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Research Paper

Inverse Square Numbers.

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Abstract:
Inverse Square Numbers are numbers whose square also inverts when the number inverts.
If XY is an inverse square number,
then $(XY)^2 = ABC$
$(YX)^2 = CBA$.
For example:
We know that 12^2 is 144, If we invert the number 12, we get 21. 21^2 is 441. If you have observed thoroughly, then you will come to know that the number 144 is also inverted = 441.Similarly, We know that 13^2 is 169, if we invert the number 13, we get 31. 31^2 is 961. If you have observed thoroughly, then you will come to know that the number 169 is also inverted = 961.
This Theory will help people in finding squares. A person can easily find the square of 31 if he know the square of 13 and similarly, he can find the square of 21 and other such numbers.

Thus,

$(12)^2 = 144$
$(21)^2 = 441$

Isn't it interesting ? Such numbers are called Inverse Square Numbers.
10,11,12,13,20,21,22,30,31 are 2 - digit inverse square numbers.

For example :

$(10)^2 = 100$
$(01)^2 = 001$

$(11)^2 = 121$
$(11)^2 = 121$

$(12)^2 = 144$
$(21)^2 = 441$

$(13)^2 = 169$
$(31)^2 = 961$

$(20)^2 = 400$
$(02)^2 = 004$

$(22)^2 = 484$
$(22)^2 = 484$

$(30)^2 = 900$
$(03)^2 = 009$

These are 2 - digit Inverse Square Numbers.
If we add these numbers with 100, then 3-digit Inverse Square Numbers are formed. 100 is also a Inverse Square Number.

For example :

$(100)^2 = 10000$
$(001)^2 = 00001$

$(110)^2 = 12100$
$(011)^2 = 00121$

$(111)^2 = 12321$
$(111)^2 = 12321$

$(112)^2 = 12544$
$(211)^2 = 44521$

$(113)^2 = 12769$
$(311)^2 = 96721$

$(120)^2 = 14400$
$(021)^2 = 00441$

$(122)^2 = 14884$
$(221)^2 = 48841$

$(130)^2 = 16900$
$(031)^2 = 00961$

These are 3-digit Inverse Square Numbers.
To obtain 4-digit Inverse Square Numbers we can just add 1000 in 2-digit Inverse Square Numbers.
Here, 1000 is also a Inverse Square Number.

$(1000)^2 = 1000000$
$(0001)^2 = 0000001$

$(1010)^2 = 1020100$
$(0101)^2 = 0010201$

$(1011)^2 = 1022121$
$(1101)^2 = 1212201$

$(1012)^2 = 1024144$
$(2101)^2 = 4414201$

$(1013)^2 = 1026169$
$(3101)^2 = 9616201$

$(1020)^2 = 1040400$
$(0201)^2 = 0040401$

$(1022)^2 = 1044484$
$(2201)^2 = 4844401$

$(1030)^2 = 1060900$
$(0301)^2 = 0090601$

You might be thinking that, there is a zero in hundredth place. Can we write any number there ?

We can write 1 or 2 in the hundredth place but when the last two digits are 13 or 30, then we are suppose to write 0 or 1 only. (at the hundredth place).

For example :

$(1100)^2 = 1210000$
$(0011)^2 = 0000121$

$(1110)^2 = 1232100$
$(0111)^2 = 0012321$

$(1111)^2 = 1234321$
$(1111)^2 = 1234321$

$(1112)^2 = 1236544$
$(2111)^2 = 4456321$

$(1113)^2 = 1238769$
$(3111)^2 = 9678321$

$(1120)^2 = 1254400$
$(0211)^2 = 0044521$

$(1122)^2 = 1258884$
$(2211)^2 = 4888521$

$(1130)^2 = 1276900$
$(0311)^2 = 0096721$

$(1200)^2 = 1440000$
$(0021)^2 = 0000441$

$(1210)^2 = 1464100$
$(0121)^2 = 0014641$

$(1211)^2 = 1466521$
$(1121)^2 = 1256641$

$(1212)^2 = 1468944$
$(2121)^2 = 4498641$

$(1220)^2 = 1488400$
$(0221)^2 = 0048841$

When the last two digits are 13 or 30 and 2 is used in hundredth place, the Inverse Square Law will fail.

$(1213)^2 = 1471369$
$(3121)^2 = 9740641$ ✕

$(1230)^2 = 1512900$
$(0321)^2 = 00103041$ ✕

Also Note: When a digit rather than 1 is repeated more than twice in a number, then that number is not a Inverse Square Number.

