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Research Paper

Development/Application of PG-Inverse Weighted Multi-Objective Geometric Programming Model for Oil and Gas Industry in Nigeria

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ABSTRACT: *In this paper, we developed a method called PGWMOGP to solve MOGP problems. The method is robust and not restricted by either positive or negative degrees of difficulty in geometric programming. We applied the method on the oil, oil & condensate and natural gas production in Nigeria from the year 2017 to 2021 to maximize the revenue of the NNPC. The data used for the study is a secondary data collected from the BP Statistical Review of World Energy 2022. We applied the developed method and obtained the optimal compromised solution to be 21192000 million dollars; this is the least that can be attained by the objective function for the period. We observed from the analysis that the products progressively contributed to the optimal objective function as the year progresses; hence, year 2017 contributed 96742.74 million dollars, year 2018 contributed 101753.7 million dollars, year 2019 contributed 1213265 million dollars, year 2020 contributed 25416540 million dollars and the year 2021 contributed the largest which was 4564527 million dollars. The contribution per year was dependent on the level of output for that year. These results will help the management of NNPC and others to take appropriate decisions.*

KEYWORDS: *Non-linear optimization, positive g-inverse weighted multi-objective geometric programming, Pareto optimal objective function, Pareto optimal primal decision variables, Pareto optimal dual decision variables.*

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I. INTRODUCTION

In today's world, decision makers are looking for how to get a model that can answer as much as possible their numerous decision problems. In this case, instead of finding an optimal solution to various problems, they can combine the problems and find a common solution to it. This is more economical and effective than when the solutions for such problems are sought individually. In this paper, we seek to combine these objective functions and find a compromised objective function. The author [1] observed that most of the real-world decision-making problems in economic, environmental, social and technical areas are multi-dimensional and multi-objective ones. The author developed fuzzy geometric programming technique to solve Multi-Objective Geometric Programming (MOGP) problem; however, multi-objective optimization problems differ from single-objective optimization problem. Generally, engineering design and management science problems have multi-objective functions. In this case it is not suitable to use any single objective programming to find an optimal compromise solution. The application of MOGPP in a case of maximizing the crude oil, oil & condensate and natural gas production is the desire of the Nigerian National Petroleum Company (NNPC) Limited. The motive is to optimize these objectives at once and find a compromised (pareto) solution to save cost.

As a limited liability company, the interest of NNPC Limited is to optimize the production of these products for the overall interest of the company and environmental safety. The natural gas industry consists of three main parts; production and processing, transmission and storage, and distribution, [2]. Natural Gas in liquid form is transported in Liquefied Natural Gas Vessels over a long distance without major accidents or safety problems. In the same vein, crude oil is transported using pipe lines and vessels but its transportation is not as bulky and costly as gas, [3]. We are not interested on the mechanism of the production but on how the output of the three products could be optimized to achieve optimal benefit for the company. We are concerned with the yearly output of oil and gas from (2017 – 2021). We want to apply our newly developed positive g-inverse weighted multi-objective geometric programming technique (PGWMOGP) to optimize the revenue accrued from the production of the products over a period of five years based on the available data.

II. LITERATURE REVIEW

Management is faced with the problem on how to develop a model that can handle their numerous managerial decision problems relating to optimization. The application and extension of Geometric programming gave birth to MOGPP. Some authors in the recent past have worked on the MOGP in order to find a solution to multi-objective geometric programming problem. Some researchers, [4], observed that multi-objective geometric programming (MOGP) is a powerful optimization technique developed for solving various non-linear programming problems subject to linear and non-linear constraints. MOGP has been applied by many researchers to several optimization and engineering problems such as integrated circuit design, engineering design, project management and inventory management. MOGP is a special type of non-linear programming problem with multiple objective functions. In many real-life optimization problems, multiple objectives have to be taken into account, which may be related to the social, economic and technical aspects of real optimization problems.

