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Research Paper

Numerical and Qualitative Features of a Longitudinal Fin with Temperature –Dependent Properties and Internal Heat Generation

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ABSTRACT*A numerical investigation of longitudinal rectangular fin with temperature dependent thermal conductivity and internal heat generation is carried out. For enhanced visualizations and assessment,quantitative as well as qualitative impact of certain thermal parameters on the fin temperature profile are determined. This is necessary in order to better comprehend realistic combinations of fin thermo-geometric parameters in thermal engineering applications.Quite often it is very challenging to arrive at analytic or numerical solutions to nonlinear systems of equations. For such problems a qualitative approach not only avails us a rich information concerning the physics of the problem, but also allows us to make valid conclusions irrespective of whether we know the solution or not.*

Keywords: *qualitative, numerical,fins, temperature dependent thermal conductivity, thermo-geometric parameter, internal heat generation.*

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I. INTRODUCTION

Mechanical devices such as air conditioner in automobiles, processors in computers,electronic chips,oil pipelines etc, generate a lot of heat during operation This often results in inefficiency and a decrease in performance. Heat generated wiithin these equipments needs to be dissipated inorder for them to work properly. As a result , extended surfaces or fins are designed to contribute additional surface areas to enhance heat transfer between their surfaces and the ambient especially by convection.

Fins with different thermal and geometric configurations are designed to achieve the most convinient, effective , efficient and economic output. They can be circular or rectangular in shape, or can also be made of different materials like aluminum or copper to optimize their thermal properties. But due to its convinience, economy and larger convective area, the rectangular longitudinal fin is favored over other geometrical configurations.

If the temperature between the tip of the fin and the base is not large, a linear approach is generally used in design. But if large, thermal parameters play a huge role because both the convective and conductive heat transfer coefficients become functions of temperature and the problem beign essentially nonlinear evokes a numerical solution.

Internal heat generation is of a practical importance especially for equipments involving considerable amount of heat transfer during operation for example in electrical current carrying conductors,nuclear rods or any other heat generating electrical equipments. It is therefore essential to identify and analyze its causes so that fin design can be optimized.

As mentioned, heat transfer in fins is highly nonlinear and in a majority of cases defies analytic or closed form solutions. Aziz and Enamul-Huq [1] applied regular perturbation expansion to examine pure convectionin fins with temperature dependent thermal conductivity. Later on Aziz [2] extended this study to include uniform internal heat generation within the fin. Other attempts include those of Chowdhury and Hashim[3], where they applied the Adomian decomposition technique to determine the temperature and flux profiles of a straight rectangular fins with nonlinear thermal conductivity.

We hasten to point out that as numerical solution techniques started gaining prominence in nonlinear applications, semi-exact analytical methods were also developing. And to a fairly large extent the combination of both methods has been found to be very useful in resolving the nonlearity in fin problems (Moitsheki et al.

[4]).These techniques include the homotopy analysis method (HAM) (Khani and Aziz[5]). Hosseini et al.[6] applied HAM to quantify ballpack estimates of heat transferer in fins with temperature-dependent internal heat generation and thermal conductivity. Similar applications involving the homotopy perturbation method (HPM) is recorded in Ganji et al.[7], Ganji[8], Ganji and Rajabi[9], and Rajabi and Ganji. [10]). Another well known technique that falls under this categoryis the adomian decomposition method ADM). An example of its application to the nonlinear fin problem can be found in Arslanturk[11].

Current litterature search in the numerical and semi-exact solution of fin problems reveals that very little has been done to qualitatively analyze heat transfer process in fins. The few include work done by Harley [12] Harley and Moitsheki [13]. Otherwise not much has been done to obtain a qualitative insight into thermal varaible parameters which are associated with fins in practical situations. Our task in this paper is therefore twofold namely, to numerically determine the the effects of various combinations of fin parameters on the temperature profile in a fin (Sobamowo[14], Onyejekwe [15]) and to qualitatively evaluatethese effects using the tools of dynamical analysis.

