m,|E_2|=m(m-2) and |E_3|=$\binom{m}{2}$.By using the proposition if u∈ V(G) then $d_{(\overline{G^{-+}})}$(u) = m and if e∈ E(G) then $d_{(\overline{G^{-+}})}(e)$= m+n-3.

Therefore SK($\overline{G^{-+}}$)=$\sum_{uv\in E_1(\overline{G^{-+}})}\frac{d_{(\overline{G^{-+}})}(u)+d_{(\overline{G^{-+}})}(v)}{2}$

=$\sum_{uv\in E_1}\frac{d_{(\overline{G^{-+}})}(u)+d_{(\overline{G^{-+}})}(v)}{2}+\sum_{ue\in E_2}\frac{d_{(\overline{G^{-+}})}(u)+d_{(\overline{G^{-+}})}(e)}{2}+\sum_{ef\in E_3}\frac{d_{(\overline{G^{-+}})}(e)+d_{(\overline{G^{-+}})}(f)}{2}$

=$\sum_{uv\in E_1}\frac{m+m}{2}+\sum_{ue\in E_2}\frac{m+(m+n-3)}{2}+\sum_{ef\in E_3}\frac{(m+n-3)+(m+n-3)}{2}$

=m^2+m(n-2)($\frac{2m+n-3)}{2}$)+$\frac{m(m-1)}{2}(m+n-3)$.

Theorem 1.7: Let G be a graph with n vertices and m edges.

Then SK($\overline{G^{--}}$)= m $[d_{(G)}(u)+d_{(G)}(v)]$+ m$[2d_{(G)}(u)+(m+1)]$+$\frac{m(m-1)(m+1)}{2}$.

Proof. Partition the edge set E($\overline{G^{--}}$)in three sets E_1,E_2 and E_3, where E_1 = {uv | u,v∈ E(G)},E_2 = {ue |the vertex u is incident to the edge e in G}and E_3={ef|e,f∈ $E(G)$} and so |E_1|=m,|E_2|=2m and |E_3|=$\binom{m}{2}$.By using the proposition if u∈ V(G) then $d_{(\overline{G^{--}})}$(u) = 2$d_{(G)}(u)$and if e∈ E(G)then $d_{(\overline{G^{--}})}(e)$= m+1.

Therefore SK($\overline{G^{--}}$)=$\sum_{uv\in E_1(\overline{G^{--}})}\frac{d_{(\overline{G^{--}})}(u)+d_{(\overline{G^{--}})}(v)}{2}$

=$\sum_{uv\in E_1}\frac{d_{(\overline{G^{--}})}(u)+d_{(\overline{G^{--}})}(v)}{2}+\sum_{ue\in E_2}\frac{d_{(\overline{G^{--}})}(u)+d_{(\overline{G^{--}})}(e)}{2}+\sum_{ef\in E_3}\frac{d_{(\overline{G^{--}})}(e)+d_{(\overline{G^{--}})}(f)}{2}$

=$\sum_{uv\in E_1}\frac{2d_{(G)}(u)+2d_{(G)}(v)}{2}+\sum_{ue\in E_2}\frac{2d_{(G)}(u)+(m+1)}{2}+\sum_{ef\in E_3}\frac{(m+1)+(m+1)}{2}$

=m $[d_{(G)}(u)+d_{(G)}(v)]$+m$[2d_{(G)}(u)+(m+1)]$+$\frac{m(m-1)(m+1)}{2}$.

SK index of transformation graphs G^{xyz}

Theorem 2.1: Let G be a graph with n vertices and m edges.
Then SK(G^{+++})=2SK(G)+$\sum_{st\in E_y}\frac{d_G(a)+2d_G(b)+d_G(c)}{2}+\sum_{eu\in E_z}\frac{3d_G(u)+d_G(v)}{2}$,where s = ab, t = bc$\in E(G)$,e = uv $\in E(G)$; u,v,a,b,c $\in V(G)$ and are distinct.

Proof. Partition the edge set E(G^{+++})in three sets E_x,E_y and E_z, where E_x = {uv | u,v∈V(G)},E_y = {st | s,t∈E(G)} and E_z = {ue| u∈ V(G),e∈ E(G)}.By using the proposition if u∈ V(G) then$d_{(G^{+++})}$(u) = $2d_{(G)}(u)$and if e∈ E(G) then $d_{(G^{+++})}(e)$=$d_{(G)}(u)+d_{(G)}(v)$.

Therefore SK (G^{+++})=$\sum_{uv\in E(G^{+++})}\frac{d_{(G^{+++})}(u)+d_{(G^{+++})}(v)}{2}$

$$=\sum_{uv\in E_x}\frac{d_{(G^{+++})}(u)+d_{(G^{+++})}(v)}{2}+\sum_{st\in E_y}\frac{d_{(G^{+++})}(s)+d_{(G^{+++})}(t)}{2}+\sum_{eu\in E_z}\frac{d_{(G^{+++})}(u)+d_{(G^{+++})}(e)}{2}$$

$$=\sum_{uv\in E_x}\frac{2d_G(u)+2d_G(v)}{2}+\sum_{st\in E_y}\frac{d_G(a)+d_G(b)+d_G(b)+d_G(c)}{2}+\sum_{eu\in E_z}\frac{2d_G(u)+d_G(u)+d_G(v)}{2}$$

$$=2SK(G)+\sum_{st\in E_y}\frac{d_G(a)+2d_G(b)+d_G(c)}{2}+\sum_{eu\in E_z}\frac{3d_G(u)+d_G(v)}{2}.$$

Theorem 2.2: Let G be a graph with n vertices and m edges.
Then SK(G^{++-}) =$\sum_{uv\in E_x}m+\sum_{st\in E_y}\frac{d_G(a)+2d_G(b)+d_G(c)+2(n-4)}{2}+\sum_{eu\in E_z}\frac{m+d_G(v)+d_G(w)+n-4}{2}$,where s= ab, t = bc$\in E(G)$,e = vw $\in E(G)$; u,v,w,a,b,c $\in V(G)$ and are distinct.

Proof. By using the proposition if u∈ V(G) then$d_{(G^{++-})}$(u) = m and if e∈ E(G) then$d_{(G^{++-})}$(e) =d_G(u)+d_G(v)+n-4.

