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Research Paper

A Study on Regula Falsi Graphs to Clear Road Blockage

Ambrish Pandey[1] and Chitra Singh[2]
[1]*(Research Scholar, Department of Mathematics, Rabindranath Tagore University, Raisen-464993, Bhopal, Madhya Pradesh, India)*
[2]*(Associate Professor, Department of Mathematics, Rabindranath Tagore University, Raisen-464993, Bhopal, Madhya Pradesh, India)*
Corresponding Author: Ambrish Pandey

ABSTRACT: *Regula falsi graph plays an important role in the field of graph theory and numerical techniques. In this research paper, we study regula falsi graph to overcome road blockages. If the roads of a city get blocked due to snow, we plan to pour salt on the city roads. But to put salt on the roads, planning has to be done in advance. In this situation, we use Euler paths or circuits to traverse the roads in the most efficient way. Here, we represent the cities by the vertices of the graph and roads connecting the cities are represented by the edges of the graph.*

KEYWORDS: *Euler circuits, Euler paths, Regula Falsi Graph, Road blockage.*

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I. INTRODUCTION

Graph theory is the fastest growing branch of modern mathematics. Its origin can be traced back to Euler 250 years ago. He worked on the famous problem known as 'Seven Bridges of Konigsberg problem', which are quoted as the origin of the graph theory. He made discoveries in many areas of science and technology and studied the applications of those fields. Simply, graph theory is the study of lines and points. It pictorially represents mathematical truths. So, it has grown rapidly in recent times with a lot of research activities. Today, at the international level, one third of the mathematics research papers are from graph theory and combinatorics. Therefore, the importance and significance of graph theory is its immense applications of other fields.

Regula Falsi Graphs are widely used in providing problem solving techniques, as it gives an intuitive way before a formal definition is presented. The concept of a regula falsi graph plays an important role in the field of science and technology. Consider that $G(x) = 0$ is a graphical equation. The solution of this equation is a fundamental task in many branches of science and engineering. It has never been of interest to numerical practitioners, regardless of the age of the subject, because no method is perfect in all cases. Very important for this problem is a set of methods that guarantee that the solution lies within a certain interval $[p, q]$ of $\mathbb{R}$, called convergence of regula falsi graphs. In this case, it must be true that $G \in C([p, q])$, $G(p)G(q) < 0$. Regula Falsi Graph finds the roots of a graphical equation by graphical interpolation between the current projections.

Regula Falsi Graphs provide a convenient means of handling the graphical and related information describing a road-block, and review the brief analysis and generalization of road blockage. The theoretic techniques of regula falsi, such as the shortest path between blockage cities and spanning trees, have been shown to provide a solution to the important problem of extracting of financial relevance measures of blocked road segments. Spanning trees can be used in addition to maintain connectivity between destination cities during attenuation. The method of regula falsi graph allows efficient extraction and handling of the relevant graphical properties of a road blockage.

The aim of this work is to construct hierarchical levels in which graph theoretic methods are applied. Functional classifications have been derived on the basis of access and communication between destination cities. These rely on the derivations on the shortest path spanning trees. These techniques provide the foundation within the spatial and attribute domains needed to implement contextually based generalization. The focus in these processes is to produce coherent, logical and contextually relevant results for the future.

In this paper, we have considered that the roads of a city get blocked due to snow. Then we have to plan to remove road blockages and so we have planned to put salt on city roads. For this, we have already made a plan to put salt on the roads. In this situation, we use Euler paths or circuits to traverse the roads in the most efficient way. We denote cities by the vertices of the graph and roads by the edges of the graph, which connect the cities.

II. PRELIMINARIES AND METHODS

In this section, we discuss the basic definitions and concepts.

Definition 1. Let G be an undirected graph then G is said to be a regula falsi graph if any two vertices are connected by exactly one path.

Definition 2. A Regula Falsi path is a path that uses every edge of a graph G exactly once. This path starts and ends at different vertices.

Definition 3. When the starting vertex of the Euler path in a graph G is also connected with the ending vertex of that path in G, then it is called the Euler Circuit or Euler Tour or Eulerian Cycle.

Definition 4. Consider G is a connected graph. If exactly two vertices have odd degree, it is a regula falsi path. If no vertices of an undirected graph have odd degree, i.e. have even degree, it is an Euler circuit.

Definition 5. Consider G is a graph. Then G is said to be a regula falsi based graph if it contains an euler circuit. That is the graph is either only isolated points or contains isolated points as well as exactly one group of connected vertices, where vertices are connected by edges.

Definition 6. A barrier or obstacle used to block or limit the movement of hostile vehicles along a route is known as the road blockage.

Definition 7. The valency of a vertex is the number of edges incident.

Definition 8. A path is a sequence of edges, with consecutive edges sharing an end vertex.