II. PROBLEM FORMULATION

Consider convective heat transfer in a straight 1D longitudinal fin. The fin has cross-sectional area A, length l, and perimeter P as shown in Fig.1. The fin projects from a base at temperature T_b into a surrounding fluid at a temperature T_a . It is assumed that at the fin surface heat loss occurs mostly by convection. The tip of the fin is insulated and has a zero flux since the heat loss is assumed to be small and negligible in that region. The length to width ratio is relatively largeand as a result, heat flow through the fin is basically one-dimensional. Finally the fin thermo-geometric properties is assumed not to vary with time.

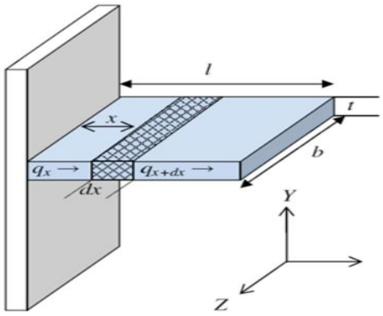

Fig 1 .A Schematic drawing for a rectangular longitudinal Fin

Given the above characteristics, the one-dimensional governing ordinary differential equation describing heat transfer in a fin, is written as follows:

$$\frac{d}{dx}\left[k(T)\frac{dT}{dx}\right]-\frac{h(T)}{A}P(T-T_{\infty})+q(T)=0 \qquad (1)$$

where K(T) is the temperature dependent thermal conductivity , h(T) the convective heat transfer coefficent, the internal heat generation parameter is q(T).
The Boundary Conditions are

$$x=1;\frac{dT}{dx}=0$$
$$x=0;T=T_B \qquad (2)$$

Temperature dependent thermal properties is represented as:

$$K(T)=K_0(1+\lambda(T-T_0) \qquad (3)$$

where K_0 is the thermal conductivity when the temperature is T_0 and λ is a heat transfer parameter. The convection coefficient is:

$$h(T) = h_b \left[\frac{T - T_\infty}{T_b - T_\infty} \right]^n \quad (4)$$

Internal heat generation parameter is given as

$$q(T) = q_0[1 + \psi(T - T_\infty)] \quad (5)$$

The exponent constant n practically varies between -3 and 3, and represents different modes of convection. For example n=-1/4 and n=1/4, represent natural boiling and condensation respectively. For thermal radiation n=2, for constant heat transfer n=0 and n=1 for linear heat distribution.
Inserting equations (3-5) into equation(1), we obtain

$$\frac{d}{dx}\left[k_0[1 + \lambda(T - T_\infty)] \frac{dT}{dx} \right] - \frac{h_b P(T - T_\infty)^{n+1}}{A(T_b - T_\infty)^n} + q_0[1 + \psi(T - T_\infty)] = 0 \quad (6)$$

Employing the following dimensionless parameters:

$$X = \frac{x}{l}, \theta = \left[\frac{T - T\infty}{T_b - T\infty} \right], H = \frac{h}{h_b}, M^2 = \frac{Ph_b L^2}{AK_a} \quad (7a)$$

$$Q = \frac{q_0 A}{h_b P(T_b - T_\infty)}, \gamma = \psi(T_b - T_\infty), and, \beta = \lambda(T_b - T_\infty) \quad (7b)$$

The governing equation reduces to:

$$\frac{d^2\theta}{dx^2} + \beta\theta \frac{d^2\theta}{dX^2} + \beta \left[\frac{d\theta}{dX} \right]^2 - M^2\theta^{n+1} + M^2 Q(1 + \gamma\theta) = 0 \quad (8a)$$