Therefore SK(G^{++-})=$\sum_{uv\in E(G^{++-})}\frac{d_{(G^{++-})}(u)+d_{(G^{++-})}(v)}{2}$

$$=\sum_{uv\in E_x}\frac{d_{(G^{++-})}(u)+d_{(G^{++-})}(v)}{2}+\sum_{st\in E_y}\frac{d_{(G^{++-})}(s)+d_{(G^{++-})}(t)}{2}+\sum_{eu\in E_z}\frac{d_{(G^{++-})}(u)+d_{(G^{++-})}(e)}{2}$$

$$=\sum_{uv\in E_x}\frac{m+m}{2}+\sum_{st\in E_y}\frac{d_G(a)+d_G(b)+n-4+d_G(b)+d_G(c)+n-4}{2}+\sum_{eu\in E_z}\frac{m+d_G(v)+d_G(w)+n-4}{2}$$

$$=\sum_{uv\in E_x}m+\sum_{st\in E_y}\frac{d_G(a)+2d_G(b)+d_G(c)+2(n-4)}{2}+\sum_{eu\in E_z}\frac{m+d_G(v)+d_G(w)+n-4}{2}.$$

Theorem 2.3: Let G be a graph with n vertices and m edges.
Then SK(G^{+-+})= 2SK(G)+$\sum_{st\in E_y}\frac{2m-d_G(a)-d_G(b)-d_G(c)-d_G(d)+6}{2}+\sum_{eu\in E_z}\frac{d_G(u)-d_G(v)+m+3}{2}$,where s = ab, t = cd$\in E(G)$,e = uv $\in E(G)$; u,v,a,b,c,d $\in V(G)$ and are distinct.

Proof. By using the proposition if u∈ V(G) then $d_{(G^{+-+})}$(u) = $2d_G$(u) and if e∈ E(G) then $d_{(G^{+-+})}$(e) =m-d_G(u)-d_G(v)+3.

Therefore SK($\boldsymbol{G}^{+-+}$)=$\sum_{uv\in E(G^{+-+})}\frac{d_{(G^{+-+})}(u)+d_{(G^{+-+})}(v)}{2}$

$$=\sum_{uv\in E_x}\frac{d_{(G^{+-+})}(u)+d_{(G^{+-+})}(v)}{2}+\sum_{st\in E_y}\frac{d_{(G^{+-+})}(s)+d_{(G^{+-+})}(t)}{2}+\sum_{eu\in E_z}\frac{2d_{(G^{+-+})}(u)+d_{(G^{+-+})}(e)}{2}$$

$$=\sum_{uv\in E_x}\frac{2d_G(u)+2d_G(v)}{2}+\sum_{st\in E_y}\frac{m-d_G(a)-d_G(b)+3+m-d_G(c)-d_G(d)+3}{2}+\sum_{eu\in E_z}\frac{2d_G(u)+m-d_G(u)-d_G(v)+3}{2}$$

$$=2SK(G)+\sum_{st\in E_y}\frac{2m-d_G(a)-d_G(b)-d_G(c)-d_G(d)+6}{2}+\sum_{eu\in E_z}\frac{d_G(u)-d_G(v)+m+3}{2}.$$

Theorem 2.4: Let G be a graph with n vertices and m edges.
Then SK(G^{-++}) =$[\binom{n}{2}-m](n-1)$+$\sum_{st\in E_y}\frac{d_G(a)+2d_G(b)+d_G(c)}{2}+\sum_{eu\in E_z}\frac{(n-1)+d_G(u)+d_G(v)}{2}$,
where s = ab, t = bc$\in E(G)$,e = uv $\in E(G)$; u,v,a,b,c $\in V(G)$ and are distinct.

Proof. By using the proposition If u∈ V(G) then$d_{(G^{-++})}$(u) =n-1 andif e∈ E(G) then$d_{(G^{-++})}$(e) =d_G(u)+d_G(v).

Therefore SK(G^{-++}) =$\sum_{uv\in E(G^{-++})}\frac{d_{(G^{-++})}(u)+d_{(G^{-++})}(v)}{2}$

$$=\sum_{uv\in E_x}\frac{d_{(G^{-++})}(u)+d_{(G^{-++})}(v)}{2}+\sum_{st\in E_y}\frac{d_{(G^{-++})}(s)+d_{(G^{-++})}(t)}{2}+\sum_{eu\in E_z}\frac{d_{(G^{-++})}(u)+d_{(G^{-++})}(e)}{2}$$

$$=\sum_{uv\in E_x}\frac{(n-1)+(n-1)}{2}+\sum_{st\in E_y}\frac{d_G(a)+d_G(b)+d_G(b)+d_G(c)}{2}+\sum_{eu\in E_z}\frac{(n-1)+d_G(u)+d_G(v)}{2}$$

$$=[\binom{n}{2}-m](n-1)+\sum_{st\in E_y}\frac{d_G(a)+2d_G(b)+d_G(c)}{2}+\sum_{eu\in E_z}\frac{(n-1)+d_G(u)+d_G(v)}{2}.$$

Analogous to above we get the following theorems.

Theorem 2.5: Let G be a graph on vertices and m edges. Then

(i) SK(G^{---})= $\sum_{uv\in E_x}[(n+m-1)-d_G(u)-d_G(v)]$ + $\sum_{st\in E_y}\frac{(n+m-1)-d_G(a)-d_G(b)-d_G(c)-d_G(d)}{2}+\sum_{vw\in E_z}\frac{2(n+m-1)-2d_G(u)-d_G(v)-d_G(w)}{2}$,where s = ab,t = cd∈ $E(G)$,e = vw ∈ $E(G)$; u,v,w,a,b,c,d ∈ $V(G)$ and are distinct.

(ii) SK(G^{--+})= $[\binom{n}{2}-m](n-1)$ + $\sum_{st\in E_y}\frac{2(m+3)-(d_G(a)+(b)+d_G(c)+d_G(d))}{2}+\sum_{eu\in E_z}\frac{[n+m-d_G(u)-d_G(v)+2]}{2}$,where s = ab, t = cd∈ $E(G)$, e = uv ∈ $E(G)$; u,v,a,b,c,d ∈ $V(G)$ and are distinct.

(iii) SK(G^{-+-})=$\sum_{uv\in E_x}[n+m-1-d_G(u)-d_G(v)]$ +$\sum_{st\in E_y}\frac{2(n-4)+d_G(a)+2d_G(b)+d_G(c)}{2}+\sum_{eu\in E_z}\frac{2n+m-2d_G(u)+d_G(v)+d_G(w)-5}{2}$,where s = ab, t = bc∈ $E(G)$,e = vw ∈ $E(G)$; u,v,w,a,b,c ∈ $V(G)$ and are distinct.

