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Research Paper

Signomial-Geometric Programming Model of Marketing Mix Problem

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ABSTRACT: *Marketing mix is the interaction among the marketing decision variables. The marketing decision variables include: advertising, price, demand, in-store promotion, disposable income, sales, quality of the product, consumption pattern etc. The interactions among these variables are non-linear and vary from one variable to the other. There are so many obstructions to quantifying the interactions (elasticity) among these variables and to determine the optimal combination of these variables to produce optimal profit. Consequently, this paper aims to develop Signomial- Geometric Programming Model to proffer solution to the marketing mix problem using Zikko Waters Ltd as our case study. The data collected from Zikko Waters Ltd was specific to 2021 where the developed model was applied to determine the optimal profit for the year. From this paper, we discovered that quality and consumption* $(QC)_t$*; disposable income and demand* $(I_nD)_{ti}$ *did not contribute significantly to the optimal profit of the business for the period because there was no interaction between them. Some of the interactions were negative and others were positive but these signs do not affect the optimality and the optimal profit was 5,566,769.00 naira only for the period.*

KEYWORD: *Marketing mix, Optimal profit, Elasticity, Optimal primal decision variables, Signomial-Geometric programming.*

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I. INTRODUCTION

In marketing, the management is faced with the problem of decision making. This problem stems from the fact that management need to combine marketing mix variables in the proportion that will yield optimal profit. Marketing mix is the interaction among the marketing decision variables, these decision variables include; price, quality of product and advertising and so on. Marketing mix [1] is broadly classified into two namely: the "marketing mix" variables such as price, advertising, salesmen's salaries and product quality, which may be adjusted at the discretion of the firm; and the environmental variables such as disposable income, age composition, and population growth, which are outside the control of the firm. These variables influence the customer's utility. The ability to combine these marketing decision variables in a proportion that will maximize profit fall within the purview of optimization; hence, the optimal combination of these variables results in increased sales and expansion of revenue. As a decision problem, the firm is faced with the problem of either to increase quality of the products, lower the price of the product, increase advertising or increase the number of salesmen. In taking this type of decision, cost will be incurred on combination of these marketing decision variables. The neglect of the optimization stage means that managers often do not obtain any obvious recommended course of action from the study, nor can they easily compare alternatives against their objectives and constraints, or test the sensitivity of the solutions to the model specifications [2], these underline the importance of optimization in marketing. Other methods have been used to model marketing mix problem but optimization appears most suitable to solve the marketing mix problem. In marketing, the relationship between market performance and marketing efforts is influenced by interaction mechanisms. Advertising may influence sales levels directly and could also affect customers' price sensitivity. Likewise, sales may be a function of the competitiveness of the sales environment. Such interactions were popularized several years ago with the introduction of the "marketing mix" concept, which emphasizes that marketing efforts create sales synergistically rather than independently [3].

A marketing mix interaction effect exists if the firm's decision level for one marketing mix instrument affects sales sensitivity to changes of the firm's decision level for another instrument. There are several methods used in modelling marketing mix problems. One of them is the Classical method which is applied to obtain the optimal price [4]. But in this paper, we use an optimization technique known as signomial-geometric programming. Signomial programming is an extension of geometric Programming (GP) [5]. This is an optimization model that was derived from generalizing the geometric programming. Some authors proposed geometric programming model for marketing mix problem. The only restriction with this method is that it allows only the positive coefficient of polynomial function known as posynomial, while in the real life scenario; a marketing function has a form of generalized polynomial function. Those activities that bring money contribute positive coefficient while those that take money contribute negative coefficient to the marketing problem. Hence, the optimization model that accommodates a general polynomial function having both positive and negative coefficients is called (complimentary geometric programming) signomial programming.

Some authors also suggested signomial programming [6] but in most cases, their models were so complex to the understanding of marketing professionals and signomial programming does not produce the global optimal solution. In this paper, we have developed a simplified combination of signomial-geometric programming model that optimize profit from marketing mix problem and achieve global optimal solution. From literature, the nature of function that results from marketing mix problem is concave. While some previous optimization models assume convex function, that is, concave objective function and convex constraint equations; others assume independent decision variables. This makes solutions from such model incorrect. The marketing decision variables are not only dependent and non-linear but interact with one another. Hence, this requires a polynomial function with either positive or negative coefficients or combination of both with non-linear interactive decision variables.