With the following boundary conditions

$$X = 0, \ \frac{d\theta}{dx} = 0$$

$$X = 1, \theta = 1 \quad (8b)$$

III. NUMERICALSOLUTION

Equation (8) is solved with the shooting-secant method. Rearrangement yields:

$$\frac{d}{dX}\left[(1 + \beta\theta) \frac{d\theta}{dX} \right] = M^2\theta^{n+1} - M^2 Q(1 + \gamma) \quad (9a)$$

subsequently

$$\frac{d^2\theta}{dX^2} = \frac{-\beta \left[\frac{d\theta}{dX} \right]^2 + M^2\theta^{n+1} - M^2 Q(1 + \gamma)}{1 + \beta\theta} \quad (9b)$$

Equation (9b) is decomposed into two coupled nonlinear ordinary differential equaions

$$P(X) = \theta, \ S = \theta' = P', P'' = S' = \theta''$$

$$P' = S$$

$$S' = \frac{-\beta\left[P'\right]^2 + M^2 P^{n+1} - M^2 Q(1 + \gamma)}{1 + \beta P} \quad (10)$$

where : X, θ are dimensionless distance and temperatute; M is the thermo-geometric parameter, β is the nonlinear thermal conductivity parameter. Q is dimensionless heat transfer parameter, γ is dimensionless internal heat generation parameter.

IV. FIN DESIGN PARAMETERS

IV-1. Fin Efficiency

This is the ratio of the actual heat transferred fron the fin surface to the sorrounding environment or fluid to the actual amount of heat transferred from the entire fin area ; that is if we assume that the entire fin area is situated at the base of the fin. Simply stated, it is the ratio of actual heat transferred to that of the ideal heat tranferred

$$Q_f = \int_0^1 ph(T)(T - T_b)dX \tag{11}$$

The maximum heat transfer rate is given by

$$Q_{\max} = ph_b \mathrm{L}(T_b - T_\infty)dX \tag{12}$$

Therefore, the efficiency is given by

$$\eta = \frac{Q_f}{Q_{\max}} = \frac{\int_0^1 ph(T)(T - T_b)dX}{ph_b L(T_b - T_\infty)} \tag{13}$$

Simplifying the above equation yields

$$\eta = \int_0^1 \theta^{n+1} dX \tag{14}$$

IV-2. Fin Effectiveness

Effectiveness of a fin can be defined as the ratio of heat transfer rate of the fin to the rate of fin to the rate of heat transfer if the fin is not there.

$$\varepsilon = \frac{Q_f}{Q_{fb}} \tag{15}$$

Q_{fb} is the amount of heat dissipation from the area of the fin base and is given by

$$Q_{fb} = \int_0^1 ph_b \frac{\delta}{2}(T_b - T_\infty)dX \tag{16}$$

$$\varepsilon = \frac{\int_0^1 2ph(T)(T - T\infty)}{ph_b \delta (T_b - T_\infty)dX} \tag{17}$$

Therefore, the dimensionless effectiveness is given as

$$\varepsilon = 2a_r \int_0^1 \theta^{n+1} dX \tag{18}$$

Where a_r is the aspect ratio given as the dimensionless length (L) to the thickness of the fin (δ).

V. RESULTS and DISCUSSION

In what follows, we carry out a numerical investigation of the effect of certain parameters on fin performance. Fig.2 illustrates the temperature profiles for an increase of the thermo-geometric parameter M and fixed values of β, Q, γ. It can be seen that an increase in M results in a steeper temperature profile. Equation (7a) reveals a proportional relationship between M and the fin length L. Hence relatively small values of M yield smaller fin lengths and higher values of temperature profiles. The reverse is the case for higher values.

