



# Common Fixed Points Theorems for Rational Contractions in Generalized Fuzzy Metric Spaces

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**Abstract:** In this paper, we shall introduce rational type contractions and prove common fixed point theorems for the above defined contractions in generalized fuzzy metric spaces. Some results of literature, which are the immediate consequences of our main results are also mentioned.

**Keywords:** Common fixed point, Generalized Fuzzy Metric Space, Rational Contractions.

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## I. INTRODUCTION

Fixed Point Theory is one of the most interesting branch of non-linear analysis. Fixed point theory provides the most important and traditional tools for proving the existence of solutions of many problems in both pure and applied mathematics. In metric fixed point theory, the interplay between contractive conditions and existence and the uniqueness of a fixed point has been very strong. Showing the existence and uniqueness of a fixed point has many applications, in different fields, such as computer science, engineering, etc; see[1]. As it is known to all that the proverbial fixed point theorem of Banach [3] has been widely used in many branches of Mathematics and Physics. There are a large number of generalizations for such a theorem. These generalizations do not just include metric spaces, they include contraction as well. In general, this theorem has been extended in two directions. On the one hand, the usual contractive condition is replaced with weakly contractive conditions. And on the other hand, the action spaces are replaced with different types of metric spaces. The study of fixed points of mappings which satisfy certain contractive conditions has primary applications in the solution of differential and integral equations (see[2], [4-8], [10] and [12-16]).

In 1965, the concept of fuzzy set was given by Zadeh. Then to get fixed point theorems in 1975, Kramosil and Michalek [9] introduced the concept of fuzzy metric space. It's stronger form was given by George and Veeramani [5]. Since then, a number of fixed point theorems are proved in fuzzy metric space. In 2014, Tripathy *et al.*[18] gave the notion of generalized fuzzy metric space. Recently in 2021, Rehman *et al.* [11] gave rational type contractions in fuzzy metric spaces. In this paper, we shall extend the work of Rehman *et al.* [11] in generalized fuzzy metric space. To prove our main results, we need some definitions from literature as follows:

**Definition 1.1.**[17] A binary operation  $*$ :  $[0,1]^2 \rightarrow [0,1]$  is called a continuous  $t$  - norm if  $([0,1],*)$  is an Abelian topological monoid with unit 1 such that  $a_1 * a_2 \leq b_1 * b_2$  whenever  $a_1 \leq a_2, b_1 \leq b_2 \forall a_1, a_2, b_1, b_2 \in [0,1]$

**Definition 1.2.**[18] Let  $X$  be an arbitrary set,  $*$  be a continuous  $t$  - norm and  $M$  - be a fuzzy set in  $X^2 \times [0, \infty)$  such that for all  $x, y \in X$  and for distinct points  $z, w \in X$  each of them different from  $x$  and  $y$  &  $t_1, t_2, t_3, t > 0$ , one has

1.  $M(x, y, 0) = 0$ ;
2.  $M(x, y, t) = 1$  iff  $x = y$ ;
3.  $M(x, y, t) = M(y, x, t)$ ;
4.  $M(x, y, t_1) * M(y, z, t_2) * M(z, w, t_3) \leq M(x, w, t_1 + t_2 + t_3)$ ;

5.  $M(x, y, *) : [0, \infty) \rightarrow [0, 1]$  is left continuous.

Then we say that  $(X, M, *)$  is a generalized fuzzy metric space. Throughout the paper, we shall denote generalized fuzzy metric space by g.f.m.s.

**Definition 1.3.**[18] Let  $(X, M, *)$  be g. f. m. s., then

(i) A sequence  $\{x_n\}$  in  $X$  is said to be converge to  $x$  if for each  $\varepsilon \in (0, 1)$ ,  $t > 0 \exists n_0 \in \mathbb{N}$  such that  $M(x_n, x_m, t) > 1 - \varepsilon \forall n, m \geq n_0$ .

(ii) A sequence  $\{x_n\}$  in  $X$  is said to be a Cauchy if for each  $\varepsilon \in (0, 1) \& t > 0$ ,  $\exists n_0 \in \mathbb{N}$  such that  $M(x_n, x_m, t) > 1 - \varepsilon, \forall n, m \geq n_0$ .