Because of the non-linearity of the decision variables, linear programming is not a good model for marketing mix problems. In this kind of model, economy of scale and elasticity play a vital role in sales. The price elasticity of demand is the percentage change in the quantity of a good demanded divided by the corresponding percentage change in its price, while the cross elasticity of demand for good i with respect to changes in the price of good j is the percentage change in the quantity of good i demanded, divided by the corresponding percentage change in the price of good j [7]. In this paper, the aim is to develop a Signomial-Geometric programming model for marketing mix problem. The model will be used to determine the optimal profit, while the specific objectives are:

i. To determine the elasticity (optimal dual decision variables).
ii. To filter and discard some irrelevant marketing mix variables from the model.
iii. To determine the nature of relationship among the interacting variables.
iv. To Determine the optimal profit from the interaction of the relevant marketing mix variables.
v. To determine the optimal primal decision variables.

This paper is divided into five sections. Section one above gives the general introduction of the work. Section two is devoted to literature review. Section three takes care of materials and methods. Section four takes care of implementation of the developed model. Section five treats result and discussion.

II. LITERATURE REVIEW

Many authors used different methods such as linear programming; Regression, classical method etc to model marketing mix problem but each of these methods has some short comings. For regression technique [3], the behaviour of these marketing mix variables is at variance with the use of regression model which requires the decision variables to be independent and uncorrelated, but the variables in marketing mix problem are dependent and highly interactive. Hence, the response yield is not the true picture of the problem. There is substantial empirical support for the presence of marketing interactions.

Some authors use classical optimization method [8 and 9]. This approach uses the standard maximization procedure of differential calculus to obtain the values of the decision variables that optimize the objective function. There are two major limitations of this method; the First is that the optimal solutions are not generally guaranteed because only the first order (necessary) conditions are taken into account because the second order conditions are difficult to check. In the Second case, the optimal marketing mix decisions can only be achieved if one assumes concave objective functions and convex feasible regions; conditions that cannot readily be assumed in marketing [10]. Sales function is concave and predictor variables consist of n terms, each term being a set of interacting variables, where the interaction among the variables is often nonlinear and because of concave sales function resulting naturally from marketing mix problem [1]; the requirement of all positive coefficients prevents the direct application of the nonlinear regression techniques.

In many cases, researchers have avoided optimization models in marketing, partly because empirical estimates of typical marketing response functions suggest non-convex maximization problems not easily handled by conventional nonlinear programming. GP now offers a convenient structure for modeling and optimizing the realistic complexity of many important marketing problems [6]. The authors [1] and [2] observed that some coefficients in the sales function bear negative values which are out of the scope of geometric programming. Geometric programming gives globally optimal solution, whereas signomial programming gives only a local optimal solution. GP appears capable of handling many typical problems in marketing, some of which have been considered too complex for analytical solution.

In some other cases, researchers [11] used linear programming to model marketing mix problem. This method cannot be effectively used because of the limitations associated with it. In the first case, the marketing decision variables interact with one another and cannot have linear relationship; rather, they have nonlinear relationship. Secondly, the objective function of the model is actually concave with convex constraint equation, but the marketing mix problem does not follow this trend, it follows a concave function which cannot be solved with linear programming, see [6]. Hence, solving this kind of problem using linear programming means ignoring the basic nature of the problem and hence will not give the required and true result. In the same view as [1] and [2]; [12] were of the opinion that optimizing the manufacturer's allocation of resources among a set of alternative channels is a problem which is of central importance in marketing, but it is one which has proved insoluble by conventional programming procedures.

Geometric programming can provide a theoretically attractive and practical method for solving marketing mix problem, but as we have pointed out, geometric programming restricts the coefficients of the polynomial to positive values and does not accommodate negative coefficients of the polynomial. On the other hand, decision on the proper combination of marketing variables to optimize profit is a subset of decision theory, and as such, Bayesian methods have become widespread in marketing literature [13]. Different authors observed that the appeal of the Bayesian approach has long been noted by researchers because of developments in computational methods and expanded availability of detailed marketplace data. They saw Bayesian method as having advantage in modeling marketing mix problem especially in situations where there is limited information about a large number of units or where the information comes from different sources. Here, it is believed that Bayesian statistical method cannot handle a nonlinear concave function with highly interactive decision variables.