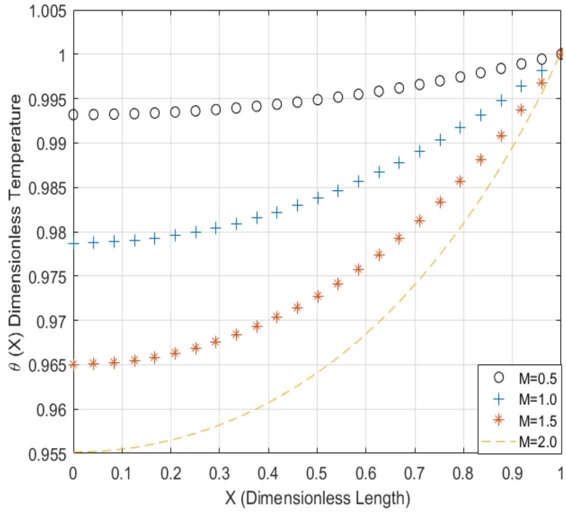

Fig 2 Temperatureprofile for β=1,Q=0.8,γ=0.1

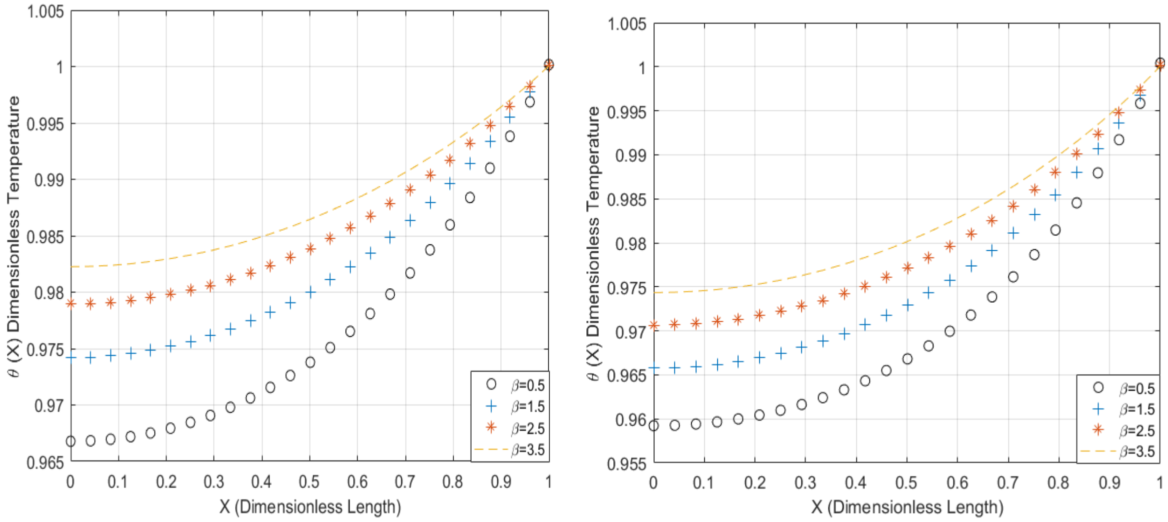

Fig 3. Temperature profile for (a) M=1.5, n=1, Q=0.8, γ=0.1 and (b) M=2.0, n=1, Q=0.8, γ=0.1

Figures 3a and 3b illustrate the effect nonlinear thermal conductivity on temperature profiles. The more the value of β the more the heat is lost as reflected by the values of the dimensionless temperature. We should also note that the temperature of the fin at the base is uniform and the heat transfer at the tip is neglected according to the specification of the boundary conditions. A slight increase in M resulted in little or no change in temperature distribution.

