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Research Paper

Topological Near Homomorphisms

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ABSTRACT: *Here, some properties related to topological near subgroups are discussed. It is prove that the product of topological near groups is a topological near group. In this paper, Topological near group homomorphisms are introduced and studied. Finally, near action, near homogenous space and near kernel are defined and studied.*
Keywords: *Near groups, topological near groups, topological sub near groups, product of topological near groups, topological near group homomorphisms, topological near homogeneous space,near kernel.*
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I. INTRODUCTION

Rough set, as a mathematical theory for dealing with imprecise, uncertain and in-complete data, was first introduced by pawlak (1982).Its main idea is to use the known incomplete information or knowledge to approximately describe the concept of imprecise or uncertain, or to deal with ambiguous phenomena and problems ac-cording to the results of observation and measurement. After more than 30 years of research, the theory of rough set has been continuously improved and widely expanded in applications, see Wu and Mi (2019). At present, it has been successfully applied in machine learning and knowledge discovery, information system analysis, data mining, decision support system, fault detection, process control, pattern recognition, etc.

In [7],Bagirmaz et al. Introduced the concept of topological rough groups they ex-tended the notion of a topological group to include algebraic structures of rough groups.In addition they presented some examples and properties.

In2002,J.F.Peters developed the nearest theory as a generalization of rough set theory. peters utilized the features of objects to develop the nearness of objects [23]and consequently, the classified our universal, set with respect to the object information available. The near set approach leads to partitions of ensembles of sample objects with measurable information content and an approach to feature selection. A probe function is are a valued function representing a feature of physical objects such as images or behaviors of individual biological organisms.

The main purpose of this paper is to introduce some basic definitions and results about topological near groups and topological near subgroups. We also introduce the Cartesian product of topological near groups.

In this paper, we present near action and near homogenous spaces, and discuss someof their properties we also define a near kernel. We organize the paper as follows, In Section 2 we collect the needed material about near groups and near homomorphisms. Then the definitions of topological near groups and important properties have been recalled in section3,section 4 presents our main results where we introduce, near action and homogenous spaces.

II. PRELIMINARIES

In this section, some definitions and results about near sets, near groups and topological groups used in this paper are given

Object Description [21]

Table1: Description Symbols

Symbol	Interpretation
$\mathcal{R}$	Set of Real numbers
O	Set of perceptual objects
X	$X \subseteq O$,Set of sample objects
x	$x \in O$, Sample perceptual objects
F	A set of functions representing object features
B	$B \subseteq F$ set of functions representing object features
ψ	$\psi: O \to \mathcal{R}^L$,object description
L	L is a description length
i	$i \leq L$
ψ_i	$\psi \in B$ where $\psi: X \to \mathcal{R}$, probe function
$\psi(x)$	$\psi(x) = (\psi_1(x), \psi_2(x), \psi_3(x) \dots \psi_i(x) \dots . \psi_L(x)$

Objects are known by their description. An object description is defined by means of a tuple offunction values ψ (x) associated withanobject x ∈ X .The important thing to notice is the choiceof functions $\psi_i \in$ B used to describe an object of interest.

The intuition underlying a description ψ (x) is recording of measurements from sensors, where each sensor is modeled by a function ψ_i.

Assume that B ⊆ F is a given set of functions representing features of sample objectsX∈O. Let $\varphi_i \in$ B, whereφ_i :O → $\mathfrak{R}$. The value of ψ_i(x) is a measurement associated with a feature of an object x ∈X .The function $\psi_i$is called a probe. In combination, the functions representing object features provide a basis for an object description $\psi : O \to \mathcal{R}^L$, a vector containing measurements (returned values) associated with each functional value ψ_i(x), where the description length $|\psi|$=L.

Object Description:$\psi(x) = (\psi_1(x), \psi_2(x), \psi_3(x) \dots \psi_i(x) \dots . \psi_L(x))$

Nearness Objects [21]

Sample objects X ⊆ O are near each other if, and only if the objects have similar descriptions. Recall that each description ψ^1defines a description of an object. Then let Δ_{ψ_i}denote

$$\Delta_{\psi_i} = \psi_i(x') - \psi_i(x)$$

where x, x′∈ O. The difference $\Delta_\psi$leads to a definition of the indiscernibility relation $\sim_B$ introduced by ZdzislawPawlak[12].