Haven seen the short comings from different models, we propose a signomial-geometric programming model that will not only take care of concave objective function and convex constraint equations and the interactions (elasticity) among the marketing mix variables but will produce the required global optimal solution and determine the optimal primal decision variables.

III. MATERIALS AND METHODS

Optimization in marketing mix problem is of the form $g(t) = S(t) - C(t)$, see [1, 6] where the interest is to maximize the profit $g(t)$ composed of sum of sales, S(t) and cost, C(t), over the number of units. A function of this nature is called signomial or complimentary geometric programming [5] which allows the negative coefficient of the function. Signomial programming is an extension of geometric (posynomial) programming [14], [15] which permits only the positive coefficients of the program. In optimizing the marketing mix problem, it is desired to attain global optimal solution, which is guaranteed by geometric programming. Signomial programming produces local optimal solution. The maximization of the dual problem is the same thing as minimizing the primal problem and from the fundamental duality theory [16], the maximum of the dual problem is equals to the minimum of the primal problem and at optimality, both converge to the same optimal solution. Instead of minimizing the primal objective function, [17] maximize the dual objective function subject to linear constraints to obtain optimal dual decision variables.

$$\text{Maximize } g(y) = \prod_{k=0}^{m} \prod_{j=1}^{n_k} \left(\frac{C_{kj}}{y_{kj}} \sum_{j=1}^{n_k} y_{kj} \right)^{y_{kj}} \tag{1}$$

Subject to:

$$\sum_{j=1}^{n_0} y_{0j} = 1 \tag{2}$$

$$\sum_{k=0}^{m}\sum_{j=1}^{n_k} a_{kij} y_{kj} = 0 \ ; \quad i = 1, 2, \ldots, n \tag{3}$$

Where y_{0j} = the dual decision variable from the objective function, y_{kj} = dual decision variables from the constraint equation, m = number of variables, n_k = number of terms in the constraint equation, n_0 = number of terms in the objective function. Normality condition (2) and orthogonality condition (3) are sufficient conditions for optimality and equations (2) and (3) are combined to form equation (4) given as:

$$Ay = B \tag{4}$$

where

A is an (m x n) coefficient matrix, y is a vector of dual decision variables of order (n x 1) and B is a vector of constants of order (m x 1). The stationary point (optimal solution) of the objective function for constrained geometric programming model is given as

$$g^*(t) = g^*(y) = \prod_{k=0}^{m}\prod_{j=1}^{n_k}\left(\frac{C_{kj}}{y_{kj}*}\sum_{i=1}^{n_k} y_{kj}^*\right)^{y_{kj}^*} \tag{5}$$

where g*(t) is the minimum of the primal objective function and g*(y) is the maximum of the dual objective function [18]. If the function, g(t), is known to possess a minimum, the stationary value, g*(t), will be the global minimum (optimal) solution of g(t) since, in this case, there is a unique solution for y*. If the problem has a greater than zero degree of difficulty; the minimum of the primal problem can be obtained by maximizing the corresponding dual function given in equation (1).

3.1. Optimal Weights of the Dual Decision Variables

We determine the optimal weight of the dual decision variable from equation (4) as follows:

$$y^* = A^- B \tag{6}$$

where y* is the optimal dual decision variables, A^- is the Moore-Penrose generalized inverse. Using MATLAB to compute the optimal dual decision variables, we have

$$\begin{aligned} &>> A = [a_{11}, a_{12}, \ldots; a_{21}, a_{22}; \ldots; a_{m1}, a_{m2}; \ldots; a_{mn}] \\ &>> B = [b_1; b_2; \ldots; b_n] \\ &>> y* = \text{Pinv}(A) * B \end{aligned} \tag{7}$$