(iv) SK(G^{+--})=$\sum_{uv\in E_x} m$+$\sum_{st\in E_y}\frac{2(n+m-1)-d_G(a)-d_G(b)-d_G(c)-d_G(d)}{2}+\sum_{eu\in E_z}\frac{2m+n-1-d_G(v)-d_G(w)}{2}$,where s = ab, t = cd∈ $E(G)$,e = vw ∈ $E(G)$; u, v,w, a, b, c,d ∈ $V(G)$ and are distinct.

Theorem 3.1: Let G be a graph with n vertices and m edges.
Then
SK($\overline{G^{+++}}$)= $\sum_{uv\in E_x}[2(n-1)-d_G(u)-d_G(v)]$ + $\sum_{st\in E_y}\frac{4(n-1)-d_G(a)-2d_G(b)-d_G(c)}{2}+\sum_{eu\in E_z}\frac{4(n-1)-3d_G(u)-d_G(v)}{2}$,where s = ab, t = bc∈ $E(G)$,e = uv ∈ $E(G)$; u, v, a, b, c ∈ $V(G)$ and are distinct.

Proof. By using proposition if u∈ V(G) then $d_{(\overline{G^{+++}})}$(u) = 2(n-1-d_G(u)) and if e∈ E(G) then $d_{(\overline{G^{+++}})}$(e) = 2(n-1)-d_G(u)-d_G(v).

Therefore SK($\overline{G^{+++}}$)=$\sum_{uv\in E(\overline{G^{+++}})}\frac{d_{(\overline{G^{+++}})}(u)+d_{(\overline{G^{+++}})}(v)}{2}$

=$\sum_{uv\in E_x}\frac{d_{(\overline{G^{+++}})}(u)+d_{(\overline{G^{+++}})}(v)}{2}$+$\sum_{st\in E_y}\frac{d_{(\overline{G^{+++}})}(s)+d_{(\overline{G^{+++}})}(t)}{2}+\sum_{eu\in E_z}\frac{d_{(\overline{G^{+++}})}(u)+d_{(\overline{G^{+++}})}(e)}{2}$

= $\sum_{uv\in E_x}\frac{2(n-1-d_G(u))+2(n-1-d_G(v))}{2}$ + $\sum_{st\in E_y}\frac{2(n-1)-d_G(a)-d_G(b)+2(n-1)-d_G(b)-d_G(c)}{2}+\sum_{eu\in E_z}\frac{2(n-1-d_G(u))+[2(n-1)-d_G(u)-d_G(v)]}{2}$

=$\sum_{uv\in E_x}[2(n-1)-d_G(u)-d_G(v)]$+$\sum_{st\in E_y}\frac{4(n-1)-d_G(a)-2d_G(b)-d_G(c)}{2}+\sum_{eu\in E_z}\frac{4(n-1)-3d_G(u)-d_G(v)}{2}$.

Theorem 3.2: Let G be a graph on vertices and m edges. Then SK($\overline{G^{++-}}$)= $\sum_{uv\in E_x}({}^{n}_{2}C-m)$+$\sum_{st\in E_y}\frac{6(n-2)-d_G(a)-2d_G(b)-d_G(c)}{2}+\sum_{eu\in E_z}\frac{({}^{n}_{2}C-m)+[3(n-2)-d_G(v)-d_G(w)]}{2}$, where s = ab, t = bc∈ $E(G)$,e = vw ∈ $E(G)$; u, v, w,a, b, c ∈ $V(G)$ and are distinct.

Proof. By using proposition if u∈ V(G) then $d_{(\overline{G^{++-}})}$(u) = ${}^{n}C_{2}$- m and if e∈ E(G) then $d_{(\overline{G^{++-}})}$(e) =3(n-2)-d_G(u)-d_G(v).

Therefore SK($\overline{G^{++-}}$)=$\sum_{uv\in E(\overline{G^{++-}})}\frac{d_{(\overline{G^{++-}})}(u)+d_{(\overline{G^{++-}})}(v)}{2}$

=$\sum_{uv\in E_x}\frac{d_{(\overline{G^{++-}})}(u)+d_{(\overline{G^{++-}})}(v)}{2}$+$\sum_{st\in E_y}\frac{d_{(\overline{G^{++-}})}(s)+d_{(\overline{G^{++-}})}(t)}{2}+\sum_{eu\in E_z}\frac{d_{(\overline{G^{++-}})}(u)+d_{(\overline{G^{++-}})}(e)}{2}$

=$\sum_{uv\in E_x}\frac{({}^{n}_{2}C-m)+({}^{n}_{2}C-m)}{2}$+$\sum_{st\in E_y}\frac{[3(n-2)-d_G(a)-d_G(b)]+[3(n-2)-d_G(b)-d_G(c)]}{2}+\sum_{eu\in E_z}\frac{({}^{n}_{2}C-m)+[3(n-2)-d_G(v)-d_G(w)]}{2}$

=$\sum_{uv\in E_x}({}^{n}_{2}C-m)$+$\sum_{st\in E_y}\frac{6(n-2)-d_G(a)-2d_G(b)-d_G(c)}{2}+\sum_{eu\in E_z}\frac{({}^{n}_{2}C-m)+[3(n-2)-d_G(v)-d_G(w)]}{2}$.

Theorem 3.3: Let G be a graph with n vertices and m edges. Then SK($\overline{G^{+-+}}$)= $\sum_{uv\in E_x}\frac{4(n-1)-d_G(u)-d_G(v)}{2}$ + $\sum_{st\in E_y}\frac{2[\binom{n}{2}(n-5)-m+5]+d_G(a)+d_G(b)+d_G(c)+d_G(d)}{2}+\sum_{eu\in E_z}\frac{[2(n-1-d_G(u)]+\binom{n}{2}(n-5)-m+5+d_G(u)+d_G(u)}{2}$,where s = ab, t = cd∈ $E(G)$,e = uv ∈ $E(G)$; u, v, a, b, c,d ∈ $V(G)$ and are distinct.

Proof. By using the proposition if u∈ V(G) then $d_{(\overline{G^{+-+}})}$(u) = 2(n-1-d_G(u)) and if e∈ E(G) then $d_{(\overline{G^{+-+}})}$(e) = $\binom{n}{2}$(n-5)-m+5+d_G(u)+d_G(v).

