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Research Paper

Thermodynamics Analysis for Interacting Holographic Dark Energy in Brans-Dicke Theory

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Abstract
The ground breaking reality of accelerating expansion of the universe is becoming a most important topic of research at present in cosmology. In the present work, the thermodynamics analysis is applied to interacting holographic dark energy model in the framework of Brans-Dicke theory of gravity. We have assumed the universe to be isotropic and homogenous. The Brans-Dicke scalar field ϕ is assumed to be of logarithmic form. It is observed that the model satisfied the GSL of thermodynamics and ˙Stot remains positive for early and late evolution of the universe. The result has been shown graphically.

Keywords: *Accelerating, homogenous, gravity, thermodynamics.*

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Introduction

Recently modified theories of gravity paid much attention to explain the recent accelerated expansion of the universe which was confirmed with observation in 1998 by Type Ia supernovae [1, 2, 3, 4]. This accelerated expansion also confirmed by cosmic microwave background radiation [5, 6], large-scale structure [7, 8], baryon acoustic oscillations [9] and Planck data [10, 11]. Now, it is interesting to find the factor responsible for this accelerating expansion of the universe. There must exists some unknown type of component with negative pressure known as Dark Energy(DE). The observation suggests that 70% of the cosmos is filled with this unknown type of DE. In the literature, a large number of theories of gravity were proposed to explain the mysterious concept of accelerated expansion of the universe. Brans-Dicke theory of gravity is one of the important theory of gravity to spot the present accelerated expansion of the universe. The DE models namely quint- essence [12, 13], K-essence [14, 15], tachyon[16, 17], phantom [18, 19, 20], quintom [21, 22, 23], holographic dark energy [24, 32], agegraphic dark energy [26, 27] have been discussed in the literature to solve the mysteries of DE.

’t Hooft [28] introduced the DE models based on the holographic principle which was studied by L. Susskind [29], have obtained a lot of attention. These models of DE, are known as holographic dark energy (HDE) models. [30] shows that the formation of black holes puts an upper bound on the DE density in the formalism of quantum field theory. Li [31] proposed that if ρ_h is the quantum zero-point energy density caused by a short distance cut-off, the total energy in an area of size L should not exceed the mass of a black hole of the same size. Taking in to consideration the largest IR cut-off L, the author obtained $\rho_h = 3c^2M_p^2L^{-2}$, where c denotes a dimensionless constant and M_p is the reduced Planck mass. A number of authors have discussed various aspects of HDE [32, 33, 34, 35, 36, 37, 38, 39]. It was shown [32] that the Hubble horizon as IR cut off leads to an incorrect Equation of State (EOS) of DE. The interaction between dark sector of the universe i.e Dark Matter (DM) and DE may give correct EOS when identified L with Hubble horizon $L = H^{-1}$ [33]. The HDE models have been studied extensively in literature to discuss the evolution of the universe [40, 41, 42].

The Brans-Dicke theory (BD) proposed by Brans and Dicke [43] in 1961 is natural extension to General Theory of Relativity which is able to explain the present accelerated expansion of the universe.

In this theory the gravitational constant G is not assumed to be constant but is proportional to inverse of the scalar field ϕ known as BD scalar field. The BD theory gives appropriate framework to study HDE. The BD theory has got a lot of attention due to it's association with string theory and extra dimensional theory. The HDE in BD theory is discussed in literature to explain the recent accelerated expansion of the universe [44, 45, 46, 47, 48]. In literature, it is shown in numbers of models the power law form $\phi \propto a^n$ of BD scalar field leads to constant deceleration parameter [49, 50, 51]. The authors [48] introduced logarithmic form of BD scalar field which is able to resolve the problem of constant deceleration parameter and evolve slowly to show slow variation of G. In the present study, we extend the work of [48] and shows the thermodynamics analysis of the model in BD theory.

This paper is organized as follows: Section 2. is devoted to model and field equation in presence of interaction between DM and DE. Section 3. discusses with logarithmic form of BD scalar field. Section 4. focuses on thermodynamics analysis of the model. We end with conclusion of the work in section 5.