(iii) A g. f. m. s. in which every Cauchy sequence is convergent is said to be complete .

**Definition 1.4.** [11] Let  $(X, M, *)$  be a g. f. m. s. Then fuzzy metric is  $M$ - triangular if

$$\frac{1}{M(x,y,t)} - 1 \leq \left(\frac{1}{M(x,z,t)} - 1\right) + \left(\frac{1}{M(z,y,t)} - 1\right) \forall x, y, z \in X .$$

## II. Main Results

In this section, we shall prove common fixed point theorems.

**Theorem 2.1.** Let  $(X, M, *)$  be a g. f. m. s. where  $M$  is triangular. Let  $S, T : X \rightarrow X$  be two self maps on  $X$  such that

$$\frac{1}{M(Sx, Ty, t)} - 1 \leq \psi \left( a \left( \frac{1}{M(x, y, t)} - 1 \right) + b \left( \frac{M(x, y, t)}{M(x, Sx, t) * M(y, Ty, 2t)} - 1 \right) \right), \tag{2.1}$$

where  $a, b \in (0, 1)$  and  $a + b < 1$  and  $\psi \in \Psi$ .

Then  $S, T$  have a unique common fixed point.

**Proof :** Let  $x_0 \in X$  and  $x_{2n+1} = Sx_{2n}, x_{2n+2} = Tx_{2n+1} \forall n \in \mathbb{N}$  Then from equation (2.1) with  $t > 0$  we have

$$\begin{aligned} \left(\frac{1}{M(x_{2n+1}, x_{2n+2}, t)} - 1\right) &\leq \left(\frac{1}{M(Sx_{2n}, Tx_{2n+1}, t)} - 1\right) \\ &\leq \psi \left[ a \left( \frac{1}{M(x_{2n}, x_{2n+1}, t)} - 1 \right) + b \left( \frac{M(x_{2n}, x_{2n+1}, t)}{M(x_{2n}, Tx_{2n}, t) * M(x_{2n+1}, Tx_{2n}, 2t)} - 1 \right) \right] \\ &= \psi \left[ a \left( \frac{1}{M(x_{2n}, x_{2n+1}, t)} - 1 \right) + b \left( \frac{M(x_{2n}, x_{2n+1}, t)}{M(x_{2n}, x_{2n+1}, t) * M(x_{2n+1}, x_{2n+1}, 2t)} - 1 \right) \right] \\ &\leq \psi \left[ a \left( \frac{1}{M(x_{2n}, x_{2n+1}, t)} - 1 \right) \right] \end{aligned}$$

Continuing this process, we get

$$\frac{1}{M(x_{2n+1}, x_{2n+2}, t)} - 1 \leq \psi^n \left( \frac{1}{M(x_0, x_1, t)} - 1 \right) \tag{2.2}$$

( $\because a < 1$ )

Taking limit  $n \rightarrow \infty$  in equation (2.2), we get

$$\lim_{n \rightarrow \infty} M(x_{2n+1}, x_{2n+2}, t) = 1 \text{ for all } t > 0. \tag{2.3}$$

Now we will prove that  $\{x_n\}$  is a Cauchy sequence.

Let  $n \in \mathbb{N}$  and there is a  $p \in \mathbb{N}$  such that

$$M(x_n, x_{n+1}, t) \geq M(x_n, x_{n+1}, \frac{t}{p}) * M(x_{n+1}, x_{n+2}, \frac{t}{p}) * \dots * M(x_{n+p-1}, x_{n+p}, \frac{t}{p})$$

Taking limit as  $n \rightarrow \infty$ , we get

$$\begin{aligned} M(x_n, x_{n+p}, t) &\geq 1 * 1 * \dots * 1 = 1 \\ \Rightarrow \lim_{n \rightarrow \infty} M(x_n, x_{n+p}, t) &= 1. \end{aligned} \tag{2.4}$$

Hence  $\{x_n\}$  is a Cauchy sequence.