Therefore SK($\overline{G^{+-+}}$)=$\sum_{uv\in E(\overline{G^{+-+}})}\frac{d_{\overline{(G^{+-+})}}(u)+d_{\overline{(G^{+-+})}}(v)}{2}$

=$\sum_{uv\in E_x}\frac{d_{\overline{(G^{+-+})}}(u)+d_{\overline{(G^{+-+})}}(v)}{2}$+$\sum_{st\in E_y}\frac{d_{\overline{(G^{+-+})}}(s)+d_{\overline{(G^{+-+})}}(t)}{2}+\sum_{eu\in E_z}\frac{d_{\overline{(G^{+-+})}}(u)+d_{\overline{(G^{+-+})}}(e)}{2}$

= $\sum_{uv\in E_x}\frac{2[(n-1-d_G(u)]+2[n-1-d_G(v)]}{2}$ + $\sum_{st\in E_y}\frac{[\binom{n}{2}(n-5)-m+5+d_G(a)+d_G(b)]+[\binom{n}{2}(n-5)-m+5+d_G(c)+d_G(d)]}{2}$
+$\sum_{eu\in E_z}\frac{[2(n-1-d_G(u)]+[\binom{n}{2}(n-5)-m+5+d_G(u)+d_G(v)]}{2}$

= $\sum_{uv\in E_x}\frac{4(n-1)-d_G(u)-d_G(v)}{2}$ + $\sum_{st\in E_y}\frac{2[\binom{n}{2}(n-5)-m+5]+d_G(a)+d_G(b)+d_G(c)+d_G(d)}{2}$ + $\sum_{eu\in E_z}\frac{[2(n-1-d_G(u)]+\binom{n}{2}(n-5)-m+5+d_G(u)+d_G(u)}{2}$.

Theorem 3.4: Let G be a graph with n vertices and m edges. ThenSK($\overline{G^{-++}}$)= $\sum_{uv\in E_x}(n-1)$+$\sum_{st\in E_y}\frac{4(n-1)-d_G(a)-2d_G(b)-d_G(c)]}{2}+\sum_{eu\in E_z}\frac{3(n-1)-d_G(u)-d_G(v)}{2}$,

where s = ab, t = bc∈ $E(G)$, e = uv ∈ $E(G)$; u, v, a, b, c ∈ $V(G)$ and are distinct.

Proof. By using the proposition if u∈ V(G) then $d_{\overline{(G^{-++})}}(u)=n-1$ and if e∈ E(G) then $d_{\overline{(G^{-++})}}(e)$ =2(n-1)-d_G(u)-d_G(v).

Therefore SK($\overline{G^{-++}}$)=$\sum_{uv\in E(\overline{G^{-++}})}\frac{d_{\overline{(G^{-++})}}(u)+d_{\overline{(G^{-++})}}(v)}{2}$

=$\sum_{uv\in E_x}\frac{d_{\overline{(G^{-++})}}(u)+d_{\overline{(G^{-++})}}(v)}{2}$+$\sum_{st\in E_y}\frac{d_{\overline{(G^{-++})}}(s)+d_{\overline{(G^{-++})}}(t)}{2}+\sum_{eu\in E_z}\frac{d_{\overline{(G^{-++})}}(u)+d_{\overline{(G^{-++})}}(e)}{2}$

=$\sum_{uv\in E_x}\frac{(n-1)+(n-1)}{2}$+$\sum_{st\in E_y}\frac{[2(n-1)-d_G(a)-d_G(b)]+[2(n-1)-d_G(b)-d_G(c)]}{2}+\sum_{eu\in E_z}\frac{(n-1)+2(n-1)-d_G(u)-d_G(v)}{2}$

=$\sum_{uv\in E_x}(n-1)$+$\sum_{st\in E_y}\frac{4(n-1)-d_G(a)-2d_G(b)-d_G(c)]}{2}+\sum_{eu\in E_z}\frac{3(n-1)-d_G(u)-d_G(v)}{2}$.

Similar to above we get the following theorems.

Theorem 3.5: Let G be a graph with n vertices and m edges. Then

(i)SK($\overline{G^{---}}$) = $\sum_{uv\in E_x}[{}^{n}_{2}\text{C}-\text{n}-\text{m}+1]+[d_G(\text{u})+d_G(\text{v})]$ + $\sum_{st\in E_y}\frac{2[{}^{n}_{2}\text{C}-\text{m}-\text{n}+1]+d_G(\text{a})+d_G(\text{b})+d_G(\text{c})+d_G(\text{d})}{2}$ + $\sum_{eu\in E_z}\frac{2({}^{n}_{2}\text{C}-\text{m}-\text{n}+1)+2d_G(\text{u})+d_G(\text{v})+d_G(\text{w})}{2}$,where s = ab, t = cd∈ $E(G)$,e = vw ∈ $E(G)$; v,w, a, b, c,d ∈ $V(G)$ and are distinct.

(ii)SK($\overline{G^{--+}}$)= $\sum_{uv\in E_x}(n-1)$ + $\sum_{st\in E_y}\frac{2[\binom{n}{2}(\text{n}-5)-\text{m}+5]+d_G(\text{a})+d_G(\text{b})+d_G(\text{c})+d_G(\text{d})}{2}$ + $\sum_{eu\in E_z}\frac{(n-1)+\binom{n}{2}(\text{n}-5)-\text{m}+5+d_G(\text{u})+d_G(\text{v})}{2}$,where s = ab,t = cd∈ $E(G)$, e = uv ∈ $E(G)$; u, v, a, b, c,d ∈ $V(G)$ and are distinct.

(iii) SK($\overline{G^{-+-}}$)= $\sum_{uv\in E_x}[{}^{n}_{2}\text{C}+d_G(\text{u})-\text{n}-\text{m}+1+d_G(\text{v})]$ + $\sum_{st\in E_y}\frac{2[3(\text{n}-2)]-d_G(\text{a})-2d_G(\text{b})-d_G(\text{c})}{2}$ + $\sum_{eu\in E_z}\frac{{}^{n}_{2}\text{C}+2d_G(\text{u})-\text{n}-\text{m}+1+3(n-2)-d_G(\text{v})-d_G(\text{w})}{2}$,where s = ab, t = bc∈ $E(G)$,e = vw ∈ $E(G)$; v,w, a, b, c ∈ $V(G)$ and are distinct.

(iv)SK($\overline{G^{+--}}$) = $\sum_{uv\in E_x}({}^{n}_{2}C-m)+\sum_{st\in E_y}\frac{2[{}^{n}_{2}C-\text{m}-\text{n}+1]+d_G(\text{a})+d_G(\text{b})+d_G(\text{c})+d_G(\text{d})}{2}+\sum_{eu\in E_z}\frac{2({}^{n}_{2}C-\text{m})-\text{n}+1+d_G(\text{v})+d_G(\text{w})}{2}$,where s = ab, t = cd∈ $E(G)$,e = vw ∈ $E(G)$; v,w,a,b,c,d ∈ $V(G)$ and are distinct.