Table2:Set,Relation,ProbeFunctionSymbols

Symbol	Interpretation
$\sim_B$	$\{(x,x')\|f(x) = f(x')\forall f \in B\}$,indiscernibility relation
$[x]_B$	$\{[x]_B = \{x \in X\|x' \sim_B x\}$, elementary granule (class)
$O/\sim_B$	$O/\sim_B = \{ [x]_B\|x \in O\}$,quotient set
ξ_B	Partition $\xi_B = O/\sim_B$
Δ_{ψ_i}	$\Delta_{\psi_i} = \psi_i(x') - \psi_i(x)$, probe functions difference

Definition2.1.[21]

Let X,X′⊆O, B⊆F. Set X is near X′ if and only if there exists x∈X, x′∈X′, ψ_i∈B such that x$\sim_{\psi_i}$x′.

Remark 2.2.[21] If X is near X ′, then X is a near set relative to X ′ and X ′ is anear set relative to X.

Definition 2.3.[21]Let X ⊆ O and x, x′ ∈X. If x is near x′, then X is called anear set relative to itself for the reflexive nearness of X.

Definition 2.4. [21] Let B ⊆ F be a set of functions representing features of objects x,x′∈O. Objects x, x′ are called minimally near each other if there exists ψ_i∈B such that x$\sim_{\psi_i}$}x′,

Δψi=0.

Definition2.5.[21]Let x, x′∈ O, B⊆F. Then

$\sim_B = \{(x,x') \in O \times O | \forall \psi_i \in B, \Delta_{\psi_i} = 0\}$

Is called the indiscernibility relation on O, where the description length i≤| ψ|.

Theorem2.6.[21]The objects in a class $[x]_B \in \xi_B$ are near objects.

Definition 2.7. [3]A topological group is a group (G, ∗) together with a topologyon G that satisfies the following two properties:

1. The mapping f:G×G→G defined by f(x,y)=xy is continuous when G is endowed with the product topology.
2. The inverse mapping g:G→G defined by $g(x)=x^{-1}$is continuous.

We remark that item (1) is equivalent to the statement that, whenever W ⊆ G is open, and W ∈ $N(x_1x_2)$, then there exists open sets V_1 and V_2 such that $V_1 \in N(x_1)$;$V_2 \in N(x_2)$ and $V_1V_2 = \{x_1x_2/x_1 \in V_1; x_2 \in V_2\} \subseteq W$. Also, item (2) is equivalent to showing that wheneverV⊆G is open, then $V^{-1}=\{x^{-1}|x \in V\} \in N(x^{-1})$ is open. Let G be a topological group and let H be a subgroup of G. Then H becomes a Topological group whenEndowed with the topology induced by G.

Definition 2.8.[12] Let NAS=(O, F, $\sim_{B_r}$, N_r, ν_{N_r}), be a nearness approximation space and let · be a binary operation defined on O.A subset G of perceptual objectsO is called a near group if the following properties are satisfied

1. ∀x,y∈G, x·y∈$N_r(B)*G$
2. ∀x,y,z∈G,(x·y)·z=x·(y·z) property holds in $N_r(B)*G$.
3. ∃e ∈$N_r(B)*G$such that∀x ∈ G, x · e = e · x = x, e is called the near identityelement of the group G.
4. ∀x∈G, ∃y∈Gsuch that ,x·y=y·x=e,y is called the near inverse element of x in G.

Proposition2.9. [12] Let G be a near group

(1)∀x,y∈H,x·y∈$N_r(B)*H$

(2)∀x∈H,x^{-1}∈H

(3) There is one and only one identity element in near group G.

(4) ∀x∈G,there is only one y such that x·y=y·x=e; we denote it by x^{-1}.

(5) $(x^{-1})^{-1}=x$.

(6) $(x \cdot y)^{-1}=y^{-1} \cdot x^{-1}$.