Definition 9. A graph is connected if a path can be found linking any pair of its vertices.

Definition 10. A cycle is a path which leads from a vertex to the same vertex.

Definition 11. A tree is a connected graph with no cycles.

Definition 12. A spanning tree for a vertex set is a tree of edges in the graph in which a path can be found linking any pair of vertices.

Clearly graphs can represent the topology of road blockages in a natural way, with vertices corresponding to road segments between cities. The representation can be enhanced by associating cost values with each edge to represent road length or travel time.

Graph theory can be used in the generalization of road blockages in two different ways: to derive quantitative measures of graphical properties of roads and cities [7; 10], or to identify and represent important graphical information which is needed for the effective application of generalization procedures [12].

Given a set of cities of particular interest in road blockages, with associated arc costs giving information on distance or travel time. In the absence of other data, road segments used for travel between cities can reasonably be assumed to be the shortest routes between cities. A set of weighting values can be created for roads that directly reflect their relevance to this given city. This overview leads to a process of deriving road weights that can serve to support generalization of road blockages across different contexts defined by a set of cities of particular interest. When graphical information such as road surface type or number of road blockages is also available, this can be used in conjunction with length information to produce road cost that reflects relative travel times. These are more likely to give an accurate reflection of the relative use of roads and therefore provide better road weights to support normalization. The fact that spanning trees have been established for the cities can also be exploited as a means of maintaining connectivity between cities while normalizing road blockage and reducing density. This is due to the retention of a spanning tree of roads during normalization by ensuring that no city is isolated from the rest of the road block.

III. THE LITERATURE SURVEY

Y. Hosseini et al, (2023) study Resource-based seismic resilience optimization of the blocked urban road network in emergency response phase considering uncertainties. In this paper, they modeled uncertainties based on probabilistic models to estimate the expected value of path restoration time. They proposed Topology-Demand base network functionality index to evaluate network performance after the earthquake. They designed Simulated Annealing algorithm that determines the maximum required resources and optimal task sequences. The proposed framework is showcased by a road network in Tehran metropolis. They showed that the effect of resource number on reducing the completion time decreases with increasing resources [9].

T. Takabatake et al, (2022) review the Influence of road blockage on tsunami evacuation: A comparative study of three different coastal cities in Japan. In this paper, they aim to clarify the influence that seismic damage to the buildings and the debris created will have on the road network, and how this in turn will influence tsunami evacuation and the expected number of casualties. They developed an agent-based tsunami evacuation simulation model that can consider the changes in evacuation behaviour due to road blockage. They showed that considering the effects of building collapse and associated road blockage significantly increased the expected mortality rates. They correlated with the percentage of collapsed buildings and blocked roads [20].

Y. C. Yu and P. Gardoni (2022) introduce the Predicting road blockage due to building damage following earthquakes. In this paper, they proposed a probabilistic model using the data from the 2010 Haiti Earthquake and calibrated by Bayesian approach to predict the debris distance from undamaged buildings. They used to construct fragility curves that give the conditional probability of road blockage at a given road section for a given seismic intensity. They proposed model considers the relevant factors affecting the road blockage probability, including building types, damage level and road characteristics. They calculated the probability of road blockage for an entire road by system and parallel reliability analysis. They proposed models apply to any general urban area without the dependence on historical data from past earthquakes [22].

S. Samanta, (2021) study Measure of influences in social networks. In this study, they developed a measure of the influence of an individual (node) on/from individuals in the focal network and that in the context of associated networks using fuzzy systems. They developed Mathematical formulations for the notion of the influence of a node based on the structure of a network. Also, fuzzy parameters that capture real-life situation-based characteristics have been included. Thus, the objective (structure-based) and subjective (using fuzzy membership parameters) nature of the network have been captured. They collected Facebook data to illustrate the proposed approach. They consider new features: (a) a fuzzy definition of centrality measures, (b) power measure, (c) notion of associated network and a measure for linking it to the main network, (d) in addition, they provide a mechanism (through subjective parameters) to adapt our approach to a given situation. Thus their approach becomes adaptable to a wide variety of applications [18].

P. Brimblecombe and Z. Ning (2015) focus on the Effect of road blockages on local air pollution during the Hong Kong protests and its implications for air quality management. In this paper, they evaluate the air quality before, during and after a temporary roadway blockage event in Hong Kong that took place during Hong Kong protests from late September to mid-December, 2014. The local regulatory air quality monitoring data from both roadside and general ambient stations were used to assess the impact of roadway blockages on the air quality. They showed some benefits deriving from road blockages on the local air quality, but the impact was not always apparent because of seasonal variation in meteorological conditions and synoptic transport of pollutants [2].