Similar to theorems 1.1 to 1.7 and using the analogous technique we have the following for SK_1(G) indices:

Theorem 4.1: Let G be a graph with n vertices and m edges. Then:

(i) $SK_1(G^{++})$= $4SK_1(G^{++})+\sum_{ue\in E_2}2d_{(G)}(u)$.

(ii) $SK_1(G^{+-})$= $\frac{m^3}{2}$ +m(n-2)$\frac{m*(n-2)}{2}$.

(iii) $SK_1(G^{-+})$=($\frac{n(n-1)-2m}{2}$)$\frac{(n-1)^2}{2}$ +2m(n-1).

(iv) $SK_1(\overline{G^{++}})$=$\sum_{uv\notin E(G)}\frac{[n+m-1-2d_G(u)]*[n+m-1-2d_G(v)]}{2}$+$\sum_{ue\in E_2}\frac{[n+m-1-2d_G(u)]*[n+m-3]}{2}+\frac{m(m-1)}{2}\frac{(n+m-3)^2}{2}$.

(v) SK_1 ($\overline{G^{+-}}$)= $\frac{[n(n-1)-2m](n-1)^2}{2}$+m$(n-1)(m+1)$+$\frac{m(m-1)*(m+1)^2}{4}$.

(vi) SK$_1$($\overline{G^{-+}}$)= $\frac{m^3}{2}$+m(n-2)($\frac{m*(n+m-3)}{2}$)+$m(m-1)\frac{(n+m-3)^2}{4}$.

(vii) SK$_1$($\overline{G^{--}}$)= m [$2d_{(G)}(u)d_{(G)}(v)$]+ 2m[$d_{(G)}(u)(m+1)$]+$\frac{m(m-1)(m+1)^2}{4}$.

SK$_1$index of transformation graphs G^{xyz}

Theorem 5.1: Let G be a graph with n vertices and m edges.

Then SK$_1$ (G^{+++}) =4SK$_1$(G)+$\sum_{st\in E_y}\frac{[d_G(a)+d_G(b)]*[d_G(b)+d_G(c)]}{2}+\sum_{eu\in E_z}d_G(u)*[d_G(u)+d_G(v)]$,

where s = ab, t = bc$\in E(G)$,e = uv $\in E(G)$; u,v,a,b,c $\in V(G)$ and are distinct.

Proof. Partition the edge set E(G^{+++})in three sets E$_x$, E$_y$ and E$_z$, where E$_x$ = {uv | u,v∈V(G)},E$_y$ = {st | s,t∈E(G)} and E$_z$ = {ue| u∈ V(G),e∈ E(G)}.By using the proposition if u∈ V(G) then $d_{(G^{+++})}$(u) = $2d_{(G)}(u)$ and if e∈ E(G) then $d_{(G^{+++})}(e)=d_{(G)}(u)+d_{(G)}(v)$.

Therefore SK$_1$(G^{+++})=$\sum_{uv\in E(G^{+++})}\frac{d_{(G^{+++})}(u)*d_{(G^{+++})}(v)}{2}$

$=\sum_{uv\in E_x}\frac{d_{(G^{+++})}(u)*d_{(G^{+++})}(v)}{2}+\sum_{ue\in E_y}\frac{d_{(G^{+++})}(s)*d_{(G^{+++})}(t)}{2}+\sum_{ef\in E_z}\frac{d_{(G^{+++})}(u)*d_{(G^{+++})}(e)}{2}$

$=\sum_{uv\in E_x}\frac{2d_G(u)*2d_G(v)}{2}+\sum_{st\in E_y}\frac{[d_G(a)+d_G(b)]*[d_G(b)+d_G(c)]}{2}+\sum_{eu\in E_z}\frac{2d_G(u)*[d_G(u)+d_G(v)]}{2}$

=4SK$_1$(G)+$\sum_{st\in E_y}\frac{[d_G(a)+d_G(b)]*[d_G(b)+d_G(c)]}{2}+\sum_{eu\in E_z}d_G(u)*[d_G(u)+d_G(v)]$.

Theorem 5.2: Let G be a graph with n vertices and m edges.

Then SK$_1$(G^{++-})=$\sum_{uv\in E_x}\frac{m^2}{2}+\sum_{st\in E_y}\frac{[d_G(a)+d_G(b)+n-4]*[d_G(b)+d_G(c)+n-4]}{2}+\sum_{ue\in E_z}\frac{m*[d_G(v)+d_G(w)+n-4]}{2}$,where s= ab, t = bc$\in E(G)$,e = vw $\in E(G)$; u,v,w,a,b,c $\in V(G)$ and are distinct.

Proof. By using the proposition if u∈ V(G) then$d_{(G^{++-})}$(u) = m and if e∈ E(G) then$d_{(G^{++-})}$(e) =d_G(u)+d_G(v)+n-4.

Therefore SK$_1$(G^{++-})=$\sum_{uv\in E(G^{++-})}\frac{d_{(G^{++-})}(u)*d_{(G^{++-})}(v)}{2}$

$=\sum_{uv\in E_x}\frac{d_{(G^{++-})}(u)*d_{(G^{++-})}(v)}{2}+\sum_{st\in E_y}\frac{d_{(G^{++-})}(s)*d_{(G^{++-})}(t)}{2}+\sum_{eu\in E_z}\frac{d_{(G^{++-})}(u)*d_{(G^{++-})}(e)}{2}$

$=\sum_{uv\in E_x}\frac{m*m}{2}+\sum_{st\in E_y}\frac{[d_G(a)+d_G(b)+n-4]*(b)+d_G(c)+n-4]}{2}+\sum_{ue\in E_z}\frac{m*[d_G(v)+d_G(w)+n-4]}{2}$

$=\sum_{uv\in E_x}\frac{m^2}{2}+\sum_{st\in E_y}\frac{[d_G(a)+d_G(b)+n-4]*[d_G(b)+d_G(c)+n-4]}{2}+\sum_{ue\in E_z}\frac{m*[d_G(v)+d_G(w)+n-4]}{2}$.

Theorem 5.3: Let G be a graph with n vertices and m edges.

Then SK$_1$(G^{+-+})=4SK$_1$(G)+ $\sum_{st\in E_y}\frac{[m-d_G(a)-d_G(b)+3]*[m-d_G(c)-d_G(d)+3]}{2}+\sum_{eu\in E_z}d_G(u)*[m-d_G(u)-$ *dGv+3]*,where s = ab, t = cd$\in E(G)$,e = uv $\in E(G)$; u,v,a,b,c,d $\in V(G)$ and are distinct.