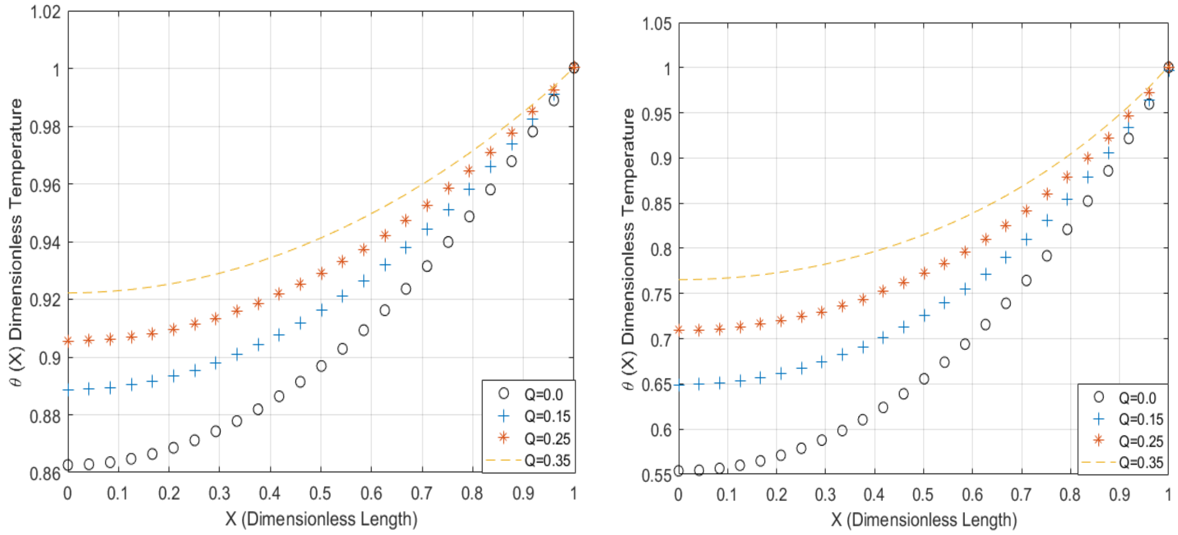

Fig 4. Temperature profile for (a) M=1, n=1, γ=0.2, β=2.0 and (b) M=2.0, n=1, γ=0.2, β=1.0

Figures (4a) and (4b) show the effect of increasing the internal heat generation in a fin. As Q increases the temperature profile becomes flatter, less nonlinear and displays a higher temperature distribution along the fin. The fin operation becomes less efficient. There is however a minor change when the value of the thermo-geometric parameter is slightly increased. This implies a longer fin length L and higher convection coefficient.

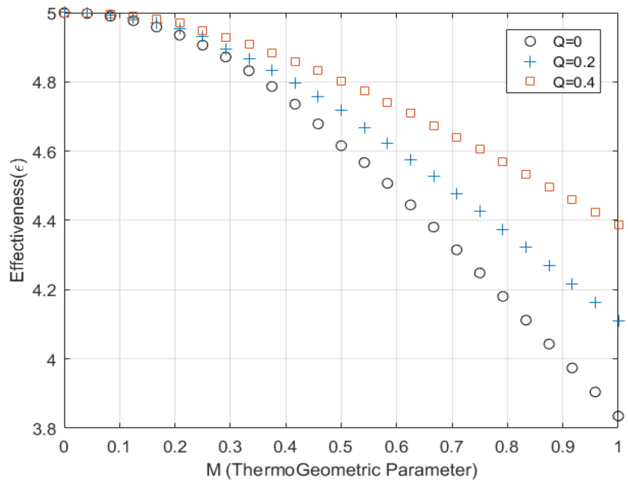

Fig.5: Fin effectiveness versus thermo-geometric parameter for various values of Q , $n=1$,

$$\gamma = 0.2,\ n = 1,\ \beta = 1.0\ ,\ a_r = 2.5$$

Fig.5 shows that overall for the given fin parameter values adopted, the fin is more effective for relatively smaller values of thermo-geometric parameter M and internal heat generation parameter Q.The trend is the same for the values of Q valueschosen. The following remarks can be made as well

(i) Low values of thermo-geometric parameter M favor efficiency
(ii) The above can be interpreted as a fin of small length (Equation 7a)
(iii) High values of internal heat generation does not favor efficient fin operation

Nest we carry out a dynamical study of the fin model to gain a better insight of the impact of some of the parameters on the overall performance.