Other researchers, [5], observed that engineering design problem has multiple objective-functions. In this case, it is not suitable to use any single-objective programming to find an optimal compromise solution. Thus, in multiple-objective geometric program there are number of minimization type objective function, number of inequality type constraints and n number of strictly positive decision variables. They added a constant to the MOGP weighted objective function as their developed method to have a zero degree of difficulty. They were of the opinion that there is no single global solution, and it is often necessary to determine a set of points that fit all predetermined definition for an optimum. The predominant concept in defining an optimal point is that of Pareto optimality, which they defined that, $x^* \in X$, is Pareto optimal if and only if there does not exist another point, $x \in X$, such that $f_k(x) \leq f_k(x_k^*)$ for at least one function. Still contributing in this area of knowledge, [6], developed a method for finding a compromise optimal solution of certain multi-objective geometric programming problem by using weighting method. First they transformed the multiple objective functions to a single objective by considering it as the linear combination of the multiple objectives along with suitable constants called weights. By changing the weights, the most compromise optimal solution was arrived at by using GP techniques. The authors were of the opinion that the weighting method is the simplest multi-objective optimization which has been widely applied to find the non-inferior optimal solution of multi-objective function within the convex objective space.

Contributing their own quota, [7], were of the opinion that the real world problems are multi choice problems, and that optimizing a combination of objectives has the advantages of producing a single compromise solution required. They emphasized on a theory that has been developed for locating the points of maxima and minima of constrained and unconstrained nonlinear optimization problems, popularly known as Kuhn−Tucker theory. They acknowledged that, it is not well suited for computational purposes, but it provides a set of necessary and sufficient conditions for locating the point of optimality. They applied constraint method to solve a class of multi-objective Geometric programming problems. Using constraint method, they can optimize one of the objective functions at a time where other objectives are kept in the constraint part of the model. This method is found more suitable than other generating methods used for obtaining pareto optimal solutions. After obtaining lower and upper bounds of each objective function with the given constraints, they have generated a set of pareto optimal solution. In multi-objective optimization problems, what is optimal in terms of one of the objectives is usually non-optimal for the remaining objectives. The multi-objective Geometric programming problem is considered as a Vector-minimization problem and assumed to have optimal compromise solution. The authors transformed a MOGP by incorporating Kuhn – Tucker conditions for optimality into the GPP model, the new model has a MOGP form which they solve using the dual form of GP.

Then, [8] observed that when there are multiple objectives in the GPP, the problem is defined as the Multi-Objective Geometric Programming Problem (MOGPP). In general, there are two methods of solutions to MOGP namely: fuzzy GPP and weighted mean methods in the literature. Numerical approximations are widely used to solve the Multi-objective programming problems. One of the numerical approximations is the Taylor series expansion which the authors also applied in their study. This numerical approach minimizes the weighted objective function subject to Kuhn-Tucker Conditions. The solution obtained at the end of the iterative processing gives the pareto optimal solution The results obtained are compared to the results of the weighted mean method and the same results were found. The authors were of the opinion that a multi-objective problem is often solved by combining its multiple objectives into one single objective scalar function. This approach is in general known as the weighted-sum or scalarization method. In more detail, the weighted sum method minimizes a positively weighted convex sum of the objectives that represents a new optimization problem with a single objective function.

The researchers, [9], developed a method for solving MOGP which was able to calculate the bounds of objective value for the problems where the cost, the constraint coefficients and right-hand sides are interval parameters. Their technique was basically the Weighting method and by the use of interval-valued function, they were able to solve the problem by geometric programming technique. The authors observed that the weighting method is one of the most popular techniques for solving MOGPPS, which can be applied to obtain the non inferior optimal solution with the convex objective space. The parameters of models for many real world problems are usually stated imprecisely and this leads to the formulation of the MOGP models with interval values. By applying weighted method, the authors converted a multi-objective geometric programming problem to a single objective geometric programming problem in which parameters are interval valued numbers. The authors were of the opinion that their technique takes minimal time and that their procedure will help researchers for wider application in the field of engineering problems.