Since  $(X, M, *)$  is complete space. So,  $\exists v \in X$  such that  $x_n \rightarrow v$  as  $n \rightarrow \infty$  that is

$$\lim_{n \rightarrow \infty} M(x_n, v, t) = 1 \forall t > 0. \tag{2.5}$$

Since  $M$  is triangular, then

$$\begin{aligned} \left(\frac{1}{M(v, Tv, t)} - 1\right) &\leq \left(\frac{1}{M(v, v_{n+1}, t)} - 1\right) + \left(\frac{1}{M(v_{n+1}, Tv, t)} - 1\right) \\ &= \left(\frac{1}{M(v, v_{n+1}, t)} - 1\right) + \left(\frac{1}{M(v_n, Tv, t)} - 1\right) \end{aligned}$$

Using equation (2.1) we obtain that

$$\leq \left(\frac{1}{M(v, v_{n+1}, t)} - 1\right) + \psi \left[ a \left( \frac{1}{M(v_n, v, t)} - 1 \right) + b \left( \frac{M(v_n, v, t)}{M(v_n, Sv_n, t) * M(v, Tv_n, 2t)} - 1 \right) \right]$$

Taking limit as  $n \rightarrow \infty$ , we get

$$\begin{aligned} M(v, Tv, t) &= 1 \text{ for all } t > 0. \\ \Rightarrow Tv &= v. \end{aligned} \tag{2.6}$$

Again, using that  $M$  is triangular  $\left(\frac{1}{M(v, Sv, t)} - 1\right) \leq \left(\frac{1}{M(v, Tv_n, t)} - 1\right) + \left(\frac{1}{M(Sv, Tv_n, t)} - 1\right)$ ,

$$\leq \left(\frac{1}{M(v, Tv, t)} - 1\right) + \psi\left[a\left(\frac{1}{M(v, v, t)} - 1\right) + b\left(\frac{M(v, v, t)}{M(v, Sv, t) * M(Tv, v, 2t)} - 1\right)\right]$$

Again taking limit  $n \rightarrow \infty$

we obtain that

$$\lim_{n \rightarrow \infty} M(v, S_v, t) = 1 \text{ for all } t > 0.$$

$$\Rightarrow Sv = v. \text{ (2.7)}$$

From equations(2.6) and (2.7), we obtain

$$Tv = Sv = v.$$

Uniqueness :

Let's assume that if possible  $u \& v$  are two different common fixed points.

Then by equation (2.1)

$$\left(\frac{1}{M(Su, Tv, t)} - 1\right) \leq \psi\left[a\left(\frac{1}{M(u, v, t)} - 1\right) + b\left(\frac{M(u, v, t)}{M(u, v, t) * M(u, v, t)} - 1\right)\right]$$

$$< \left(\frac{1}{M(u, v, t)} - 1\right)$$

a contradiction.

So  $S$  and  $T$  have a unique common fixed point.

**Corollary 2.2.** Let  $(X, M, *)$  be g.f.m.s, where  $M$  is triangular. Assume  $T$  is a self map on  $X$  such that

$$\frac{1}{M(Tx, Ty, t)} - 1 \leq \psi\left[a\left(\frac{1}{M(x, y, t)} - 1\right) + b\left(\frac{M(x, y, t)}{M(x, Tx, t) * M(y, Ty, 2t)} - 1\right)\right],$$

$$\forall x, y \in X \quad a + b < 1, \quad a, b \geq 0.$$

Then  $T$  has a unique fixed point

**Proof:** Taking  $S = T$  in Theorem 1 ,we get the proof .

**Corollary 2.3.** Let  $(X, M, *)$  be a g.f.m.s.  $T$  be a self map on  $X$  such that

$$\left(\frac{1}{M(Tx, Ty, t)} - 1\right) \leq a\left(\frac{1}{M(x, y, t)} - 1\right) + b\left(\frac{M(x, y, t)}{M(x, Tx, t) * (M(y, Ty, 2t))} - 1\right)$$

$$\forall x, y \in X \quad a + b < 1, \quad a, b \geq 0.$$

Then  $T$  has a unique fixed point.