3.2. Optimal Objective Function (g*(t))

From the stationary point stated in equation (5), we determine the optimal value of the objective function as follows:

$$\begin{aligned} g^*(y) = &\left\{\left(\frac{C_1}{y_1^*}\right)\wedge y_1^*\right\} \times \left\{\left(\frac{C_1}{y_2^*}\right)\wedge y_2^*\right\} \times \ldots \times (y_3^* + y_4^*)\wedge(y_3^* + y_4^*) \\ &\times \ldots \times (y_{n-1}^* + y_n^*)\wedge(y_{n-1}^* + y_n^*) \end{aligned} \tag{8}$$

where $g^*(y)$ is the optimal value of the objective function and $(y_{n-1}^* + y_n^*)$ is the sum of the optimal weight of the dual decision variables corresponding to each constraint equations. We observed that there is a subscript, k, attached to y at the optimal objective function, y_k; k indicates where the decision variable is coming from. If it assumes 0, that is y_0, it indicates that it is coming from the objective function and if it assumes 1, that is y_1, it indicates that it is coming from the first constraint equation of the Geometric program, etc.

3.3. Optimal Weight of the Primal Decision Variables (t*).

We applied the optimal value of the objective function, g*(t) and the optimal weights of the dual decision variables, y^* with the relationship that exist between them and the original geometric program to determine the optimal weights of the primal decision variables of the geometric program, see equation (14).

$$C_j \prod_{i=1}^{m} (t_i)^{a_{ij}} = y_j^* \, g^*(t) \tag{9}$$

$$\frac{y_j^* \, g^*(t)}{C_j} = (t_1)^{a_{1j}} . (t_2)^{a_{2j}} ... (t_n)^{a_{mj}} \tag{10}$$

$$\ln\left[\frac{y_j^* \, g^*(t)}{C_j}\right] = a_{1j} \ln t_1 + a_{2j} \ln t_2 + ... + a_{mj} \ln t_n \;\; ; i = 1, ... , m. \tag{11}$$

$$\text{Let } w_i = \ln t_i \tag{12}$$

we have

$$\ln\left[\frac{y_j^* \, g^*(t)}{C_j}\right] = a_{1j} w_1 + a_{2j} w_2 + ... + a_{mj} w_n \tag{13}$$

where $\ln\left[\frac{y_j^* \, g^*(t)}{C_j}\right]$ is the vector of constant, B. To obtain the optimal weights of the primal decision variable, take the exponential of equation (12) to have:

$$t_i{}^* = e^{w^*} \tag{14}$$

where the elements of the column vector w* are subset of real number R, see [19]. Proportion of sales is presented in table 1; demand due to advert, price, in-store promotion and disposable income is presented in table 2, while the proportion of limits for some marketing variables is presented in table 3.

IV. DATA PRESENTATION AND ANALYSIS

4.1. DATA PRESENTATION

Table 1. Proportion of Sales (10,000.00 naira)

YEAR	2019	2020	2021
Value	0.2	0.35	0.45

Source: Zikko Waters Ltd (2021)

Table 2 Demand due to Advert., Price, In-store promo., and Disposable Income (10,000)

Quality, consumption	Adv., Demand	Demand, Price, In-store Promo.	Demand, Disposable income
1	4.40	4.58	1.54

Source: Zikko Waters Ltd (2021)

Table 3. Proportion of Limits for some Marketing Variables

Variables	Lower Limit	Upper Limit
Advertising	0.35	-
Price	0.65	0.92

Demand	0.3	0.65
In-store promotion	0.2	0.35
Advertising + Disposable income	0.6	-
Price + Advertising	0.6	-

Source: Zikko Waters Ltd (2021)

4.2. Model Development

In developing the marketing mix model for Zikko waters Ltd, we consider specifically the following marketing mix variables as presented in the Tables above: Advertising (A_t), Price (P_t), Demand (D_t), In-store promotion (I_{pt}), Disposable income (I_{nt}), Sales (S_t), Quality (Q_t), and consumption (C_t). Each of them will be assigned values as follows: (A_t) = t_1, (P_t), = t_8, (D_t) = t_3, (I_{pt}) = t_5, (I_{nt}) = t_6, (S_t) = t_4, (Q_t) = t_7, and (C_t) = t_2; our focus is on the data for 2021.