Specifically, we aim to be acquainted with the behavior of the linearized governing equation in the neighborhood of the equilibrium point. This not only provides an insight into the model problem parameters close to the equilibrium but also gives a clearer picture of the range or combinations of certain fin parameters. For example if the real part of the computed eigenvalue of the nonlinear system t is nonzero, the linearized analysis will correctly predict if the equilibrium is stable or unstable. Taken together, this type of analysis provides amore vivid information concerning fin operation that would have been very difficult to obtain by application of numerical techniques alone.

Firstly, computer generated phase-portraits are employed to produce critical or equilibrium points of the nonlinear system of equations. These critical points are then classified according to the eigenvalues of the corresponding linearization. One fin parameter that is of interest to us here is the thermogeometric parameter M. It provides information on the relative importance of conduction and convection . (see equation 7b). The next valuable parameter is the dimensionless heat transfer coefficient Q. A comprehensive qualitative analysis of the effect of Q on the solution profile is sought. Fig. 6a shows the phase portrait produced for the following fin parameters $\left(M=2, \beta=1,\ n=1, Q=0.5,\ \gamma=0.1\right)$

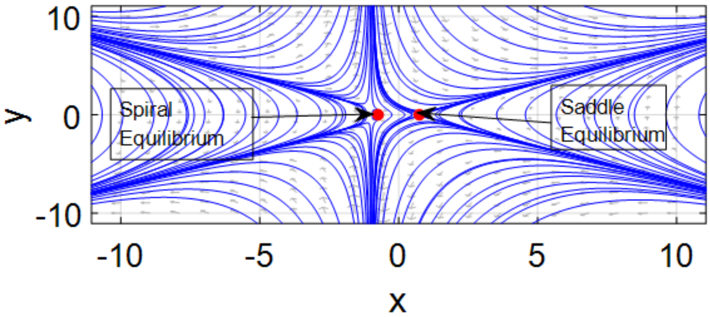

Fig. 6a Qualitative profiles for the heat transfer coefficient Q = 0.5

It comes with the following details:

Location of spiral equilibrium point: $\left(-0.74162, 0\right)$, Type: Undetermined

Jacobian matrix $\begin{bmatrix} 0 & 1 \\ -22.962 & -3.8703e-06 \end{bmatrix}$

Eigenvalues and eigenvectors:
$$\begin{matrix} -1.9351e-06+4.7819i & \left(-8.2498e-08-0.20429i,\ \ 0.9789\right) \\ 1.9351e-06-4.7819i & \left(-8.2498e-08+0.20429i\ ,\ \ 0.9789\right) \end{matrix}$$

In addition to the above, the following was also computed:

Saddle located at : $\left(0.74162, 0\right)$; Type: unstable

Jacobianmatrix : $\begin{bmatrix} 0 & 1 \\ 3.4066 & -5.7418e-07 \end{bmatrix}$

Eigenvalues and eigenvectors for saddle equilibrium:

$$1.8457 \quad \left(0.47638, \quad 0.85709\right)$$
$$-1.8457 \quad \left(-0.47638, \quad 0.85709\right)$$

Fig. 6a displays a spiral and saddle equilibria. As can be seen the eigenvalues and eigenvectors of the spiral equilibrium are complex conjugates and this lends it its spiral configuration. . The real part is almost zero asa result there is practically no motion along the direction dictated by the accompanying eigenvectors. The eigenvalues can be considered as purely imaginary. The resulting trajectory is ellipsoid and there is no net motion towards its equilibrium. The equilibriumtype is undetermined. A saddle point was also identified. Saddle equilibrium is always unstable since one of the eigenvalues is always positive. As can be observed , most of the trajectories approach the saddle equilibrium along the eigenvector corresponding to the negative eigenvalue and does exactly the opposite along the eigenvectors of positive eigenvalue.

Fig. 6b shows the phase plane when Q was increased from 0.5 to 5.