The researcher, [10], modeled a multi-response stratified sampling survey as a multi-objective geometric programming problem (MOGPP). These responses have different objectives, which was modeled into geometric programming problems with multiple objective functions (MOGP). They applied fuzzy multi-objective geometric programming approach to determine the solution to the modeled problem. The fuzzy programming approach was described for solving the formulated MOGPP and optimum allocation of sample sizes are obtained. The optimal cost of stratification was determined using the dual MOGP and the size of each stratum was determined using the primary and dual relationship in GP. The paper reflects the application of fuzzy programming for solving the multi-objective geometric programming problem (MOGPP). The problem of multiple responses in stratified sample survey has been formulated as MOGPP and the dual solution is obtained with the help of Lingo software. The optimum allocations are obtained with the help of primal-dual relationship theorem along with corresponding dual solution. A numerical example was illustrated by the author to ascertain the practical utility of the given method in multiple-response stratified sample surveys.

The researchers, [11], applied MOGP to model a multivariate double sampling design. The author modeled the multivariate double sampling problem into a geometric programming problem with multi-objective function. His solution procedure was fuzzy MOGP and the dual MOGP was used to obtain the optimal objective function. The primal decision variables, the cost of sampling and sampling sizes were determined through primal – dual relationship. The upper and lower bound of fuzzy membership was determined also. His work is innovative and shows another extension of GP to other areas of disciplines. The paper gives the insightful study of the problem of Multivariate Double Sampling Design which is formulated as a convex MOGPP with non-linear objective function and linear constraints. The fuzzy programming approach is used for converting the (MOGPP) into Single Objective Geometric Programming Problem (SOGPP) with the help of membership function. The formulated SOGPP was solved with the help of LINGO Software and the dual solution is obtained. A numerical example was given to establish the practical utility of the given method in multivariate two-stage stratified sample surveys. The researcher adopted this method for obtaining the solution of very complicated convex programming problem, which extends to multi-stage sample survey problems.

In addition to the works of other researchers as discussed above in this area, we developed and applied the positive g-inverse weighted multi-objective geometric programming (PGWMOGP) method to solve MOGPP. This method has been found to yield a global pareto optimal solution. The method is robust and not restricted by either negative or positive degrees of difficulty and can solve MOGPP with very large degrees of difficulty. It is a combination of positive g-inverse method and the weighted method of MOGP and was found to be more effective and robust in finding solution to MOGPP.

III. MATERIALS AND METHODS

Geometric programming models can be unconstrained as in equation (1) or constrained as in equation (2) and (3), see [12].

$$Minimize \quad f(x) = \sum_{j=1}^{N} C_j \prod_{i=1}^{m} x_i^{a_{ij}} \tag{1}$$

$$Minimize\ f_0(x) = \sum_{j=1}^{N_0} C_{0j} \prod_{i=1}^{m_0} x_i^{\ a_{0ij}} \quad (2)$$

Subject to $g_k(x) = \sum_{j=1}^{N_k} C_{kj} \prod_{i=1}^{m_k} x_i^{\ a_{kij}} \le 1$ (3)

Starting from equation (2), if $f_1(x)$, $f_2(x)$,… ,$f_p(x)$ are n objective functions for any vector x = (x , x ,...., x_n $)^T$, then the weighting method for their optimal solution is defined as:

Let $W = \left\{ w : w \in R^n, w > 0, \sum_{j=1}^{n} w_j \ = 1 \right\}$ be the set of non-negative weights.

The weighted objective function for the multiple objective functions defined above can be stated as

$$f_n(x) \quad (4)$$

where $f_n(x) = \min_{x \in X} \sum_{j=1}^{n} w_j f_j(x)$.

Therefore, we have

$$Minimize\ f_n(x) = \sum_{e=1}^{p} w_e \left(\sum_{j=1}^{N_0} C_{0j} \prod_{i=1}^{m_0} x_i^{\ a_{0ij}} \right) \quad (5)$$

$$Minimize\ f_n(x) = \sum_{e=1}^{p} \sum_{j=1}^{N_0} w_e C_{0j} \prod_{i=1}^{m_0} x_i^{\ A_{0ij}} \quad (6)$$

were A_{0ki} is assumed to be a rectangular matrix to be manipulated by the positive g- inverse, see [2]. Hence, the GWMOGP model for the study in the primary form becomes

$$Minimize\ f_n(x) = \sum_{e=1}^{p_0} \sum_{j=1}^{N_0} w_e C_{0j} \prod_{i=1}^{m_0} x_i^{\ A_{0ij}} \quad (7)$$