Let g(t) be the profit, then

$$g(t) = \sum S(t) - \sum C(t) \tag{15}$$

$$g(t) = 0.45(ADI_pQPS)_t - 4.40(AD)_t - 4.58(PDI_p) - (QC)_t - 1.54(I_nD)_t \tag{16}$$

We observed that sales are induced by advertising, demand, in-store promotion, quality of the product and price. Since our interest is to maximize profit from the marketing mix problem, we have

$$\text{Maximize } g(t) = 0.45(ADI_pQPS)_t - \left[4.40(AD)_t + 4.58(PDI_p) + (QC)_t + 1.54(I_nD)_t\right] \tag{17}$$

Substituting for the variables, we have

$$\text{Maximize } g(t) = 0.45t_1t_3t_5t_7t_8t_4 - \left[4.40t_1t_3 + 4.58t_8t_3t_5 + t_7t_2 + 1.54t_6t_3\right] \tag{18}$$

Subject to

1. Advertising: $t_1 \le 0.35$ (19)
2. Relative price: $0.65 \le t_8 \le 0.95$ (20)
3. Relative Demand: $0.3 \le t_3 \le 0.65$ (21)
4. Relative In-store promotion: $0.25 \le t_5 \le 0.35$ (22)

   Strategies:

5. $t_1 + t_5 \le 0.6$ (23)
6. $t_8 + t_1 \le 0.35$ (24)

The problem now becomes

$$\text{Maximize } g(t) = 0.45t_1t_3t_4t_5t_7t_8 - \left[4.40t_1t_3 + 4.58t_3t_5t_8 + t_2t_7 + 1.54t_3t_6\right] \tag{25}$$

Subject to

$$2.86t_1 \le 1 \tag{26}$$

$$0.65t^{-1}{}_{8} \leq 1 \quad (27)$$

$$1.04t_8 \leq 1 \quad (28)$$

$$0.3t^{-1}{}_{3} \leq 1 \quad (29)$$

$$1.54t_3 \leq 1 \quad (30)$$

$$0.25t^{-1}{}_{5} \leq 1 \quad (31)$$

$$2.86t_5 \leq 1 \quad (32)$$

$$1.67t_1 + 1.67t_5 \leq 1 \quad (33)$$

$$2.86t_8 + 2.86t_1 \leq 1 \quad (34)$$

Equation (25) is a signomial model subject to constraint equations (26) – (34), which does not produce a global optimal solution and we desire to have a model that will produce a global optimal solution. Therefore, we convert the signomial problem above to geometric programming problem by reformulation.

Now

Let $u(t) = 0.45t_1t_3t_4t_5t_7t_8$ (35)

$$f(t) = -[4.40t_1t_3 + 4.58t_3t_5t_8 + t_2t_7 + 1.54t_3t_6] \quad (36)$$

Hence, we minimize the inverse of the objective function subject to the reformulated constraint;

Minimize $t^{-1}{}_{0}$ (37)

Subject to $\frac{t_0}{u(t)} + \frac{f(t)}{u(t)} \leq 1$ (38)

Equations (37) and (38) are equivalent to the geometric programming primal problem (P):

The Model now becomes:

Minimize $g_0(t) = t^{-1}{}_{0}$ (39)

Subject to (1): $g_1(t)$:

$$2.22t_0t^{-1}{}_{1}t^{-1}{}_{3}t^{-1}{}_{4}t^{-1}{}_{5}t^{-1}{}_{7}t^{-1}{}_{8} + 9.78t^{-1}{}_{4}t^{-1}{}_{5}t^{-1}{}_{7}t^{-1}{}_{8} + 10.18t^{-1}{}_{1}t^{-1}{}_{4}t^{-1}{}_{7} + 2.22t^{-1}{}_{1}t_2t^{-1}{}_{3}t^{-1}{}_{4}t^{-1}{}_{5}t^{-1}{}_{8} + 3.42t^{-1}{}_{1}t^{-1}{}_{4}t^{-1}{}_{5}t_6t^{-1}{}_{7}t^{-1}{}_{8} \leq 1 \quad (40)$$