1 Model and Field Equation

The modified Einstein Hilbert action in BD theory is given by

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}(-\phi R + \frac{\omega}{\phi}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi) + \mathcal{L}_m\right], \tag{1}$$

where g, R and $\mathcal{L}_m$ represent the determinant of the metric tensor $g_{\mu\nu}$, Ricci scalar curvature, and the matter Lagrangian density. ω denotes the coupling parameter. We assume the universe to be homozenous and isotropic, the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric given by the equation

$$ds^2 = dt^2 - a^2(t)\left[dx^2 + dy^2 + dz^2\right], \tag{2}$$

where a denotes the cosmic scale factor of the universe. Let us assume that the universe is filled with pressureless DM and DE. The variation of the action (1) with respect to the metric tensor, $g_{\mu\nu}$ for the line element (2) will give the following field equations

$$H^2 + H\frac{\dot{\phi}}{\phi} - \frac{\omega}{6}\frac{\dot{\phi}^{\,2}}{\phi^{\,2}} = \frac{\rho_m + \rho_h}{3\phi}, \tag{3}$$

$$2\frac{\ddot{a}}{a} + H^2 + 2H\frac{\dot{\phi}}{\phi} + \frac{\omega}{2}\frac{\dot{\phi}^{\,2}}{\phi^{\,2}} + \frac{\ddot{\phi}}{\phi} = \frac{-p_h}{\phi}, \tag{4}$$

where ρ_m, ρ_h and p_h stand for the energy density of DM, HDE and pressure of HDE. Let us consider the interaction between HDE and DM. The conservation equations are given by [47, 53]

$$\dot{\rho}_m + 3H\rho_m = Q, \tag{5}$$

$$\dot{\rho}_h + 3H(1 + w_h)\rho_h = -Q, \tag{6}$$

where w_h denotes the equation of state(EOS) given by $w_h = \frac{p_h}{\rho_h}$. Let us take the interaction as $Q = \Gamma\rho_h$. The sign of Γ defines the interaction rate. The positive values of Γ shows the energy transfer from HDE to DM and vice versa. As Γ is proportional to Hubble parameter[53], So let us take the value of $Q = 3b^2H\rho_h$ where b^2 is coupling constant. The BD scalar field ϕ obeys the wave equation

$$\ddot{\phi} + 3H\dot{\phi} = \frac{\rho_m + \rho_h - 3p_h}{2\omega + 3}. \tag{7}$$

In BD theory, the HDE density takes the form $\rho_h = 3c^2L^{-2}$, where ϕ is a scalar field which is depending upon time which couples with gravity. We will take Hubble horizon as IR cut off which is given by $L = H^{-1}$, then HDE takes the form

$$\rho_h = 3c^2H^2 \tag{8}$$

where c^2 is dimensionless constant.

2 Logarithmic Form of BD scalar Field

In literature, it has been shown by number of authors that the power law form of BD scalar field leads to constant deceleration parameter and fails to achieve phase transition of the universe. Therefore, we consider logarithmic form of BD scalar field. The logarithmic form was proposed by Kumar [48] and resolve the problems associated with power law form. Recently, New Agegraphic Dark Energy in Brans-Dicke Theory with sign changeable interaction for flat universe was studied by Pinki [54] and obtained time dependent deceleration parameter. The value of BD scalar field is given by

$$\phi = \phi_0 \ln(\alpha + \beta a), \tag{9}$$

where ϕ_0, $\alpha > 1$ and $\beta > 0$ are constants. It is important to mention here that for $\beta = 0$, the General Relativity is recovered. From equation (6), we can obtain

$$\dot{\rho}_h + 3H(1 + w_h)\rho_h = -3b^2 H \rho_h \tag{10}$$

Using equation (8) and(9) in (10), we get

$$\frac{\beta a}{(\alpha + \beta a)\ln(\alpha + \beta a)} + 2\frac{\dot{H}}{H^2} + 3(1 + w_h) = -3b^2 \tag{11}$$

3 Thermodynamics Analysis

Thermodynamics is one of the most important branch of science as it's never changes with new discoveries and ideas. The motivation in this work follows from the fact to check thermodynamics viability of HDE Model proposed by [48]. According to generalized second law of thermodynamics (GSL), the total entropy of the universe can not decrease with time [55, 56, 57]. The total entropy (S_{tot}) is expressed as the sum of horizon entropy (S_h) and inside the horizon entropy (S_{in}) and given by

$$S_{tot} = S_h + S_{in} \tag{12}$$

The rate of change in total entropy can be given by [54]

$$\dot{S}_{tot} = \frac{(\frac{4\pi \dot{H}}{H^2})^2}{H(\frac{\dot{H}}{H^2} + 2)} \tag{13}$$

To satisfy GSL of thermodynamics, $\dot{S}_{tot} \geq 0$, otherwise the condition of GSL of thermodynamics is violated. Equation (13) suggests that we need the value of $\frac{\dot{H}}{H^2}$ for analysis of GSL of thermodynamics. The deceleration parameter is given by