IV. Subject to $g_{ki}(x) = \sum_{j=1}^{N_k} C_{kj} \prod_{i=1}^{m_k} x_i^{\ A_{kij}} \le 1$ (8)

Subject to normality and orthogonality conditions

However, if the objective space of the original problem is non-convex, then the weighting method may not be capable of generating the efficient solutions on the non-convex part of the problem, but this is not the case because all geometric programming problems are equivalent to a convex problem, see [13]. The dual form of PGWMOGP given in equation (7) and (8) is

$$Maximize\ f(y) = \prod_{e=1}^{p_0} \prod_{k=0}^{m} \prod_{j=1}^{N_k} \left(\frac{w_e C_{kj}}{y_{kj}} \sum_{j=1}^{N_k} y_{kj} \right)^{y_{kj}} \quad (9)$$

Subject to

$$\sum_{k=0}^{m} \sum_{i=1}^{n} \sum_{j=1}^{N_k} a_{kij} y_{kj} \ = 0 \quad (10)$$

$$\sum_{j=1}^{N_0} y_{0j} \ = 1 \quad (11)$$

where y_{kj} = dual decision variables, n = the number of constraint equations. Equation (10) and (11) are the orthogonality and normality condition. Both equations are combined as given in equations (12).

$$Ay = B \tag{12}$$

where **A** is a matrix of order (m x n) to be restricted to positive values; hence, the positive g-inverse, **y** is a vector of dual decision variables of order (n x 1) and **B** is a vector of constants of order (m x 1). The optimal dual decision variables y* must be strictly positive for the optimal objective function to exist. That is for f*(x) to exist, $\exists\, y^* > 0 \ni f^*(x) > 0$

At the stationary point (optimal solution) the minimum of the primal is equal to the maximum of the dual objective function. This is given in equation (13).

$$f^*(x) = f(y^*) = \prod_{e=1}^{p_0} \prod_{k=0}^{m} \prod_{j=1}^{N_k} \left(\frac{w_e C_{kj}}{y_{kj}^*} \sum_{i=1}^{N_k} y_{kj}^* \right)^{y_{kj}^*} \tag{13}$$

If the function f(x) is known to possess a minimum, the stationary value f*(x) given in equation (13) will be the global minimum of f(x) since there is a unique solution for y*. The optimal solution of the dual is related to the optimal solution of the primal problem from equation (14).

$$C_j \prod_{i=1}^{m} (x_i)^{a_{ij}} = y^*{}_j f^*(x). \tag{14}$$

Optimal primal decision variables are obtained from equation (15)

$$\ln(x_i) = w \Rightarrow x^* = e^w \tag{15}$$

IV. DATA PRESENTATION AND ANALYSIS

4.1. DATA PRESENTATION

In this section we present the data collected on the production of Oil, Crude oil and Condensate and Natural gas for the year 2017 -2021 for Nigeria from BP Statistical Review of World Energy, 2022. Our interest is to maximize the revenue from the production of the three products from the three sectors of primary energy produced by Nigerian NNPC Limited. Table 1 presented Oil Production in thousands of barrels per day, table 2 presented Crude Oil and Condensate production in thousands of barrels per day and table 3 presented Natural Gas Production in billion cubic meters per day.

Table 1. Oil Production in thousands of barrels per day

Year	2017	2018	2019	2020	2021
Qty	1968	2006	2101	1828	1626
Price ($/bbl)	54.31	72.47	64.95	42.31	69.76
Revenue	2022.31	2078.47	2165.95	1870.31	1695.76

BP Statistical Review of World Energy 2022

Table 2. Crude Oil and Condensate production in thousands of barrels per day

Year	2017	2018	2019	2020	2021
Qty	1890	1922	2014	1765	1545
Price ($/bbl)	54.31	72.47	64.95	42.31	69.76
Revenue	1944.31	1994.47	2078.95	1807.31	1614.76

BP Statistical Review of World Energy 2022

Table 3. Natural Gas Production in billion cubic meters per day

Year	2017	2018	2019	2020	2021

Qty	47.2	48.3	49.3	49.4	45.9
Price: Henry Hub ($/m.btu)	2.96	3.12	2.51	1.99	3.82
Revenue	50.16	51.42	51.81	51.39	49.72

BP Statistical Review of World Energy 2022

4.2. Analysis

It will be more economical to optimize the revenue (from the production of these three products) at once, instead of determine them separately. Hence, we model the problem as follows: we converted the revenue from each of the product to millions according to the measurements for sales; that is, dollars per barrel and dollars per million btu. Let the productions and the revenue from each year under consideration be represented by the decision variables $x_i; i = 1,...,5;$ and $x_i \geq 0$.