Subject to (2): $g_2(t)$:

$$2.86t_1 \leq 1 \quad (41)$$

$$0.65t^{-1}{}_{8} \leq 1 \quad (42)$$

$$1.04t_8 \leq 1 \quad (43)$$

$$0.3t^{-1}_{3} \le 1 \tag{44}$$

$$1.54t_{3} \le 1 \tag{45}$$

$$0.25t^{-1}_{5} \le 1 \tag{46}$$

$$2.86t_{5} \le 1 \tag{47}$$

$$1.67t_{1} + 1.67t_{5} \le 1 \tag{48}$$

$$2.86t_{8} + 2.86t_{1} \le 1 \tag{49}$$

Subject to orthogonality and normality condition of equation (4)
Equations (39) to (49) is a converted standard constrained Geometric programming model for the marketing mix problem.

4.3. Analysis

This problem has K = 7 degrees of difficulty [K = n-(m+1)]; where K = degree of difficulty, n = number of terms, m = number of variables. The degree of difficulty K = n - (m+1); hence, $n_0 = 1$; $n_k = 16$; m = 9. Hence, K = 17 – 10 = 7.

We have seen that the number of terms are greater than the number of variables plus one, this gives rise to greater than zero degree of difficulty problem [18]. Since this problem does not have a unique solution, we maximize the dual program of equation (1) subject to linear constraint of equation (4).

Forming the orthogonality and normality condition for the dual decision variables from equation (4), we have

$-y_1+y_2+0y_3+0y_4+0y_5+0y_6+0y_7+0y_8+0y_9+0y_{10}+0y_{11}+0y_{12}+0y_{13}+0y_{14}+0y_{15}+0y_{16}+0y_{17} = 0$

$0y_1 - y_2+ 0y_3-y_4 - y_5 - y_6 +y_7 + 0y_8 + 0y_9 + 0y_{10} + 0y_{11} + 0y_{12} + 0y_{13}+y_{14}+0y_{15}+0y_{16} +y_{17} = 0$

$0y_1+0y_2+ 0y_3+0y_4+y_5+0y_6+0y_7+0y_8+0y_9+0y_{10}+0y_{11}+0y_{12}+0y_{13}+0y_{14}+0y_{15}+0y_{16}+0y_{17} = 0$

$0y_1 - y_2+ 0y_3 + 0y_4 - y_5 + 0y_6 + 0y_7+0y_8+0y_9-y_{10}+y_{11}+0y_{12}+0y_{13}+0y_{14}+0y_{15}+0y_{16}+0y_{17} = 0$

$0y_1 - y_2- y_3 - y_4 – y_5 - y_6 + 0y_7 + 0y_8 + 0y_9+0y_{10}+0y_{11}+0y_{12}+0y_{13}+0y_{14}+0y_{15}+0y_{16}+0y_{17} = 0$

$0y_1 - y_2- y_3 + 0y_4 - y_5 - y_6 + 0y_7 + 0y_8 + 0y_9 + 0y_{10} + 0y_{11} - y_{12}+y_{13}+0y_{14}+y_{15}+0y_{16}+0y_{17} = 0$

$0y_1 +0y_2+0y_3+0y_4+0y_5+y_6+0y_7+0y_8+0y_9+0y_{10}+0y_{11}+0y_{12}+0y_{13}+0y_{14}+0y_{15}+0y_{16}+0y_{17} = 0$

$0y_1 -y_2 - y_3 - y_4 + 0y_5 - y_6 + 0y_7 + 0y_8+0y_9+0y_{10}+0y_{11}+0y_{12}+0y_{13}+0y_{14}+0y_{15}+0y_{16}+0y_{17} = 0$

$0y_1 - y_2 -y_3 + 0y_4 - y_5 - y_6 + 0y_7 - y_8 + y_9 + 0y_{10} + 0y_{11} + 0y_{12}+0y_{13}+0y_{14}+0y_{15}+y_{16}+0y_{17} = 0$

$y_1+0y_2+0y_3+0y_4+0y_5+0y_6+0y_7+0y_8+0y_9+0y_{10}+0y_{11}+0y_{12}+0y_{13}+0y_{14}+0y_{15}+0y_{16}+0y_{17} = 1$

$$
\begin{bmatrix}
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & -1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & -1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & -1 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & -1 & 0 & -1 & -1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \\ y_{16} \\ y_{17} \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}
$$