Definition2.10.[12] Let $G_1 \subset O_1$, $G_2 \subset O_2$ be neargroups. If there exists asurjection

$\phi: N_{r_1}(B)*G_1 \rightarrow \phi : N_{r_2}(B)*G_2$ such that ϕ(x · y) = ϕ(x) ∘ ϕ(y) for all x, y ∈$N_{r_1}(B)*G_1$

thenϕ is called a near homomorphism and G1, G2 are called nearhomomorphic groups.

Definition2.11.[17]A near group G with a topology τ on $N_r(B)*G$ is called a topological near group if the following hold

(a) f:G×G→$N_r(B)*G$ defined by f(a,b)=ab is continuous with respect to product topology on G×G and the topology $\tau_G$on G induced by τ.

(b) τ:G →G defined by $\tau(a)=a^{-1}$is continuous with respect to the topology τ_G on G induced by τ.

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Definition2.12.[17]Let G be a topological near group.For a fixed element a in G,we define

1. A mapping L_a:G→$N_r(B)*G$ which is defined by $L_a(x)$=ax,is called a left transformation from G is to $N_r(B)*G$
2. A mapping R_a:G→$N_r(B)*G$ which is defined by $R_a(x)$=xa,is called a right transformation from G is to $N_r(B)*G$

Definition2.13.[17]Let G be a topological near group.Then

1. The left transformation map L_a:G→$N_r(B)*G$ is continuous and one-to-one.
2. The right transformation map R_a:G→$N_r(B)*G$ is continuous and one-to-one.
3. The inverse mapping τ:G→G is a homeomorphisms for all x∈G

III. CARTESIAN PRODUCT OF TOPOLOGICAL NEAR GROUPS

In this section, we discuss some results on Cartesian products and introduce near action and near homogenous spaces in topology using near groups.

(X_1,F_1, $\sim_{B_{r_1}}$,N_{r_1})and(X_2,F_2,$\sim_{B_{r_2}}$,N_{r_2}) be two nearness approximation spaces and let$*_1$and$*_2$betwo binary operations on X_1 and X_2 respectively.For x,x_1 ∈ X_1 and y,y_1∈ X_2 we have(x,y) (x′,y′)∈ X_1× X_2

Define ∗ as,(x,y)∗(x_1,y_1)=(x∗$_1x_1$,y∗$_2y_1$)

Then ∗ is a binary operation on X_1×X_2. Indeed that the product of equivalencerelation , $\sim_{B_{r_1}}$ and $\sim_{B_{r_2}}$ is also an equivalence relation on X_1×X_2

Theorem3.1. Let G_1⊆X_1 and G_2⊆X_2be two near groups. Then the Cartesian product G_1×$G_2$is also anear group For,

1. ∀(a_1,b_1),(a_2,b_2)∈G_1×G_2,

∀(a_1,b_1)∗(a_2,b_2)=(a_1∗$_1a_2$;b_1∗$_2b_2$)∈N_r(B)*G_1×N_r(B)*G_2

2. Associative law is satisfied for all elements in N_r(B)*G_1×N_r(B)*G_2.

3. ∃an identity element (e,e′) ∈N_r(B)*G_1×N_r(B)*G_2 such that∀ (x,x′) ∈G_1×G_2, (x,x′) ×(e,e′) = (e,e′)×(x,x′) = (ex,e′x′) = (x,x′)

4. ∀(x,x$'$) ∈G_1×G_2,∃an element(y,y′) ∈G_1× G_2such that

(x,x$'$)∗(y,y$'$)=(x∗$_1$y,x$'$∗$_2$y$'$)=(y∗$_1$x,y$'$∗$_2$x$'$)=(y,y$'$)∗(x,x$'$)=(e,e$'$).