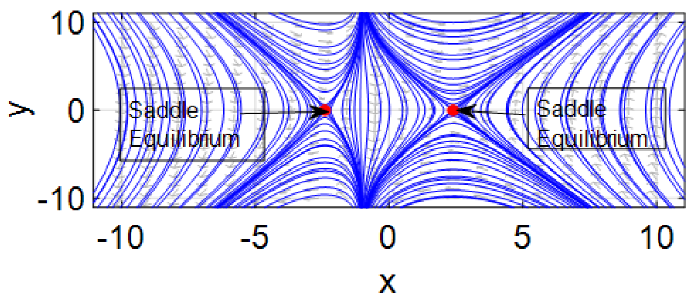

Fig. 6b Qualitative profiles for the heat transfer coefficient Q = 5

Saddle located at : $\left(-2.3452, 0\right)$; Type: unstable

Jacobian matrix : $\begin{bmatrix} 0 & 1 \\ 13.9452 & 7.4338e-07 \end{bmatrix}$

Eigenvalues and eigenvectors for saddle equilibrium:

$-3.7346 \quad \left(-0.25866, \quad 0.96597\right)$

$3.7346 \quad \left(-0.25866, \quad -0.96597\right)$

A second saddle point was also located at $\left(2.3452, 0\right)$

Jacobianmatrix : $\begin{bmatrix} 0 & 1 \\ 5.6085 & -2.9894e-07 \end{bmatrix}$

Eigenvalues and eigenvectors for saddle equilibrium:

$2.3682 \quad \left(0.43106 \quad 0.92124\right)$

$-2.3682 \quad \left(-0.43106 \quad 0.92124\right)$

For this increase,we observe a noticeable qualitative change in the profiles. Two saddle equilibrium points are identified and the spiral equilibrium point seen in Fig. 6a is totally obviated. As mentioned earlier, bothequilibria are nonlinear. A ten times increase in the value of heat transfer coefficient will definitely not be suitable for optimal fin operation. A dramatic change in profile just as seen above is likely due to bifurcation. However details of the phenomenon are outside the scope of the current study.
The next parameter to be looked at is the dimensionless heat transfer parameter Q The following fin parameters were applied

$\left(M = 0.8, \beta = 0.5, \ n = 1, Q = 0.5, \ \gamma = 0.1\right)$

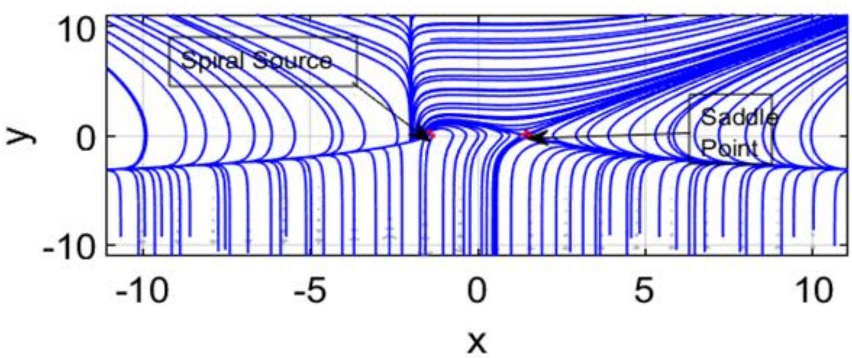

Fig. 6c Qualitative profiles for the thermogeometric parameter M= 0.8

These provided the following information

Location of spiral source: $\left(-1.4354, 0\right)$

Jacobian matrix $\begin{bmatrix} 0 & 1 \\ 6.9077 & 5.0728e-07 \end{bmatrix}$

Eigenvalues and eigenvectors: $\begin{matrix} 2.5364+0.61182i & (0.3479-0.0839i \quad 0.93377) \\ 2.5364-0.61182i & (0.3479+0.0839i \quad 0.93377) \end{matrix}$