The objective is to maximize the (production) revenue for the company over the period of five years for the three products jointly. First, we model the problem as unconstrained Gp problems with each product having its objective function. Applying equation (1), we have

$$Maxf_1(x) = 2022310x_1 + 2078470x_2 + 2165950x_3 + 1870310x_4 + 1695760x_5 \quad (16)$$

Equation (16) is the revenue function from Oil production from 2017 – 2021

$$Maxf_2(x) = 1944310x_1 + 1994470x_2 + 2078950x_3 + 1807310x_4 + 1614760x_5 \quad (17)$$

Equation (17) is the revenue function from Oil and Condensate production from 2017 – 2021

$$Maxf_3(x) = 50160000x_1 + 51420000x_2 + 51810000x_3 + 51390000x_4 + 49720000x_5 \quad (18)$$

Equation (18) is the revenue function from Natural gas production from 2017 – 2021

Applying equation (4), we have

$$w_1(f_1(x)) + w_2(f_2(x)) + .w_3(f_1(x)) = \max_{x \in X} \sum_{j=1}^{n} w_j f_j(x) \quad (19)$$

where $\sum_{j=1}^{n} w_j = 1$; let $w_1 = 0.3; w_2 = 0.4$ and $w_3 = 0.3$ and n =3

Equation (19) is written as equation (20)

$$f_n(x) = 0.3(f_1(x)) + 0.4(f_2(x)) + .0.3(f_1(x)) \quad (20)$$

$$\begin{aligned} Maxf_n(x) = {} & 606693x_1 + 623541x_2 + 64978x_3 + 561093x_4 + 508728x_5 \\ & + 777724x_1 + 797788x_2 + 831580x_3 + 722924x_4 + 645904x_5 \\ & + 15048000x_1 + 15426000x_2 + 15543000x_3 + 15417000x_4 + 14916000x_5 \end{aligned} \quad (21)$$

Equation (21) is the WMOGPP in the form of equation (6)

Applying equations (7) and (8) on equation (21), we have

$$\begin{aligned} Maxf_n(x) = {} & 606693x_1x_3 + 623541x_2x_4^{-1} + 64978x_3 + 561093x_1x_4 + 508728x_2^{-1}x_5^{-1} \\ & + 777724x_1x_2^{-1} + 797788x_2x_3^{-1} + 831580x_3x_5^{-1} + 722924x_4 + 645904x_1x_5 \\ & + 15048000x_1^{-1}x_4^{-1} + 15426000x_2x_5 + 15543000x_2^{-1}x_3 + 15417000x_1^{-1}x_4 + 14916000x_5 \end{aligned} \quad (22)$$

Subject to

$$x_1^{-1}x_2x_3^{-1}x_4^{-1}+x_1^{-1}x_2^{-1}x_3^{-1}x_5+x_1x_2x_4x_5^{-1}\le 1 \qquad (23)$$

Subject to

$$Ay = B$$

Applying equations (12) on equations (22) and (23), we have

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & 0 & -1 & -1 & 1 \\ 0 & 1 & 0 & 0 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 & -1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ \vdots \\ y_{18} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

We observed that the problem has K= N-(m+1) degrees of difficulty; that is, N= 18, m = 5, hence; K = 18 – 6 = 12. Therefore, the problem has 12 degrees of difficulty. This is a very large problem that cannot be solved using the weighted or fuzzy method because the resulting matrix is rectangular with very large degrees of difficulty. In geometric programming, one or two degrees of difficulty problem are big challenge and many authors resorted to approximating such Gpp by other methods, but here, we have twelve degrees of difficulty problem. Hence, we apply our proposed PGWMOGPP to solve the problem.