**Proof:** Taking  $\psi(t) = t$ , in corollary 2.2, we get the proof .

**Theorem 2.4.** Let  $(X, M, *)$  be a g. f. m. s. where  $M$  is triangular. Assume that  $S, T$  are two self-

Maps on  $X$  such that

$$\left(\frac{1}{M(Sx, Ty, t)} - 1\right) \leq \psi\left[a\left(\frac{1}{M(x, y, t)} - 1\right) + b\left(\frac{M(x, y, t) * M(y, Ty, t)}{M(x, Sx, t)} - 1\right) + c\left(\frac{M(x, Sx, t)}{M(x, T_y^3, t)} - 1 + \frac{M(y, Ty, t)}{M(x, T_y^2, 3, t)} - 1\right) +\right.$$

$$\left. d\left(\left(\frac{1}{M(x, Sx, t)} - 1\right) + \frac{1}{M(y, Ty, t)} - 1\right)\right] \text{ (2.8)}$$

$$\forall x, y \in X \quad a, b, c, d \geq 0 \text{ and } a + b + 2c + 2d \leq 1.$$

$S, T$  have a unique common fixed point

**Proof :** Let  $x_0 \in X$  such that  $X_{2n+1} = SX_{2n}$  and  $X_{2n+2} = TX_{2n+1}$

The equation (2.8) becomes

$$\frac{1}{M(x_{2n+1}, x_{2n+2}, t)} - 1 = \frac{1}{M(x_{2n+1}, x_{2n+2}, t)} - 1 = \frac{1}{M(Sx_{2n}, Tx_{2n+1}, t)} - 1$$

$$\begin{aligned}
 &\leq \psi \left[ a \left( \frac{1}{M(x_{2n}, x_{2n+1}, t)} - 1 \right) \right. \\
 &+ b \left( \frac{M(x_{2n}, x_{2n+1}, t) * M(x_{2n+1}, T_{2n+1}, t)}{M(x_{2n}, Sx_{2n}, t) * M(x_{2n}, T_{2n}^3, 3t)} - 1 \right) + c \left( \frac{M(x_{2n}, Sx_{2n}, t)}{M(x_{2n}, T_x^3, 3t)} - 1 \right) \\
 &+ \left. \frac{M(x_{2n}, x_{2n+1}, t)}{M(x_{2n}, T_{x_{2n+1}}^2, 3t)} - 1 \right) + d \left( \frac{1}{M(x_{2n}, Sx_{2n}, t)} - 1 + \frac{1}{M(x_{2n+1}, T_{x_{2n+1}}, t)} - 1 \right) \Big] \\
 &= \psi \left[ a \left( \frac{1}{M(x_{2n}, x_{2n+1}, t)} - 1 \right) + b \left( \frac{M(x_{2n}, x_{2n+1}, t) * M(x_{2n+1}, x_{2n+2}, t)}{M(x_{2n}, x_{2n+1}, t) * M(x_{2n}, x_{2n+3}, 3t)} - 1 \right) \right. \\
 &+ c \left[ \frac{M(x_{2n}, x_{2n+1}, t)}{M(x_{2n}, x_{2n+3}, 3t)} - 1 + \frac{M(x_{2n+1}, x_{2n+2}, t)}{M(x_{2n}, x_{2n+3}, 3t)} - 1 \right] \\
 &+ \left. d \left[ \left( \frac{1}{M(x_{2n}, x_{2n+1}, t)} - 1 \right) + \left( \frac{1}{M(x_{2n+1}, x_{2n+2}, t)} - 1 \right) \right] \right]
 \end{aligned}
 \tag{2.9}$$

Now using condition (iv) of Definition (1.2), we have

$$M(x_n, x_{n+3}t) \geq M(x_n, x_{n+1}t) * M(x_{n+1}, x_{n+2}t) * M(x_{n+2}, x_{n+3}t)$$

Using this, equation (9) becomes for  $t > 0$

$$\frac{1}{M(x_{2n+1}, x_{2n+2}, t)} - 1 \leq v \left( \frac{1}{M(x_{2n}, x_{2n+1}, t)} - 1 \right) \tag{2.10}$$

Where  $v = \frac{a+b+c+d}{1-c-d} < 1$

Similarly, for  $t > 0$ , we have

$$\frac{1}{M(x_{2n}, x_{2n+1}, t)} - 1 \leq v \left( \frac{1}{M(x_{2n-1}, x_{2n}, t)} - 1 \right) \tag{2.11}$$