Example3.2. Let X={0,1,2}be a set of perceptual objects, B={ψ1,ψ2,ψ3}be a set of functions (∗) be the binary operation, addition modulo 3. Sample values of the probe function {ψ_i} are defined as,

ψ_1:X→V_1defined byψ_1(n)=n(n−1)∀n∈X

ψ_2:X→V_2defined byψ_2(n)=n^2∀n∈X

ψ_3:X→V_3defined by ψ_3(n)=n^2−n^3∀n∈X

	0	1	2
ψ_1	0	0	2
ψ_2	0	1	4
ψ_3	0	0	-4

Let us construct the equivalence classes for each combination,thus equivalence classes are defined as,

$[0]_{\{\psi_1\}} = \{x' \in X|\psi_1(x')=\psi_1(0) = 0\}$
= {0,1}
$[2]_{\{\psi_1\}} = \{x' \in X|\psi_1(x')=\psi_1(2) = 2\}$
= {2}
Hence we have$\xi_{\{\psi_1\}} = \{[0]_{\{\psi_1\}}, [2]_{\{\psi_1\}}\}$

$[0]_{\{\psi_2\}} = \{x' \in X|\psi_2(x')=\psi_2(0) = 0\}$
= {0}
$[1]_{\{\psi_2\}} = \{x' \in X|\psi_2(x')=\psi_2(1) = 1\}$
= {1}
$[2]_{\{\psi_2\}} = \{x' \in X|\psi_2(x')=\psi_2(2) = 4\}$
= {2}
Hence we have$\xi_{\{\psi_2\}} = \{[0]_{\{\psi_2\}}, [1]_{\{\psi_2\}}, [2]_{\{\psi_2\}}\}$
$[0]_{\{\psi_3\}} = \{x' \in X|\psi_3(x')=\psi_3(0) = 0\}$
= {0,1}
$[2]_{\{\psi_3\}} = \{x' \in X|\psi_3(x')=\psi_3(2) = -4\}$
= {2}
Hence we have$\xi_{\{\psi_3\}} = \{[0]_{\{\psi_3\}}, [2]_{\{\psi_3\}}\}$
Therefore, for r=1, a classification of X is
$N_1(B) = \{\xi_{\{\psi_1\}}, \xi_{\{\psi_2\}}, \xi_{\{\psi_3\}}\}$
LetG=**{**1,2**}**be a subset of the perceptual objects,
$N_r(B)^*G=\cup_{x:[x]_{\psi_i \cap G \neq \emptyset}}[x]_{\{\psi\}_i}$

$=\{\{0,1\}\cup\{2\}\cup\{1\}\cup\{2\}\cup\{0,1\}\cup\{2\}\}$
$=\{0,1,2\}$
The Cartesian product X×X as follows
X×X=$\{(0,0),(2,2),(1,1),(0,1),(0,2),(1,2),(2,1),(1,0),(2,0)\}$
Then the new classificationis, $\{\{[0]_{\{\psi_1\}}, [2]_{\{\psi_1\}}\}, \{[0]_{\{\psi_2\}}, [1]_{\{\psi_2\}}, [2]_{\{\psi_2\}}\}, \{[0]_{\{\psi_3\}}, [2]_{\{\psi_3\}}\}\}$
Consider the near group G={1,2}then the Cartesian product G×G is
G×G= {(1,1)(1,2),(2,2)(2,1)}

Where $N_r(B)^*(G \times G) = N_r(B)^*G \times N_r(B)^*G = X \times X$. From the definition of a near group,we have that,

(i) The multiplication of elements in G×G is closed under $N_r(B)^*G \times N_r(B)^*G$,where $*_1$ and $*_2$are

(1,1)∗(1,2)=(2,0)
(1,1)∗(2,2)=(0,0)
(1,1)∗(2,1)=(0,2)
(1,2)∗(1,1)=(2,0)
(1,2)∗(2,2)=(0,1)
(1,2)∗(2,1)=(0,0)
(2,2)∗(1,1)=(0,0)
(2,2)∗(1,2)=(0,1)
(2,2)∗(2,1)=(1,0)
(2,1)∗(1,1)=(0,2)
(2,1)∗(1,2)=(0,0)
(2,1)∗(2,2)=(1,0)

(ii) The associative law is satisfied.

(iii) There exists $(0,0)\in N_r(B)^*G \times N_r(B)^*G$
Such that for every (g,g′) ∈ G×G , we have (0,0) ∗ (g,g′)=(g,g′)

(iv) For every element of G×G,∃ an inverse element in G×G,where

$(1,1)^{-1}=(2,2)\in G\times G$,
$(2,1)^{-1}=(1,2)\in G\times G$
$(1,2)^{-1}=(2,1)\in G\times G$
$(2,2)^{-1}=(1,1)\in G\times G$

Hence G×G is a near group.