L. Özdamar et al, (2014) study the Coordinating debris cleanup operations in post disaster road networks. They proposed a constructive heuristic that generates roadside debris cleanup plans for a limited number of equipment in the post-disaster road recovery planning problem. Travel times between cleanup tasks are not prefixed but depend on the blockage status of the entire road network at the time of travel. They developed a novel mathematical model that maximizes cumulative network accessibility throughout the cleanup operation and minimizes makespan. They proposed several practical and robust task selection rules that favor one or both goals that are tested on realistic size road networks with deterministic and stochastic debris cleanup times [14].

IV. IMPLEMENTATION

The practical derivation of such information requires the construction of a representation that combines thematic, spatial and graphical information about the road blockage and provides appropriate relationships between them. The procedures to test the ideas presented here have been implemented [1].

As described above, there is a natural synergy between road blockages and graph structure. However, the basic road-block structure needs some enhancements, in order to support the necessary associations between thematic and graphical information. In these implementations a distinction is made between non-structural cities (of valency 2) and structural cities (of valency I or 3 or higher). Non-structural cities may serve either to represent a point of interest, such as a settlement, or to mark a point where a thematic feature of a road changes. Roads are modeled as sequences of segments that are identical in their thematic properties, such as number of blockages or road surface type, and have no interior points of interest. In non-structural cities, roads are performed by a sequence of connecting segments. Roads end at structural cities that can also be points of interest or feature transformations.

Road blocks and cities are assigned unique identifiers, which act as keys to database records of related graphical and thematic data. The relationships between road blockage and cities are also stored in relational form. This is defining a regula falsi graph and allows for graph representation of road blockage. Thematic and graphical road information is linked to sections using relational tables. In recording road blockage it is only necessary to store the identity of the preceding city in the road or the identity of the previous road blockage in each city in the recording path. Linkage is maintained between the graph and blockage database. For example, the graphical information can be stored to allow regula falsi graph representation.

The problems of excessive computation time should also be anticipated when considering such graph search algorithms and simplification strategies. The algorithm used for these test implementations is due to Dijkstra [6]. In such cases it may be more efficient to use an algorithm such as Algorithm [16, ch.3], which is more suited to road blockages and finding the shortest path between cities.

The task of finding shortest paths in the road blockages also arises. For example, in location-allocation analyses [4]. The use of an algorithms using outward path search from both connected cities being has been suggested [17]. Clearly such developments in location-allocation studies and blockage analysis are of direct relevance to the implementation of the normalization methods presented here.

V. RESULTS

In this section, we prove some results.

Lemma 1. *Let G be a regula falsi graph in which the degree of each city is at least 2. Then G contains a cycle.*

Proof. Given that G is a regula falsi graph in which the degree of each city is at least 2. We have to show that G contains a cycle.

Let G have one road or many roads. Then the result is trivial.

Therefore, we assume that G is a simple graph.

Now, let c be any city in G. We also build a road

$$c \rightarrow c_1 \rightarrow c_2 \rightarrow \ldots\ldots\ldots c_k$$

inductively by selecting c_1 as any city adjacent to c.

We also choose c_{i+1} as any city adjacent to c_i except c_{i-1}, for each $i > 1$.

Therefore, the existence of such a city is guaranteed by our hypothesis.

Now, since G has only many cities. Hence, we must ultimately choose a city that has been chosen earlier.

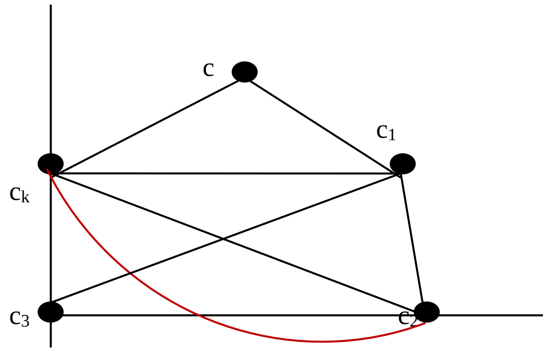

Figure 1. Regula Falsi graph G which contains a Cycle

Therefore, if c_k is the first such city, then that part of the road between two occurrences of c_k is the required cycle.

Thus, G contains a cycle.

Theorem 2. *Suppose that G is a connected graph. Then G is a Regula falsi graph if and only if the degree of each city of G is even.*

Proof. Necessary Condition:

Let the graph G be Regula falsi graph.

Then we have to show that the degree of each city of G is even.

Let $W : u \rightarrow u$ be an Euler circuit and c be an inner city such that $c \neq u$. Again let c appears k times in this regula falsi path W.

Since, every time one road comes to c, another road departs from c. Therefore,

$$d_G(c) = 2k \text{ (Even)}.$$

Also, since W starts and ends at u. Hence,

$$d_G(u) = 2.$$

Therefore, graph G contains all the cities with even degree.

Sufficient Condition:

Let us assume that G is a non-trivial connected graph such that

$d_G(c)$ is even, for every city $c \in V(G)$.