In addition to the above, the following was also computed:

Saddle located at : $(1.4534, 0)$; Type: unstable

Jacobianmatrix : $\begin{bmatrix} 0 & 1 \\ 1.0774 & 0.80285 \end{bmatrix}$

Eigenvalues and eigenvectors for saddle equilibrium:

$-0.71148 \quad (-0.81481, \quad 0.57973)$

$0.71148 \quad (0.81481, \quad 0.57973)$

In this instance, heat transfer activity within the fin is enhanced. Just like in the previous case ,we have both spiral and saddle node equilibrium points. The eigenvalues possess positive real parts; so we have an exponential growth. Points move away from the equilibrium point in an oscillatory manner.

For the next step, variables were kept the same but M was increased to M=2. The following qualitative features were generated:

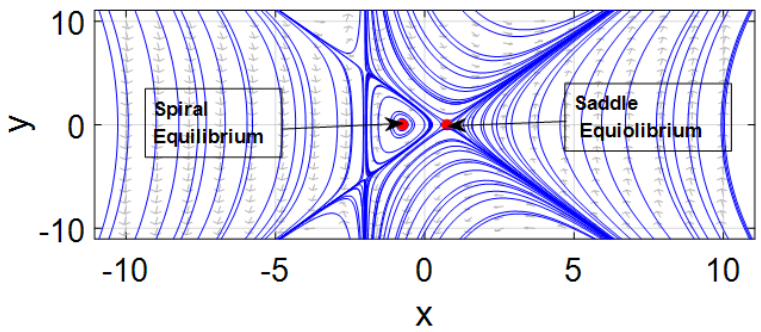

Fig. 6d: Effect of an increase in M on the solution profile.

Location of spiral equilibrium point: $(-0.74162, 0)$, Type: Undetermined

Jacobian matrix $\begin{bmatrix} 0 & 1 \\ -9.4295 & -0.7467e-07 \end{bmatrix}$

Eigenvalues and eigenvectors: $\begin{matrix} -39734e-07+3.0707i & (-4.0067e-08-0.30965i, 0.95085) \\ -39734e-07-3.0707i & (-4.0067e-08+0.30965i, 0.95085) \end{matrix}$

In addition to the above, the following was also computed:

Saddle located at : $(0.74162, 0)$; Type: unstable

Jacobianmatrix : $\begin{bmatrix} 0 & 1 \\ 4.3281 & -3.6475e-07 \end{bmatrix}$

Eigenvalues and eigenvectors for saddle equilibrium:

$2.0804 \quad (0.43323, \quad 0.90129)$

$-2.0804 \quad (-0.43323, \quad 0.90129)$

As can be observed , though spiral and saddle equilibria still exist The positive real component of the eigenvalues are almost zero in this case. As a result, rotations hardly happens. The transition between Figs. 6c and 6d is obvious. On this note we can infer that such a transition can be used to ‘sense’ a parameter changes. If for example a certain parameter is decreased a stable fixed point may be achieved, while if increased, a stable periodic orbit may be reached.

VI. CONCLUSION

In the work reported herein, heat transfer analysis in a nonlinear fin with temperature-dependent thermal conductivity and internal heat generation has been carried out. A numerical quantitative and qualitative dynamical approaches were adopted. The effects of various parameters on the overall fin performance were also assessed. The numerical results were found to be in conformity with previous work; especially [14]. Carrying out a dynamical analysis allows us to further validate the numerical results especially the role the thermo-geometric parameter M plays as a dominant physical parameter in fin design. It was obvious that a critical value of M as well as Q would lead to unstable results and bifurcation. From the work done herein it can be seen that each solution starting in a certain neighborhood of the equilibrium displays some unique characteristics within the neighborhood that can be related to engineering applications and design. Further dynamical study in this interesting observation is currently outside the scope of this study and will constitute an area of interest in a sequel to this paper.

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