$(1222)^2 = 1493284$
$(2221)^2 = 4823941$ (here, this number is an exception).

To obtain 5-digit Inverse Square Numbers, we can just add 10000 in 2-digit Inverse Square Numbers.

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Here, 10000 is also a Inverse Square Number.

For example :

$(10000)^2 = 100000000$
$(00001)^2 = 000000001$

$(10010)^2 = 100200100$
$(01001)^2 = 001002001$

$(10011)^2 = 100220121$
$(11001)^2 = 121022001$

$(10012)^2 = 100240144$
$(21001)^2 = 441042001$

$(10013)^2 = 100260169$
$(31001)^2 = 961062001$

$(10020)^2 = 100400400$
$(02001)^2 = 004004001$

$(10022)^2 = 100440484$
$(22001)^2 = 484044001$

$(10030)^2 = 100600900$
$(03001)^2 = 009006001$

You might be thinking that, there are zeroes in hundredth and thousandth places. Can we write any numbers there ?

We can write 1 or 2 in the hundredth and thousandth places but when the last two digits are 13 or 30, then we are suppose to write 0 or 1 only. (at the hundredth and thousandth places).

For example :

$(11111)^2 = 123454321$
$(11111)^2 = 123454321$

$(11013)^2 = 121286169$
$(31011)^2 = 961682121$

$(12120)^2 = 146894400$
$(02121)^2 = 004498641$

$(11130)^2 = 123876900$
$(03111)^2 = 009678321$

When the last two digits are 13 or 30 and 2 is used in hundredth or thousandth place, the Inverse Square Law will fail.

$(11213)^2 = 125731369$
$(31211)^2 = 1002216421$ ✕

$(11230)^2 = 126112900$
$(03211)^2 = 0010310521$ ✕

When a digit rather than 1 is repeated more than twice in a number, then that number is not a Inverse Square Number.

$(12212)^2 = 149132944$
$(21221)^2 = 450330841$ ✕

$(12222)^2 = 149377284$
$(22221)^2 = 493772841$ ✕

1 Digit square law.

Here, we come to know about an interesting feature of the digit 1.
Now, let's observe,

$(1)^2 = 1$

$(11)^2 = 121$

$(111)^2 = 12321$

$(1111)^2 = 1234321$

$(11111)^2 = 123454321$

$(111111)^2 = 12345654321$

$(1111111)^2 = 1234567654321$

$(11111111)^2 = 123456787654321$

$(111111111)^2 = 12345678987654321$

There is only one digit in the number 1, so it's square will be 1. There are 2 digits in the number 11, so it's square is 121, here the starting digits are 1 and 2 because 11 is a two digit number and after 1 and 2 there is reverse counting. Before 2 comes 1 so, 121. Now, there are 3 digits in the number 111, so it's square is 12321, here the starting digits are 1, 2 and 3 because 111 is a three digit number and after 1, 2 and 3 there is reverse counting. Before 3 comes 2, and before 2 comes 1, so square of 111 will be 12321. Now, there are 5 digits in the number 11111, so it's square is 123454321, here the starting digits are 1,2,3,4 and 5 because 11111 is a five digit number and after 1,2,3,4 and 5 there is reverse counting. Before 5 comes 4, before 4 comes 3, before 3 comes 2 and before 2 comes 1, so square of 11111 will be 123454321. This rule is applicable for numbers which contain only 1 as their digit/digits. This rule is applicable only till nine digits, means $(111111111)^2$.

Inverse Square Law :

10,11,12,13,20,21,22,30,31 are 2-digit inverse square numbers. We can add these numbers with 100 to get 3-digit, 1000 to get 4-digit, 10000 to get 5-digit and similarly 10000000 to get 8-digit inverse square numbers. It means that, there are infinite inverse square numbers. We can write 1 or 2 in the places of zeroes to discover new inverse square numbers. If the last two digits of the inverse square number is 13 or 30, then we can write only 1 in place of zeroes. If any digit rather than 1 is repeated more than twice in a number, then that number is not a inverse square number. **Any number made with the help of the digits = 1,2 or 1,2,0 is a inverse square number.**

References :

[1]. Maharashtra State Board, Navneet, std 10th Mathematics Digest (part 1) book – chapter 3 “Arithmetic Progression”.(Year of publication = 2022) – by **Navneet.**
[2]. Numerical Methods. (Year of publication = 2006) – **by Dr P. Kandasamy, Dr K. Thilagavathy and Dr K. Gunavathi**
[3]. **Encyclopaedia of Mathematics.**