Then we have to show that G is Regula falsi graph.

Let $W = e_1 \ldots.. e_n : c_0 \rightarrow c_n$,

where $e_i = c_{i-1}c_i$ and W is the largest path in G.

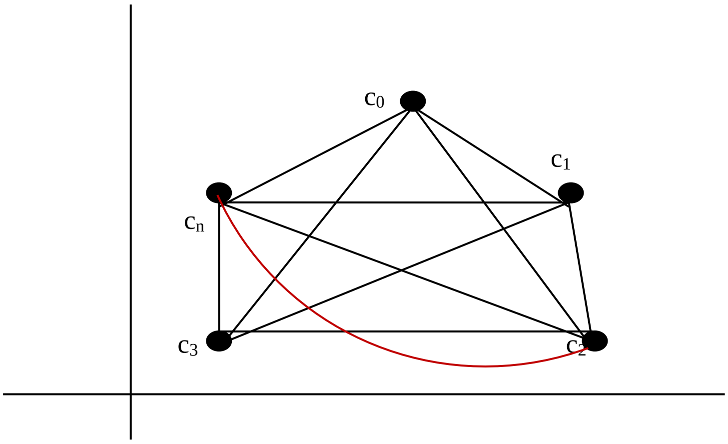

Figure 2. Regula Falsi Graph

Therefore, it follows that all $e = c_n w \in E(G)$ are among the roads of W, otherwise W_e would be the longer than W in G.
This is a contradiction.
Hence, $c_0 = c_n$, i.e., the path W is a closed path.
Now, if $c_0 \neq c_n$ then c_n can appear k times in the path W. Therefore,

$$d(c_n) = 2(k - 1) + 1 = 2k - 1 \text{ (which is odd).}$$

This is a contradiction to our hypothesis.
Hence, W should be closed path.
Now, if W is not an Euler circuit, then since G is connected, there exists a road $f = c_i u \in E(G)$ for some i, such that f is not in W.
Then, $e_{i+1}.....e_n e_1....e_i f$ is a path in G and it is longer than W.
This is a contradiction to our assumption.
So, W is a closed Euler circuit.
Therefore, G is a regula falsi graph.

Theorem 3. *Let G be a connected graph. Then G is an Regula falsi path if and only if it contains exactly two cities of odd degree.*
Proof. Necessary Condition:
Given that G is a Regula falsi path.
We have to show that G has exactly two cities of odd degree.
Let G have a Regula falsi path starting from city c and ending at city d.
Let us add a new road to the graph containing the final cities c and d, forming G'. G' also has a Euler circuit.
Then from 'Theorem 2', we can say that every city has an even degree.
Hence, the degrees of c and d in G are odd, while all others are even.
Therefore, G has exactly two cities in odd degree.
Sufficient Condition:
Given that G has exactly two cities of odd degree.
We have to show that G has a regula falsi path.
Let G have odd degrees of c and d, while all other cities have even degrees.
Let us add a new road r to the graph containing the final cities c and d, forming G'. And every city in G' has an even degree.
So, from 'Theorem 2', we can say that a Euler circuit is such that

$$c, r_1, c_2, r_2, \ldots, d, r, c$$

so that

$$c, r_1, c_2, r_2, \ldots, d$$

is a Euler walk.

Therefore, *G* is a regula falsi path.

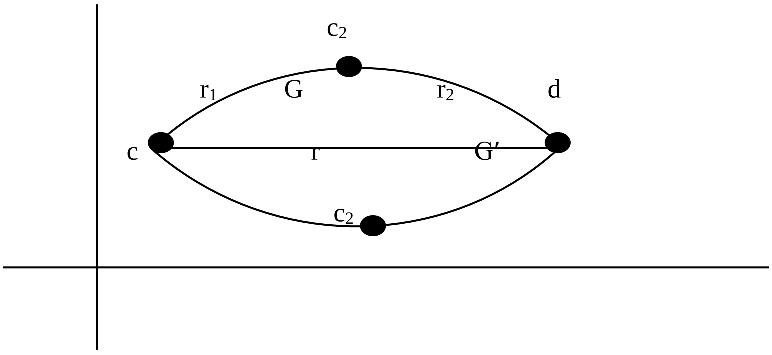

Figure 3. Regula Falsi path G of odd degree.

VI. CONCLUSION

In this paper, we have proved three results that deal with regula falsi graph. Lemma 3.1 shows that a connected graph will have a regula falsi path if it contains exactly two odd numbers of cities. Theorem 3.2 shows that a connected graph will have a Euler cycle or circuit if it has zero odd number of cities. Theorem 3.3 shows that no matter how many roads are in a connected graph, the sum of the degrees of its cities is equal to twice that number. This theorem also shows that a graph always has an odd number of cities. In future, we intend to prove many regula falsi based graphs to clear such road blockages.

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