Solving for y* from equation (6), we have

A= [-1,1,0,0,0,0;0,0,0,0,0,0,0,0,0,0,0; . . .; 1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0];

B= [0; 0; 0; 0; 0; 0; 0; 0; 0; 1];

y* = Pinv(A)*B

$$
y^* = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \\ y_{16} \\ y_{17} \end{bmatrix}
= \begin{bmatrix} 1.0000 \\ 1.0000 \\ -0.5556 \\ -0.4444 \\ -0.0000 \\ 0.0000 \\ 0.1852 \\ -0.1481 \\ 0.1481 \\ -0.5000 \\ 0.5000 \\ -0.1481 \\ 0.1481 \\ 0.1852 \\ 0.1481 \\ 0.1481 \\ 0.1852 \end{bmatrix}
$$

The above values of y* are the optimal weights of the dual decision variables. Optimal dual decision variables for y^*_5 and y^*_6 are zeros ($y^*_5 = y^*_6 = 0$) and will lead to degenerate solution at optimality. These decision variables correspond to the marketing variables $(QC)_t$ and $(I_nD)_t$. This implies that there is no interaction between them and their inclusion in the model does not contribute anything to the optimal objective function. When this kind of problem is encountered, it can be approached by striking out those variables that lead to degenerate solution out of the problem, see [20]. We can see that the signs are consistent with both constraint (1) and (2) on the respective variables. The signs (positive and negative) on the optimal dual decision variables (y*) indicates the nature of interaction (relationship) while the actual values measure the elasticity between the marketing decision variables involved. These signs will not affect the optimal objective function (profit). Profit have the optimal dual decision variable to be one (1.0000), this indicates that it depends completely on the other marketing mix variables. The interaction (elasticity) between $(AD)_t$ and $(PDI_p)_t$ have negative values while others have positive values.

From equations (8) and haven eliminated y^*_5 and y^*_6 from the model, we compute for the optimal objective function as follows:

$$g^*(y) = \left(\left(\frac{1}{1.0000}\right)^\wedge 1.0000\right) * \left(\left(\frac{2.22}{1.0000}\right)^\wedge 1.0000\right) * \left(\left(\frac{9.78}{0.5556}\right)^\wedge 0.5556\right) * \left(\left(\frac{10.18}{0.4444}\right)^\wedge 0.4444\right)$$

$$* \left(\left(\frac{2.86}{1.852}\right)^\wedge 1.852\right) * \left(\left(\frac{0.65}{0.1481}\right)^\wedge 0.1481\right) * \left(\left(\frac{1.04}{0.1481}\right)^\wedge 0.1481\right) * \left(\left(\frac{0.3}{0.5000}\right)^\wedge 0.5000\right)$$

$$* \left(\left(\frac{1.5}{0.5000}\right)^\wedge 0.5000\right) * \left(\left(\frac{0.25}{0.1481}\right)^\wedge 0.1481\right) * \left(\left(\frac{2.86}{0.1481}\right)^\wedge 0.1481\right) * \left(\left(\frac{1.67}{0.1852}\right)^\wedge 0.1852\right)$$

$$* \left(\left(\frac{1.67}{0.1481}\right)^\wedge 0.1481\right) * \left(\left(\frac{2.86}{0.1481}\right)^\wedge 0.1481\right) * \left(\left(\frac{2.86}{0.1852}\right)^\wedge 0.1852\right) * \left((2)^\wedge 2\right) * \left((0.1852)^\wedge 0.1852\right)$$

$$* \left((0.1481)^\wedge 0.1481\right) * \left((0.1481)^\wedge 0.1481\right) * \left((0.5)^\wedge 0.5\right) * \left((0.5)^\wedge 0.5\right) * \left((0.1481)^\wedge 0.1481\right)$$

$$* \left((0.1481)^\wedge 0.1481\right) * \left((0.3333)^\wedge 0.3333\right) * \left((0.3333)^\wedge 0.3333\right)$$

$$\therefore g^*(y) = 556.6769 \Rightarrow 5{,}566{,}769 \text{ naira}$$

The above is the optimal objective function.