Example3.3. LetX=$\{0,1,2\}$be a set of perceptual objects ,B=$\{\psi_1,\psi_2,\psi_3\}$be a set of functions (∗) be the binary operation, addition modulo 3. Sample values of theprobe function$\{\psi_i\}$are defined as,

ψ_1:X→V_1defined byψ_1 (n)=n(n−1)∀n∈X

ψ_2:X→V_2defined byψ_2(n)=n(n−1)(n−2)∀n∈X

ψ_3:X→V_3defined by ψ_3(n)=n(n−1) (n−2) (n−3) ∀n∈X

	0	1	2
ψ_1	0	0	2
ψ_2	0	0	0
ψ_3	0	0	0

Let us construct the equivalence classes for each combination,thus equivalence classes are definedas,
$[0]_{\{\psi_1\}} = \{x' \in X | \psi_1(x') = \psi_1(0) = 0\}$
= {0,1}
$[2]_{\{\psi_1\}} = \{x' \in X | \psi_1(x') = \psi_1(2) = 2\}$
= {2}

Hence we have $\xi_{\{\psi_1\}} = \{[0]_{\{\psi_1\}}, [2]_{\{\psi_1\}}\}$
$[0]_{\{\psi_2\}} = \{x' \in X | \psi_2(x') = \psi_2(0) = 0\}$
$= \{0,1,2\}$
Hence we have $\xi_{\{\psi_2\}} = \{[0]_{\{\psi_2\}}\}$
$[0]_{\{\psi_3\}} = \{x' \in X | \psi_3(x') = \psi_3(0) = 0\}$
$= \{0,1,2\}$
Hence we have $\xi_{\{\psi_3\}} = \{[0]_{\{\psi_3\}}\}$
Therefore, for r=1,a classification of X is,
$N_1(B) = \{\xi_{\{\psi_1\}}, \xi_{\{\psi_2\}}, \xi_{\{\psi_3\}}\}$
Let G=$\{1,2\}$ be a subset of the perceptual objects,
$N_r(B)^*G = \cup_{x:[x]_{\psi_i} \cap G \neq \emptyset} [x]_{\{\psi\}_i}$
$= \{\{0,1\} \cup \{2\} \cup \{0,1,2\} \cup \{0,1,2\}\}$
$= \{0,1,2\}$
From Definition
(1)∀a,b∈G,ab∈$N_r(B)^*(G)$
(2) The Property ∀a,b,c∈G, (a·b)·c=a·(b·c) holds in $N_r(B)^*(G)$
(3) ∃0∈$N_r(B)^*(G)$ such that ∀a∈G,a·0=0·a=a
(4) ∀a∈G,∃b∈G such that a·b=b·a=e(b is called a near inverse of a in G)G is a near group
Let $\tau = \{\emptyset, N_r(B)^*(G), \{1\}, \{2\}, \{1,2\}\}$ on $N_r(B)^*(G)$
Then, $\tau_G = \{\emptyset, G, \{1\}, \{2\}\}$ is the relative topology on GFrom Def (topological near group)
(a) 1*1=2,for T∈N(2)⊆τ, there exist open set U={1}∈N(1)⊆τ_G, such that UU⊆T
1*2=0,for T∈N(0)⊆τ, there exist open setU={1}∈N(1)⊆τ_G, and
V={2}∈N(2)⊆τ_G such that UV⊆T1∗2=1, for T∈N(1)⊆τ,there exist open set
U={2}∈N(2)⊆τ_G, such that UU⊆T
(b) $\{1\}^{-1}=\{2\}$ is open
$\{2\}^{-1}=\{1\}$ is open
Therefore G is a topological near group.
Hence the product of topological near group we have
$\tau = \{\emptyset, N_r(B)^*G, \{1\}, \{2\}, \{1,2\}\}$ as a topology on $N_r(B)^*G$.
then τ×τ is the product topology of $N_r(B)^*G \times N_r(B)^*G$.Also we have
$\tau_G = \{\emptyset, G, \{1\}, \{2\}, \{1,2\}\}$ as a relative topology on G ,then $\tau_G \times \tau_G$ a producttopology on G×G is induced by τ×τ.
Therefore consider the multiplication map
f:(G×G)×(G×G)→$N_r(B)^*G \times N_r(B)^*G$. This map is continuous with respect to the topology τ×τ and product topology on (G×G)×(G×G). Now, consider the inverse mapτ : G × G → G × G. This map iscontinuous. Hence G × G is a topological near group