$$q = -1 - \frac{\dot{H}}{H^2} \tag{14}$$

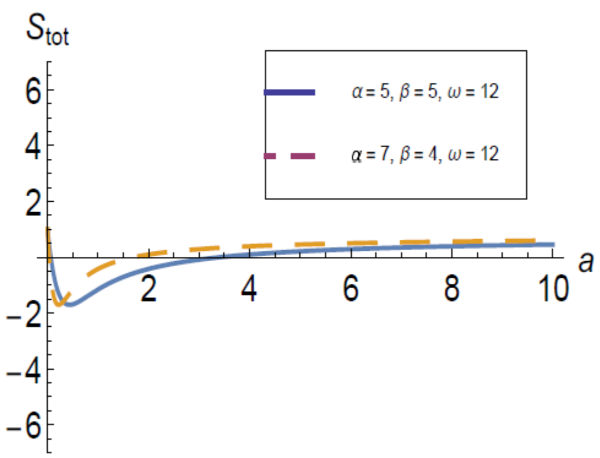

Figure 1: We have plotted $\dot{S}_{tot}$ against the scale factor a for various values of α, β and ω. We have taken $c = 0.77$, $b = 1.05$ and $H = 70$

The author [48] obtained the value of q as

$$q = \frac{3c^2 w_h + 1 + \frac{2\beta a}{(\alpha+\beta a)\ln(\alpha+\beta a)} - \frac{\beta^2 a^2}{(\alpha+\beta a)^2 \ln(\alpha+\beta a)} + \frac{\omega \beta^2 a^2}{2(\alpha+\beta a)^2[\ln(\alpha+\beta a)]^2}}{2 + \frac{\beta a}{(\alpha+\beta a)\ln(\alpha+\beta a)}} \tag{15}$$

Using equation (15) into (14), the value of $\frac{\dot{H}}{H^2}$ can be given by

$$\frac{\dot{H}}{H^2} = -1 - \frac{3c^2 w_h + 1 + \frac{2\beta a}{(\alpha+\beta a)\ln(\alpha+\beta a)} - \frac{\beta^2 a^2}{(\alpha+\beta a)^2 \ln(\alpha+\beta a)} + \frac{\omega \beta^2 a^2}{2(\alpha+\beta a)^2[\ln(\alpha+\beta a)]^2}}{2 + \frac{\beta a}{(\alpha+\beta a)\ln(\alpha+\beta a)}} \tag{16}$$

Let us check the behavior of $\dot{S}_{tot}$ in equation (13) with the help of equation (16). In the beginning of evolution of the universe when $a = 0$, the terms like $\frac{\beta a}{(\alpha+\beta a)\ln(\alpha+\beta a)}$ becomes zero, therefore the denominator becomes $H(\frac{-3}{2} - \frac{3c^2 w_h}{2})$ and numerator is always positive, therefore $\dot{S}_{tot}$ showing negative sign. Using the value of w_h given by [48] as $w_h = -b^2(1+\frac{1}{r})$, one can observe that $\dot{S}_{tot}$ is positive if $[\frac{-3}{2} + \frac{3b^2c^2}{2}(1+\frac{1}{r})] > 0$ i.e $\frac{3b^2c^2}{2}(1+\frac{1}{r}) > \frac{3}{2}$ and the value of Hubble's parameter is taken $H = 70$. The value of $b^2$is assumed to be positive. Similarly in late time evolution of the universe when $a \to \infty$, the terms like $\frac{\beta a}{(\alpha+\beta a)\ln(\alpha+\beta a)}$ approaches to zero and we obtain the same condition on $\dot{S}_{tot}$ to be positive as in case of early evolution of the universe. Therefore the HDE model obeys GSL of thermodynamics on condition of $\frac{3b^2c^2}{2}(1+\frac{1}{r}) > \frac{3}{2}$ in early and late time evolution of the universe. We have plotted the graph of $\dot{S}_{tot}$. It is clear from the Figure 1. that for various values of α, β and ω, the graph shows positive behavior in early and late time evolution of the universe as shown by the behavior of equation (13). The graph shows negative value for a very short period of time which is approximate negligible. Thus GSL is satisfied for early and late universe.

4 Conclusion and Future Scope

The accelerating expansion of the universe is becoming a most important topic of research at present. Here, in present work, we have extended the work of [48] and applied the GSL of thermodynamics to the model. We have assumed the universe to be isotropic and homogenous. The model satisfied GSL of thermodynamics and the value of $\dot{S}_{tot}$ remains positive in present as well as in future evolution of the universe. The result is also supported by graphical representation in Fig.1. The model can also be discussed with statefinder parameter and sound square speed is also the topic of future research to check the viability of the model with the existing models. One can also study the model by adding viscous effect to discuss the evolution of the universe.