Proof.By using the proposition if u∈ V(G) then $d_{(G^{+-+})}$(u) = $2d_G$(u) and if e∈ E(G) then $d_{(G^{+-+})}$(e) =m-d_G(u)-d_G(v)+3.

Therefore SK$_1$(G^{+-+})=$\sum_{uv\in E(G^{+-+})}\frac{d_{(G^{+-+})}(u)*d_{(G^{+-+})}(v)}{2}$

$=\sum_{uv\in E_x}\frac{d_{(G^{+-+})}(u)*d_{(G^{+-+})}(v)}{2}+\sum_{st\in E_y}\frac{d_{(G^{+-+})}(s)*d_{(G^{+-+})}(t)}{2}+\sum_{eu\in E_z}\frac{2d_{(G^{+-+})}(u)*d_{(G^{+-+})}(e)}{2}$

$=\sum_{uv\in E_x}\frac{2d_G(u)*2d_G(v)}{2}+\sum_{st\in E_y}\frac{[m-d_G(a)-d_G(b)+3]*[m-d_G(c)-d_G(d)+3]}{2}+\sum_{eu\in E_z}\frac{2d_G(u)*[m-d_G(u)-d_G(v)+3]}{2}$

=4SK$_1$(G)+$\sum_{st\in E_y}\frac{[m-d_G(a)-d_G(b)+3]*[m-d_G(c)-d_G(d)+3]}{2}+\sum_{eu\in E_z}d_G(u)*[m-d_G(u)-d_G(v)+3]$.

Theorem 5.4: Let G be a graph with n vertices and m edges.

Then SK$_1$(G^{-++}) =$[\binom{n}{2}-m]\frac{(n-1)^2}{2}+\sum_{st\in E_y}\frac{[d_G(a)+d_G(b)]*[d_G(b)+d_G(c)]}{2}+\sum_{eu\in E_z}\frac{(n-1)*[d_G(u)+d_G(v)]}{2}$,

where s = ab, t = bc$\in E(G)$,e = uv $\in E(G)$; u,v,a,b,c $\in V(G)$ and are distinct.

Proof. By using the proposition if u∈ V(G) then $d_{(G^{-++})}$(u) = n-1 and if e∈ E(G) then $d_{(G^{-++})}$(e) = d_G(u)+d_G(v).

Therefore SK$_1$(G^{-++}) =$\sum_{uv\in E(G^{-++})}\frac{d_{(G^{-++})}(u)*d_{(G^{-++})}(v)}{2}$

$=\sum_{uv\in E_x}\frac{d_{(G^{-++})}(u)*d_{(G^{-++})}(v)}{2}+\sum_{st\in E_y}\frac{d_{(G^{-++})}(s)*d_{(G^{-++})}(t)}{2}+\sum_{eu\in E_z}\frac{d_{(G^{-++})}(u)*d_{(G^{-++})}(e)}{2}$

$=\sum_{uv\in E_x}\frac{(n-1)*(n-1)}{2}+\sum_{st\in E_y}\frac{[d_G(a)+d_G(b)]*[d_G(b)+d_G(c)]}{2}+\sum_{eu\in E_z}\frac{(n-1)*[d_G(u)+d_G(v)]}{2}$

$=[\binom{n}{2}-m]\frac{(n-1)^2}{2}+\sum_{st\in E_y}\frac{[d_G(a)+d_G(b)]*[d_G(b)+d_G(c)]}{2}+\sum_{eu\in E_z}\frac{(n-1)*[d_G(u)+d_G(v)]}{2}$.

Similar to above we get the following theorems:

Theorem 5.5: Let G be a graph with n vertices and m edges. Then:

(i)$SK_1(G^{---})=\sum_{u,v\in E_x}\frac{[n+m-2d_G(u)-1]*[n+m-2d_G(v)-1]}{2}+\sum_{st\in E_y}\frac{[(n+m-1)-d_G(a)-d_G(b)]*[(n+m-1)-d_G(c)-d_G(d)]}{2}+\sum_{eu\in E_z}\frac{[n+m-2d_G(u)-1]*[n+m-d_G(v)-d_G(w)-1]}{2}$,where s = ab, t = cd$\in E(G)$,e = vw $\in E(G)$; u,v,w,a,b,c,$d\in V(G)$ and are distinct.

(ii) $SK(G^{--+})=[\binom{n}{2}-m]\frac{(n-1)^2}{2}+\sum_{st\in E_y}\frac{[m-d_G(a)-d_G(b)+3]*[m-d_G(c)-d_G(d)+3]}{2}+\sum_{ue\in E_z}\frac{(n-1)*[m-d_G(u)-d_G(v)+3]}{2}$,where s = ab, t = cd$\in E(G)$,e = uv $\in E(G)$; u,v,a,b,c,$d\in V(G)$ and are distinct.

(iii)$SK_1(G^{-+-})=\sum_{uv\in E_x}\frac{[n+m-1-2d_G(u)]*[n+m-1-2d_G(v)]}{2}+\sum_{st\in E_y}\frac{[n+d_G(a)+d_G(b)-4]*[n+d_G(b)+d_G(c)-4]}{2}+\sum_{eu\in E_z}\frac{[n+m-1-2d_G(u)]*[n+d_G(v)+d_G(w)-4]}{2}$,where s = ab, t = bc$\in E(G)$,e = vw $\in E(G)$; u,v,w,a,b,c $\in V(G)$ and are distinct.

(iv)$SK_1(G^{+--})=\sum_{uv\in E_x}\frac{m^2}{2}+\sum_{st\in E_y}\frac{[(n+m-1)-d_G(a)-d_G(b)]*[(n+m-1)-d_G(c)-d_G(d)]}{2}+\sum_{eu\in E_z}\frac{m*[m+n-d_G(v)-d_G(w)-1]}{2}$,where s = ab, t = cd$\in E(G)$,e = vw $\in E(G)$; u, v,w, a, b, c,d $\in V(G)$ and are distinct.

SK_1index of transformation graphs $\overline{G^{xyz}}$

Theorem 6.1: Let G be a graph with n vertices and m edges. Then $SK_1(\overline{G^{+++}})=\sum_{uv\in E_x}(n-1-d_G(u))*$ *2(n−1−dGv)+st∈Ey[2(n−1)−dGa−dGb]*[2n−1−dGb−dGc]2+eu∈Ezn−1−dGu*[2(n−1)−dGu−dGv],* where s = ab, t = bc$\in E(G)$,e = uv $\in E(G)$; u, v, a, b, c $\in V(G)$ and are distinct.