We proceed to calculate the primal decision variables.

$$250.7554 = t_0 t^{-1}{}_1 t^{-1}{}_3 t^{-1}{}_4 t^{-1}{}_5 t^{-1}{}_7 t^{-1}{}_8$$
$$31.6247 = t^{-1}{}_4 t^{-1}{}_5 t^{-1}{}_7 t^{-1}{}_8$$
$$24.3013 = t^{-1}{}_1 t^{-1}{}_4 t^{-1}{}_7$$

Taking the ln of both sides we have

$$2.3993 = \ln t_0 - \ln t_1 - \ln t_3 - \ln t_4 - \ln t_5 - \ln t_7 - \ln t_8$$
$$1.5000 = -\ln t_4 - \ln t_5 - \ln t_7 - \ln t_8$$
$$1.3856 = -\ln t_1 - \ln t_4 - \ln t_7$$

Let $Wi = \ln t_i$

Forming the matrix, we have

$W_0 - W_1 - W_3 - W_4 - W_5 - W_6 - W_7 - W_8 = 2.3993$

$0W_0 + 0W_1 + 0W_3 - W_4 - W_5 + 0W_6 - W_7 - W_8 = 1.5000$

$0W_0 - W_1 + 0W_3 - W_4 + 0W_5 + 0W_6 - W_7 + 0W_8 = 1.3856$

$$\begin{bmatrix} 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & -1 & -1 & 0 & -1 & -1 \\ 0 & -1 & 0 & -1 & 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} W_0 \\ W_1 \\ W_3 \\ W_4 \\ W_5 \\ W_6 \\ W_7 \\ W_8 \end{bmatrix} = \begin{bmatrix} 2.3993 \\ 1.5000 \\ 1.3856 \end{bmatrix}$$

A = [1,-1,-1,-1,-1,-1,-1,-1; 0,0,0,-1,-1,0,-1,-1; 0,-1,0,-1,0,0,-1,0];

B = [2.3993; 1.5000; 1.3856];

W* = Pinv(A)*B

$$W^* = \begin{bmatrix} 0.1661 \\ -0.4009 \\ -0.1661 \\ -0.4924 \\ -0.2576 \\ -0.1661 \\ -0.4924 \\ -0.2576 \end{bmatrix}$$

$$t_i^* = \exp(W^*) = \begin{bmatrix} 1.1807 \\ 0.6697 \\ 0.8470 \\ 0.6112 \\ 0.7729 \\ 0.8470 \\ 0.6112 \\ 0.7729 \end{bmatrix}$$

These are the optimal weights of the primal decision variables.

V. RESULT AND CONCLUSION

In this study, we are concerned with the optimal profit made by Zikko Waters Ltd for the year 2021 from the combination of various marketing variables of the business called marketing mix problem. We modeled the problem using Signomial-Geometric programming model and solve the modeled problem with MATLAB software. From the solution to the problem, we observed that the company made a profit of 5, 566, 769 naira (five million, five hundred and sixty-six thousand seven hundred and nine naira only) for the year. The optimal dual decision variables y* were obtained and presented above. These variables were the same as the elasticity of their respective marketing decision variables. We observed that there was no interaction between the decision y^*_5 and y^*_6 ($y^*_5 = y^*_6 = 0$) and this will lead to degenerate optimal solution. These decision variables correspond to the marketing variables $(QC)_t$ and $(I_nD)_t$. When this kind of problem is encountered, it can be approached by striking out those variables that lead to degenerate solution out of the problem, see [20]. We also observed that the signs are consistent in both constraint (1) and (2) on the respective variables. The signs (positive and negative) on the optimal dual decision variables (y*) indicates the nature of interaction (relationship) between or among the interacting variables while the actual values measure the elasticity between the marketing decision variables involved. These signs do not affect the optimal objective function (profit). The interaction (elasticity) between $(AD)_t$ and $(PDI_p)_t$ have negative values while others have positive values. Finally, we obtained the optimal primal decision variables t* which must be strictly positive because its coefficients must be non-negative according to geometric programming rule.

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