IV. NEAR ACTION AND NEAR HOMEOGENOUS SPACES IN CLASSICAL SET TOPOLOGY

This section deals with near action and near homeogenous spaces in classical settopology.
Let$(X_1, F_1, \sim_{B_{r_1}}, N_{r_1}, \nu_{r_1})and(X_2, F_2, \sim_{B_{r_2}}, N_{r_2}, \nu_{r_2})$ be a nearness approximation spaces. Let $G_1 \subseteq X_1$ and $G_2 \subseteq X_2$ be topological near groups such that τ_1 and$\tau_2$are topologies on $N_r(B)^*G_1$and $N_r(B)^*G_2$, respectively inducing $\tau_{G1}$and τ_{G2} on G_1 and G_2 respectively,
A mapping f : $N_r(B)^*G_1 \rightarrow N_r(B)^*G_2$ is called a topological near group homomorphism, if f is a near homomorphism and continuous with respect to the topology $\tau_2$on $N_r(B)^*G_2$inducing τ_{G_1}on G_2 and the topology $\tau_1$on $N_r(B)^*G_1$inducing τ_{G_1} onG_1.
A topological near group homomorphism f : $N_r(B)^*G_1 \rightarrow N_r(B)^*G_2$ is called a topological near group homeomorphism f^{-1}:$N_r(B)^*G_2 \rightarrow N_r(B)^*G_1$such that $f^{-1} \circ f = 1_{N_r(B)^*G_1}$.
Let $(X, F, \sim_{B_r}, N_r, \nu_{N_r})$ be an approximation space. Assume that,Gand Xare two subsets of $\overline{U}$ such that G is a topological near group, and X is a topological near space inducing the topology near space X i.e.near set with ordinary topology.Then we are ready to give the definition of the action of a near group G on a near space is given.

Definition4.1. Let$(X_1, F_1, \sim_{B_{r_1}}, N_{r_1}, \nu_{r_1})and(X_2, F_2, \sim_{B_{r_2}}, N_{r_2}, \nu_{r_2})$ be nearness approximation spaces and *1, *2 be binary operations on X_1 and X_2 respectively, let $G_1 \subseteq X_1$ and $G_2 \subseteq X_2$ be two near groups.If the mappingf:$N_r(B)^*G_1 \rightarrow N_r(B)^*G_2$satisfies that $f(x *_1 y) = f(x) *_2 f(y)$,for all x,y∈$N_r(B)^*G_1$ then f is called a near homomorphism.