Solving the resulting matrix using Python programming in R, we have:

```
# R program to illustrate
# Solve a linear matrix
# equation of matrices using
# Moor-Penrose pseudoinverse
# importing library for
# applying pseudoinverse library (mass)
# Representing A in
# matrix form in R
A = matrix( c(1,0,1,0,0,1,0,1,0,-1,0,1,0,0,1,0,0,1, . . . , 1,1,0,1,-1,0), nrow = 6, ncol = 18)
cat("A = :\ n")
print(A)
# Representing B in
# matrices form in R
B = matrix( c(0,0,0,0,0,1), nrow = 6, ncol = 1)
cat("B = :\ n")
print(B)
# Calculating y using ginv( )
cat("solution of linear equations using pseudoinverse :\ n")
y = ginv(A) %*% B
print(y)
Hence, we have;
A=[1,0,0,1,0,1,0,0,0,1,-1,0,0,-1,0,-1,-1,1;0,1,0,0,-1,-1,1,0,0,0,0,1,-1,0,0,1,-1,1;.                    .;
1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,0,0];
B = [0;0;0;0;0;1];
y* = Pinv(A)*B
```

$$y^* = \begin{bmatrix} 0.0336 \\ 0.0761 \\ 0.0372 \\ 0.0651 \\ 0.1115 \\ 0.0836 \\ 0.1076 \\ 0.0615 \\ 0.0687 \\ 0.0495 \\ 0.0896 \\ 0.0432 \\ 0.0471 \\ 0.0724 \\ 0.0531 \\ 0.0425 \\ 0.0294 \\ 0.0021 \end{bmatrix}$$

y* satisfies equation (12)

Applying equation (13), we obtain the optimal revenue from the optimal production of the three products simultaneously over the period of five years from 2017 – 2021 as follows:

f * =
((606693/0.0336)^0.0336)*((623541/0.0761)^0.0761)*((64928/0.0372)^0.0372)*((561093/0.0651)^0.0651)*((508728/0.1115)^0.1115)*((777724/0.0836)^0.0836)*((797788/0.1076)^0.1076)*((831580/0.0615)^0.0615)*((722924/0.0687)^0.0687)*((64590/0.0495)^0.0495)*((15048000/0.0896)^0.0896)*((15426000/0.0432)^0.0432)*((15543000/0.0471)^0.0471)*((15417000/0.0724)^0.0724)*((14916000/0.0531)^0.0531)*((1/0.0425)^0.0425)*((1/0.0294)^0.0294)*((1/0.0021)^0.0021)*((0.0740)^0.0740)

f*(x) = 21192000

Since our interest is to obtain the optimal compromised objective function (Optimal revenue for the three products over the period, the optimal compromised decision variables are not necessarily required, but to show that our method is robust and can be used to find any component part of the MOGPP, we apply equation (14) to obtain the following results;

$$\frac{y^*{}_j f^*(x)}{C_J} = x_1{}^{\pm 1}.x_2{}^{\pm 1}...x_n{}^{\pm n}$$

$$1.1737 = x_1 x_3 \tag{24}$$

$$2.5864 = x_2 x_4{}^{-1} \tag{25}$$

$$12.1325 = x_3 \tag{26}$$

$$2.4588 = x_1 x_4 \tag{27}$$

$$4.6447 = x_2^{-1} x_5^{-1} \quad (28)$$

Equations (24-28) give the optimal primal decision variables.