Proof. By using the proposition if u$\in$ V(G) then $d_{(\overline{G^{+++}})}$(u) = 2(n-1-d_G(u)) and if e$\in$ E(G) then $d_{(\overline{G^{+++}})}$(e) = 2(n-1)-d_G(u)-d_G(v).

Therefore $SK_1(\overline{G^{+++}})=\sum_{uv\in E(\overline{G^{+++}})}\frac{d_{(\overline{G^{+++}})}(u)*d_{(\overline{G^{+++}})}(v)}{2}$

$=\sum_{uv\in E_x}\frac{d_{(\overline{G^{+++}})}(u)*d_{(\overline{G^{+++}})}(v)}{2}+\sum_{st\in E_y}\frac{d_{(\overline{G^{+++}})}(s)*d_{(\overline{G^{+++}})}(t)}{2}+\sum_{eu\in E_z}\frac{d_{(\overline{G^{+++}})}(u)*d_{(\overline{G^{+++}})}(e)}{2}$

$=\sum_{uv\in E_x}\frac{2(n-1-d_G(u))*2(n-1-d_G(v))}{2}+\sum_{st\in E_y}\frac{[2(n-1)-d_G(a)-d_G(b)]*[2(n-1)-d_G(b)-d_G(c)]}{2}+\sum_{eu\in E_z}\frac{2(n-1-d_G(u))*[2(n-1)-d_G(u)-d_G(v)]}{2}$

$=\sum_{uv\in E_x}(n-1-d_G(u))*2(n-1-d_G(v))+\sum_{st\in E_y}\frac{[2(n-1)-d_G(a)-d_G(b)]*[2(n-1)-d_G(b)-d_G(c)]}{2}+\sum_{eu\in E_z}(n-1-d_G(u))*[2(n-1)-d_G(u)-d_G(v)]$.

Theorem 6.2: Let G be a graph with n vertices and m edges. Then $SK_1(\overline{G^{++-}})=\sum_{uv\in E_x}\frac{({}^{n}_{2}C-m)^2}{2}+\sum_{st\in E_y}\frac{[3(n-2)-d_G(a)-d_G(b)]*[3(n-2)-d_G(b)-d_G(c)]}{2}+\sum_{eu\in E_z}\frac{({}^{n}_{2}C-m)[3(n-2)-d_G(v)-d_G(w)]}{2}$,where s = ab, t = bc$\in E(G)$,e = vw $\in E(G)$; u, v, w,a, b, c $\in V(G)$ and are distinct.

Proof. By using the proposition if u$\in$ V(G) then $d_{(\overline{G^{++-}})}$(u) = ${}^{n}C_{2}$- m and if e$\in$ E(G) then $d_{(\overline{G^{++-}})}$(e) =3(n-2)-d_G(u)-d_G(v).

Therefore $SK_1(\overline{G^{++-}})=\sum_{uv\in E(\overline{G^{++-}})}\frac{d_{(\overline{G^{++-}})}(u)*d_{(\overline{G^{++-}})}(v)}{2}$

$=\sum_{uv\in E_x}\frac{d_{(\overline{G^{++-}})}(u)*d_{(\overline{G^{++-}})}(v)}{2}+\sum_{st\in E_y}\frac{d_{(\overline{G^{++-}})}(s)*d_{(\overline{G^{++-}})}(t)}{2}+\sum_{eu\in E_z}\frac{d_{(\overline{G^{++-}})}(u)*d_{(\overline{G^{++-}})}(e)}{2}$

$=\sum_{uv\in E_x}\frac{({}^{n}_{2}C-m)({}^{n}_{2}C-m)}{2}+\sum_{st\in E_y}\frac{[3(n-2)-d_G(a)-d_G(b)]*[3(n-2)-d_G(b)-d_G(c)]}{2}+\sum_{eu\in E_z}\frac{({}^{n}_{2}C-m)[3(n-2)-d_G(v)-d_G(w)]}{2}$

$=\sum_{uv\in E_x}\frac{({}^{n}_{2}C-m)^2}{2}+\sum_{st\in E_y}\frac{[3(n-2)-d_G(a)-d_G(b)]*[3(n-2)-d_G(b)-d_G(c)]}{2}+\sum_{eu\in E_z}\frac{({}^{n}_{2}C-m)[3(n-2)-d_G(v)-d_G(w)]}{2}$.

Theorem 6.3: Let G be a graph with n vertices and m edges. Then $SK_1(\overline{G^{+-+}})=\sum_{uv\in E_x}2[(n-1-d_G(u)]*$ *[n−1−dGv]+**st∈Ey[n2(n−5)−m+5+dGa+dGb]***[**n2(n−5)−m+5+dGc+dGd]2+eu∈Ez[2n−1−dGu*** [**n2n−5−m+5+dGu+dGv]2**,*where s = ab, t = cd*∈E(G)*,e = uv *∈E(G)*; u, v, a, b, c,d *∈V(G)* and are distinct.

Proof. By using the proposition if u∈ V(G) then $d_{\overline{(G^{+-+})}}$(u) = 2(n-1-d_G(u)) and if e∈ E(G) then $d_{\overline{(G^{+-+})}}$(e) = $\binom{n}{2}$(n-5)-m+5+d_G(u)+d_G(v).

Then $SK_1(\overline{G^{+-+}}) = \sum_{uv\in E(\overline{G^{+-+}})} \frac{d_{\overline{(G^{+-+})}}(u)*d_{\overline{(G^{+-+})}}(v)}{2}$

$= \sum_{uv\in E_x} \frac{d_{\overline{(G^{+-+})}}(u)*d_{\overline{(G^{+-+})}}(v)}{2} + \sum_{st\in E_y} \frac{d_{\overline{(G^{+-+})}}(s)*d_{\overline{(G^{+-+})}}(t)}{2} + \sum_{eu\in E_z} \frac{d_{\overline{(G^{+-+})}}(u)*d_{\overline{(G^{+-+})}}(e)}{2}$

$= \sum_{uv\in E_x} \frac{2[(n-1-d_G(u)]*2[n-1-d_G(v)]}{2} + \sum_{st\in E_y} \frac{[\binom{n}{2}(n-5)-m+5+d_G(a)+d_G(b)]*[\binom{n}{2}(n-5)-m+5+d_G(c)+d_G(d)]}{2}$
$+ \sum_{eu\in E_z} \frac{[2(n-1-d_G(u)]*[\binom{n}{2}(n-5)-m+5+d_G(u)+d_G(v)]}{2}$

$= \sum_{uv\in E_x} 2[(n-1-d_G(u)]*[n-1-d_G(v)] + \sum_{st\in E_y} \frac{[\binom{n}{2}(n-5)-m+5+d_G(a)+d_G(b)]*[\binom{n}{2}(n-5)-m+5+d_G(c)+d_G(d)]}{2} +$
$\sum_{eu\in E_z} \frac{[2(n-1-d_G(u)]*[\binom{n}{2}(n-5)-m+5+d_G(u)+d_G(v)]}{2}$.