Definition4.2. Let G_1 and G_2 be two near groups.A near homomorphism
f: $N_r(B)^*G_1 \rightarrow N_r(B)^*G_2$is said to be:
(a) A near epimorphismif f:$N_r(B)^*G_1 \rightarrow N_r(B)^*G_2$ is onto
(b) A near monomorphismif f: $N_r(B)^*G_1 \rightarrow N_r(B)^*G_2$ is one-to-one
(c) A near isomorphism if f: $N_r(B)^*G_1 \rightarrow N_r(B)^*G_2$ is both onto and one-to-one.
Definition 4.3. A continuous map f : $N_r(B)^*G \times \bar{X} \rightarrow \bar{X}$ (resp f : $\bar{X} \times N_r(B)^*G \rightarrow \bar{X}$) is called a left(resp right) near action of G on X,if it satisfies the following conditions.
(a) $g(g'x)=(gg')x$(resp.$((xg)g')= x(gg')$),for all g,g′∈ $N_r(B)^*G$ and x∈X
(b) ex = x (resp. xe = x), for every x ∈ X, where e ∈$N_r(B)^*G$ is the near identity. Then the near set X is called a near G-space.
The action f is said to be effective if gx = g′x, for every x ∈ X ⇒ g = g′. In addition the action f is said to be transitive if for every x, x′ ∈ X there exists g ∈$N_r(B)^*G$ such that gx=x′
Definition4.4. Let X be a near G-space.Then X is said to be topologically near homogeneous if for all x, y ∈X, there is a topological homeomorphismf: $\bar{X} \rightarrow \bar{X}$such that f(x)=y.
Proposition 4.5. Let G be a topological near group and X be a near G-space.Then the left transformation map L_g :$\bar{X} \rightarrow \bar{X}$ (resp right transformation map R_g :$\bar{X} \rightarrow \bar{X}$), fore very g∈G, which is defined $L_g(x)=gx(R_g(x)=xg)$,is a topological homeomorphism.
Proof.The continuity of the action f implies the continuity of L_g.The continuous
(a)and(b) in Definition 4.3 are respectively equivalent to
(i) $L_g \circ L_{g'} = L_{gg'}$
(ii) $L_e = 1_{\bar{X}}$.
Therefore, the maps L_g and $L_{g^{-1}}$ are inverse of eachother. Thus,$L_g$is a topologicalhomeomorphism from $\bar{X}$to$\bar{X}$.
Note that, the left (resp.right) transformation map $L_g(R_g)$:$\bar{X} \rightarrow \bar{X}$, is not topologically homomorphism for every g ∈ $N_r(B)^*G$.This is true only in the case that $N_r(B)^*G$ is a group.
Corollary 4.6. Let G be topological near group. Then for every open set O in X and
g∈G,$L_g(O)$=gOis openin $\bar{X}$.
Proof.By Theorem 4.5,L_g:$\bar{X} \rightarrow \bar{X}$ is a topological homeomorphism.Thus $L_g(A)$=gO is a open set in $\bar{X}$
Theorem 4.7. Let G be a topological near group such that $N_r(B)^*G$ is a group.For any open subset O of $N_r(B)^*G$,if A is a subset of$N_r(B)^*G$,then AO(respectively OA)is openin $N_r(B)^*G$.
Proof.In actual fact $N_r(B)^*G$ is a group implies G acts on itself.Thus for every g ∈$N_r(B)^*G$, L_g is a topological homeomorphism.The rest of proof follows immediately from left (right)
transformation. Because that AO =$_{a \in A} L_a(O)$ and OA=$_{a \in A} R_a(O)$.is open in $N_r(B)^*G$.
Theorem4.8. Let G be an topological near groups such that $N_r(B)^*G$ is a group.Let H be a sub near group of G such that $N_r(B)^*H$is closed under multiplication.If there is an open set O in Gsuch that e ∈ O and O ⊆ H, then$N_r(B)^*H$ is an openset in $N_r(B)^*G$.
Proof.Let O be a nonempty open set in G such that
O ⊆ Hand e ∈ O. Then for every h ∈$N_r(B)^*H$,$L_h(O)$ = hO is open in $N_r(B)^*G$.Hence $N_r(B)^*H$ =$_{h \in N_r(B)^*G}$ hO is open in $N_r(B)^*G$.
Theorem 4.9. Let $\bar{G}$ be a topological near group such that $N_r(B)^*\bar{G}$ is a groupand let H be a sub
Near group of G. Let H be a sub near group of G. Let O be an open set in G such that O⊆H. Then for every h∈H, hO is an open set in$N_r(B)^*H$
Proof. Since $N_r(B)^*H \subseteq N_r(B)^*G$ and $N_r(B)^*G$ is a group, L_h is a topological homeomorphism.By the definition of left transformation, $L_h(O)$ = hO is open in $N_r(B)^*G$ the fact that O ⊆ H implies hO∈ $N_r(B)^*H$. Hence, hO is open in $N_r(B)^*H$.
Definition 4.10. Let G_1 and G_2 be topological near groups,f : $N_r(B)^*G_1 \rightarrow N_r(B)^*G_2$ be
a topological near group homomorphism and let e_2 be the near identity element of G_2.
ThenKer(f)={g∈$N_r(B)^*G_1$:f(g)=e_2}is called the near kernel associated to the map f.
Theorem4.11. Let f be a topological near group homomorphism from $N_r(B)^*G_1$to$N_r(B)^*G_2$. Then the near kernel is a normal sub near group of $N_r(B)^*G_1$.
Proof. Let * and $*_1$ be the binary operation in G_1 and G_2 respectively. Since
$f(e_1)=e_2$,e_1∈ker(f). ker(f)≠ ∅ For every x,y∈ker(f),we have f(x)=e_2
(a) Since $f(x*y)=f(x)*_1f(y)=e_2$,we have$x*y$∈ker(f)
(b)Also $f(x^{-1})=(f(x))^{-1}=(e_2)^{-1}$.
Hence ker(f) is a sub near group of G_1
(c)For every x∈G_1 and r∈ker(f), we have $f(x*r*x^{-1}) = f(x)*_1f(r)*_1f(x^{-1})=f(x) *_1 e_2 *_1 f(x^{-1}) = f(x) *_1 (f(x))^{-1} =$