Taking the ln of equations (24 – 28), we have

$$0.1602 = \ln x_1 + 0\ln x_2 + \ln x_3 + 0\ln x_4 + 0\ln x_5$$

$$0.9502 = 0\ln x_1 + \ln x_2 + 0\ln x_3 - \ln x_4 + 0\ln x_5$$

$$2.4959 = 0\ln x_1 + 0\ln x_2 + \ln x_3 + 0\ln x_4 + 0\ln x_5$$

$$0.8997 = \ln x_1 + 0\ln x_2 + 0\ln x_3 + \ln x_4 + 0\ln x_5$$

$$1.5357 = 0\ln x_1 - \ln x_2 + 0\ln x_3 + 0\ln x_4 - \ln x_5$$

Let $\ln x_i = w_i$; hence,

$$0.1602 = w_1 + 0w_2 + w_3 + 0w_4 + 0w_5$$

$$0.9502 = 0w_1 + w_2 + 0w_3 - w_4 + 0w_5$$

$$2.4959 = 0w_1 + 0w_2 + w_3 + 0w_4 + 0w_5$$

$$0.8997 = w_1 + 0w_2 + 0w_3 + w_4 + 0w_5$$

$$1.5357 = 0w_1 - w_2 + 0w_3 + 0w_4 - w_5$$

Writing the above in the form of equation (12), we have

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{bmatrix} = \begin{bmatrix} 0.1602 \\ 0.9502 \\ 2.4959 \\ 0.8997 \\ 1.5357 \end{bmatrix}$$

Solving for w_i using R-software, we have:

```
c(1,0,0,1,0,0,1,0,0,1,1,0,1,0,0,0,1,0,1,0,0,0,0,0,1)->A
matrix(A,c(5,5))->K
t(K)->B
B%*%K->L
solve(L)->P
P%*%B->T
c(0.1602,0.9502,2.4959,0.8997,1.5357)->Y
```

matrix(Y,c(5,1))->N

T%*%N->W

$$w = \begin{bmatrix} -2.3357 \\ -2.2852 \\ 2.4959 \\ 3.2342 \\ 3.8209 \end{bmatrix}$$

Applying equation (15), we have

$$x^* = \begin{bmatrix} 96742.74 \\ 101753.7 \\ 1213265 \\ 2541654 \\ 4564527 \end{bmatrix}$$

x* is the contribution of each year $x_1, \ldots, x_5$ in millions of dollars to the optimal compromised objective function

V. RESULTS AND CONCLUSION

5.1 Results

In this paper, we developed a method that solves MOGPP using the known weighted approach in combination of positive g-inverse method. The combination of the two methods give rise to a new method called PGWMOGP. This method is robust and solves MOGP with ease. The method is not restricted to any known degree of difficulty. Here, we were able to obtain the optimal compromise solution for all the products per year. From the respective contributions of the products per year, we observed that the year 2017 contributed the least in the total revenue over the period on the production of the three products, followed by 2018 in that other and the most was in the year 2021. This may not be unconnected with the relative peace in the oil and gas production areas in Nigeria in the recent time. The optimal objective function cut across all the three products and a global minimum to all of them. With this, the management can determine the position of their business over the period of time without putting about five times the resources used to collectively determine the position of the business by PGWMOGP their by saving cost and time. The computation of the dual decision variables from the resulting rectangular matrix was made easy by python program in R- code and we were able to solve a MOGPP with twelve degrees of difficulty. The same R-programming was applied to determine the pareto optimal objective function and the pareto optimal primal decision variables. We call it pareto because the variable represents and cut across each of the production for the period. There may be other variables that produced better result but may not fit in because each of the pareto variables is the minimum any of the three products can attain for the period. These results will guide the company to take appropriate decision regarding their production in the future and to reflect on the adopted policies that lead to where they are now to make necessary adjustments.

5.2 Conclusion

In this paper, we have developed a method called PGWMOGP to solve MOGP problems. The method is robust and is not restricted by either positive or negative degrees of difficulty in geometric programming. We applied the method on the production of oil, oil & condensate and natural gas production in Nigeria from 2017 to 2021 to maximize the profit of the NNPC Limited. The data used for the study is a secondary data collected from the BP Statistical Review of World Energy 2022. We applied the developed method and obtained the pareto optimal solution to be 21192000 million dollars; this is the least that can be attained by the objective function for the period. We observed from the analysis that the products progressively contributed to the optimal objective function as the year progresses; hence, year 2017 contributed 96742.74 million dollars, year 2018 contributed 101753.7 million dollars, year 2019 contributed 1213265 million dollars, year 2020 contributed 2541654 million dollars and the year 2021 contributed the largest which was 4564527 million dollars. Application of this method is recommended to solve numerous decision and optimization problems confronting managements from day to day in running their respective businesses. This model can be applied across disciplines to solve practical optimization problems which usually occur with multiple objectives.

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