References

[1] A.G. Riess, et al., Astron. J. **116**, 1009 (1998)

[2] S. Perlmutter, et al., Astrophys. J. **517**, 565 (1999)

[3] A.G. Riess, et al., Astrophys. J. **607**, 665 (2004)

[4] A.G. Riess, et al., Astrophys. J. **659**, 98 (2007)

[5] C.L. Bennett, et al., Astrophys. J. **148**, 1 (2003)

[6] E. Komatsu, et al., Astrophys. J. Suppl. **192**, 18 (2011)

[7] K. Abazajian, et al., Astron. j. **128**, 502 (2004)

[8] M. Tegmark, et al., Phys. Rev. D **69**, 103501 (2004)

[9] W.J. Percival et al., Mon. Not. R. Astro. Soc. **401**, 2148 (2010)

[10] P.A.R. Ade et al., A & A **571**, A16 (2014)

[11] N. Aghanim, et al., A & A **641**, A6 (2020)

[12] Wetterich C., Nucl. Phys. B, 302 (1998), 668.

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[13] Ratra B., et al. Phys. Rev. D, 37 (1988), 3406.

[14] Armendariz-Picon C., et al. Phys. Rev. Lett., 85 (2000), 4438.

[15] Armendariz-Picon C., et al. Phys. Rev. D, 63 (2001), 103510.

[16] Padmanabhan T., Phys. Rev. D, 66 (2002), 021301.

[17] Sen A., JHEP, 207 (2002), 065.

[18] Caldwell R., Phys. Lett. B, 45 (2002), 23.

[19] Nojiri S., et al. Phys. Lett. B, 562 (2003), 1.

[20] Nojiri S., et al. Phys. Lett. B, 565 (2004), 702.

[21] Elizalde E., et al. Phys. Rev. D, 70 (2004), 43539.

[22] Nojiri S., Odinstov S.D., Tsujikawa S., Phys. Rev. D, 71 (2005), 63004.

[23] Anisimov A., et al., JCAP, 506 (2005), 6.

[24] Li M., Phys. Lett. B, 657 (2007), 228.

[25] Hsu S.D., Phys. Lett. B, 594 (2004), 13.

[26] Cai. R.G., Phys. Lett. B, 657 (2007), 228.

[27] Wei H., Cai R.G., Phys. Lett. B, 660 (2008), 113.

[28] G. 't Hooft, arXiv:gr-qc/9310026 (1993)

[29] L. Susskind, J. Math. Phys. **36**,6377 (1995)

[30] A.G. Cohen, D.B. Kaplan and A.E. Nelson, Phys. Rev. Lett. **82**, 4971 (1999)

[31] M. Li, Phys. Lett. B **603**, 1 (2004)

[32] S.D.H. Hsu, Phys. Lett. B 594 (2004) 13.

[33] D. Pavón and W. Zimdahl, Phys. Lett. B **628**, 206 (2005)

[34] C.P. Singh, P. Kumar, Astrophys. Space Sci. **361**, 157 (2016)

[35] M. Li, C. Lin and Y. Wang, J. Cosmol. Astropart. Phys. **05**, 023 (2008)

[36] L. Xu, J. Cosmol. Astropart. Phys. **09**, 016 (2009)

[37] F. Yu et al., Phys. Lett. B **688**, 263 (2010)

[38] A. Sheykhi et. al, Gen. Relativ. Gravit. **44**, 623 (2012)

[39] Pinki et. al, Eur. Phys. Gen. C **82**, 143 (2022)

[40] M.R.Setare, Phys. Lett. B **642**, 421 (2022)

[41] M.R.Setare, Phys. Lett. B **653**, 116 (2022)

[42] M.R.Setare, Phys. Lett. B **654**, 1 (2022)

[43] Brans, C., Dicke, R.H.: Phys. Rev. **124**, 925 (1961)

[44] A.Sheykh, M., Jamil, Phys. Lett. B**694**, 284 (2011)

[45] N. Banerjee, D. Pavon, Phys. Lett. B **647**, 477 (2007)

[46] L.Xu, W.Li, J.Lu, Eur. Phys. J. C **60**, 145 (2009)

[47] A.Sheykhi, Phys. Rev. D **681**, 205 (2009)

[48] P.Kumar, C.P Singh, Int.J.Th. Phys. **56**, 3297 (2007)

[49] M.S. Berman, F.M. Gomide, Gen. Relativ. Gravit. **20**, 191 (1988)

[50] V.B. Johri, D. Kalyani, Gen. Relativ. Gravit. **26**, 1217 (1994)

[51] G.P. Singh, A. Beesham, Aust. J. Phys. **52**, 1039 (1999)

[52] P.Kumar, C.P.Singh, Astrophys. Space Sci. **362**, 52 (2017)

[53] A. Sheykhi, Phys. Rev. D **81**, 023525 (2010)

[54] Pinki and P.Kumar, Gen. Rel. Grav. **55**, 46 (2023)

[55] J.D. Bekenstein, Lett. Nuov. Cim. **4**, 737 (1972)

[56] J.D. Bekenstein, Phys. Rev. D **7**, 2333 (1973)

[57] J.D. Bekenstein, Phys. Rev. D **9**, 3292 (1974)