e_2.Therefore, x * r * x^{-1}∈ker(f) thus ker(f) is a normal sub near group of G_1.
Example 4.12. Consider the map f: $N_r(B)^*G_U \rightarrow N_r(B)^*G_X$, where G_1 and G_2 are near groups as above respectively,
Defined as follows,
f(0)=0,f(1)=0,f(2)=0
clearly,f is a continuous and homomorphism.Hence f is a topological near group homomorphism from Def 4.10, it is easy to see thatker(f)={0,1,2}is a subset of $N_r(B)^*G_U$. Moreover ker(f) is a normal sub near group of $N_r(B)^*G_U$.
Definition4.13. Let G be a topological near group and B⊆τ be a base for τ.
For g ∈G, the family
B_g={O∩G:O ∈B,g∈O}⊆B is called a base at g in τ_G.
Theorem 4.14. Let G be a topological near group such that the identity element e ∈ G and $N_r(B)^*G$ is Closed under multiplication. Let G be an open set in $N_r(B)^*G$. For g∈G the base ofg in $N_r(B)^*G$is equal toB_g={gO :O ∈B_e}
Where $B_e$is the base of the identity e in τ_G
Proof.Since g ∈ G, we have g ∈$N_r(B)^*G$. Let O_1bean open set in $N_r(B)^*G$and let g∈O_1.Since e∈G, and G is a topological near group, there are two open setsO_2 and O_3 such that g ∈O_2, e ∈O_3 and f (O_2 ×O_3) ⊆O_1. We have G is an open set in τ. Then O_3 is a neighbourhood of e in τ. Then there is a basic open set O ∈B_e such that e∈O⊆O_3.Hence
Lg(O) = gO⊆ f (O_2× O) ⊆f(O_2×O_3)⊆O_1.
Definition4.15. Let G be a near group and A⊂G.Then A is an symmetric if $A = A^{-1}$
Proposition 4.16. Let G be a topological near group. If e ∈ G, then for each open neighbourhood O of e in G, there exists a symmetric open neighbourhood P of e in G such that P^2∩G⊂O
Proof. Take an arbitrary open neighbourhood O of e in G.Then there exists an open neighbourhood Wof e in $N_r(B)^*G$such that O = W ∩ G. Sincef: G × G →$N_r(B)^*G$ is continuous at point (e,e)and the inverse mapping is a homeomorphism,there exists a symmetric openneighbourhood P of ein G such that P^2⊂ W :HenceP^2∩G⊂O.
Proposition4.17. Let f be a near homomorphism between near groups G_1andG_2. If G_1 is a topological group,then $G_2$is also a topologicalg roup.
Proof.It suffices to prove thatG_2^2 = G_2.Take arbitrary x, y ∈ G_2.Then, there existsg,h∈G_1such that f(g)=x, f(h)=y hence,f(gh)=f(g)f(h)=xy. Since G1is a topological group, it follows thatgh∈ G_1; thus f (gh) ∈ G_2. i.exy∈ G_2. Therefore,G_2 is a topological group.

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