Theorem 6.4: Let G be a graph with n vertices and m edges. Then $SK_1(\overline{G^{-++}})$ $= \sum_{uv\in E_x} \frac{(n-1)^2}{2} + \sum_{st\in E_y} \frac{[2(n-1)-d_G(a)-d_G(b)]*[2(n-1)-d_G(b)-d_G(c)]}{2} + \sum_{eu\in E_z} \frac{(n-1)*[2(n-1)-d_G(u)-d_G(v)]}{2}$,
where s = ab, t = bc∈ $E(G)$, e = uv ∈ $E(G)$; u, v, a, b, c ∈ $V(G)$ and are distinct.

Proof. By using the proposition if u∈ V(G) then $d_{\overline{(G^{-++})}}(u) = n-1$ and if e∈ E(G) then $d_{\overline{(G^{-++})}}(e)$ =2(n-1)-d_G(u)-d_G(v).

Then $SK_1(\overline{G^{-++}}) = \sum_{uv\in E(\overline{G^{-++}})} \frac{d_{\overline{(G^{-++})}}(u)*d_{\overline{(G^{-++})}}(v)}{2}$

$= \sum_{uv\in E_x} \frac{d_{\overline{(G^{-++})}}(u)*d_{\overline{(G^{-++})}}(v)}{2} + \sum_{st\in E_y} \frac{d_{\overline{(G^{-++})}}(s)*d_{\overline{(G^{-++})}}(t)}{2} + \sum_{eu\in E_z} \frac{d_{\overline{(G^{-++})}}(u)*d_{\overline{(G^{-++})}}(e)}{2}$

$= \sum_{uv\in E_x} \frac{(n-1)*(n-1)}{2} + \sum_{st\in E_y} \frac{[2(n-1)-d_G(a)-d_G(b)]*[2(n-1)-d_G(b)-d_G(c)]}{2} + \sum_{eu\in E_z} \frac{(n-1)*[2(n-1)-d_G(u)-d_G(v)]}{2}$
$= \sum_{uv\in E_x} \frac{(n-1)^2}{2} + \sum_{st\in E_y} \frac{[2(n-1)-d_G(a)-d_G(b)]*[2(n-1)-d_G(b)-d_G(c)]}{2} + \sum_{eu\in E_z} \frac{(n-1)*[2(n-1)-d_G(u)-d_G(v)]}{2}$.

Similar to above we get the following theorems:

Theorem 6.5: Let G be a graph with n vertices and m edges. Then

(i) $SK_1(\overline{G^{---}}) = \sum_{uv\in E_x} \frac{[{}^{n}_{2}C+2d_G(u)-n-m+1]*[{}^{n}_{2}C+2d_G(v)-n-m+1]}{2} + \sum_{st\in E_y} \frac{[{}^{n}_{2}C-m-n+1+d_G(a)+d_G(b)]*[{}^{n}_{2}C-m-n+1+d_G(c)+d_G(d)]}{2} + \sum_{eu\in E_z} \frac{({}^{n}_{2}C+2d_G(u)-m-n+1)*({}^{n}_{2}C-m-n+1+d_G(v)+d_G(w)}{2}$,where s = ab, t = cd∈ $E(G)$,e = vw ∈ $E(G)$; v,w, a, b, c,d ∈ $V(G)$ and are distinct.

(ii) $SK_1(\overline{G^{--+}}) = \sum_{uv\in E_x} \frac{(n-1)^2}{2} + \sum_{st\in E_y} \frac{\left[\binom{n}{2}(n-5)-m+5+d_G(a)+d_G(b)\right]*[\binom{n}{2}(n-5)-m+5+d_G(c)+d_G(d)]}{2} + \sum_{eu\in E_z} \frac{(n-1)*[\binom{n}{2}(n-5)-m+5+d_G(u)+d_G(v)]}{2}$,where s = ab, t = cd∈ $E(G)$, e = uv ∈ $E(G)$; u, v, a, b, c,d ∈ $V(G)$ and are distinct.

(iii) $SK_1(\overline{G^{-+-}}) = \sum_{uv\in E_x} \frac{[{}^{n}_{2}C+2d_G(u)-n-m+1]*[{}^{n}_{2}C+2d_G(v)-n-m+1]}{2} + \sum_{st\in E_y} \frac{[3(n-2)-d_G(a)-d_G(b)]*[3(n-2)-d_G(b)-d_G(c)]}{2} + \sum_{eu\in E_z} \frac{[{}^{n}_{2}C+2d_G(u)-n-m+1]*[3(n-2)-d_G(v)-d_G(w)]}{2}$,where s = ab, t = bc∈ $E(G)$,e = vw ∈ $E(G)$; v,w, a, b, c ∈ $V(G)$ and are distinct.

(iv) $SK_1(\overline{G^{+--}}) = \sum_{uv\in E_x} \frac{({}^{n}_{2}C-m)^2}{2} + \sum_{st\in E_y} \frac{[{}^{n}_{2}C-m-n+1+d_G(a)+d_G(b)]*[{}^{n}_{2}C-m-n+1+d_G(c)+d_G(d)]}{2} + \sum_{eu\in E_z} \frac{({}^{n}_{2}C-m)*[{}^{n}_{2}C-m-n+1+d_G(v)+d_G(w)]}{2}$,where s = ab, t = cd∈ $E(G)$,e = vw ∈ $E(G)$; v,w,a,b,c,d ∈ $V(G)$ and are distinct.

IV. CONCLUSION

The complement of G is denoted by $\overline{G}$.If G has n vertices and m edges then the number of vertices of $G^{xy}$are n+m. On the basis of point vertex and line vertex of generalized transformation graphs and their complements the SK index and SK_1 index for G^{xy}, $\overline{G^{xy}}$, G^{xyz}, $\overline{G^{xyz}}$ generalized transformation graphs are studied.

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