From equation (10) and (11), we have

$$\frac{1}{M(x_{2n+1}, x_{2n+2}, t)} \leq \beta^n \left( \frac{1}{M(x_0, x_1, t)} - 1 \right) \rightarrow 0 \text{ as } n \rightarrow \infty$$

$$\Rightarrow \lim_{n \rightarrow \infty} M(x_{2n+1}, x_{2n+2}, t) = 1 \tag{2.12}$$

For  $t > 0$

Now we have to show that  $(x_n)$  is a Cauchy sequence

So,  $M(x_n, x_{n+p}, t) \geq M(x_n, x_{n+1}, t/p) * \dots * M(x_{n+p-1}, x_{n+p}, t/p) \rightarrow |x| * |x| \dots = 1$  as  $n \rightarrow \infty$ . So,  $(x_n)$  is a Cauchy sequence.

Since  $(X, M, *)$  is complete,  $\exists v \in V$  as  $n \rightarrow \infty$  that is

$$\lim_{n \rightarrow \infty} M(x_n, v, t) = 1 \quad t > 0 \tag{2.13}$$

Since  $M$  is triangular

$$\frac{1}{M(v, S_v, t)} - 1 \leq \left( \frac{1}{M(v, T_{x_n}, t)} - 1 \right) + \left( \frac{1}{M(S_v, T_{x_n}, t)} - 1 \right) \text{ for } t > 0. \tag{2.14}$$

Now by using contradiction of eq<sup>n</sup> (2.8)

$$\begin{aligned}
 \left( \frac{1}{M(S_v, T_{x_n}, t)} - 1 \right) &\leq \Psi \left[ a \left( \frac{1}{M(v, x_n, t)} - 1 \right) + b \left( \frac{M(v, x_n, t) * M(x_n, x_{n+1}, t)}{M(v, S_v, t) * M(v, x_{n+3}, 3t)} - 1 \right) \right. \\
 &+ c \left( \frac{M(v, S_v, t)}{M(v, x_{n+3}, 3t)} - 1 \right) + \frac{M(x_n, x_{n+1}, t)}{M(v, x_{n+3}, 3t)} - 1 \\
 &\left. + d \left[ 1Mv, Sv, t-1 + 1M(x_n, x_{n+1}, t) - 1 \right] \right] \tag{2.15}
 \end{aligned}$$

From condition (14) of def (2)

$$M(x_n, S_v, 3t) \geq M(x_n, x_{n+1}, t) * M(x_{n+1}, v, t) * M(v, S_v, t)$$

Using this inequality in equation (2.15) as taking  $n \rightarrow \infty$  we get

$$\lim_{n \rightarrow \infty} \frac{1}{M(S_v, x_{n+1}, t)} - 1 \leq (c + d) \left( \frac{1}{M(v, S_v, t)} - 1 \right)$$

Since  $c + d < 1$  this implies that  $\lim_{n \rightarrow \infty} \frac{1}{M(S_v, v, t)} = 1 \Rightarrow Sv = v$  (2.16)

In the similar way one can prove easily that  $Tv = v$  (2.17)

From equation (2.16) and (2.17) we have

$$Sv = Tv = v$$

Uniqueness:  $\rightarrow$  To prove uniqueness let's assume that  $u$  &  $v$  are two distinct common fixed points for maps  $S$  &  $T$ .

$$\begin{aligned} \left(\frac{1}{M(u, v, t)} - 1\right) &= \left(\frac{1}{M(S_u, T_v, t)} - 1\right) \\ &\leq \Psi \left[ a \left(\frac{1}{M(u, v, t)} - 1\right) + b \left(\frac{M(u, v, t) * M(v, T_v, t)}{M(u, S_u, t) * M(u, T_v^3, 3t)} - 1\right) \right. \\ &\quad \left. + c \left(\left(\frac{M(u, S_u, t)}{M(u, T_v^3, 3t)} - 1 + \frac{M(v, T_v, t)}{M(u, T_v^3, 3t)}\right) - 1\right) + d \left(\left(\frac{1}{M(u, S_u, t)} - 1\right) + \frac{1}{M(v, T_v, t)} - 1\right) \right] \end{aligned}$$

Now using

$M(u, v, 3t) \geq M(u, S_u, t) * M(S_u, T_v, t) * M(T_v, v, t)$  in the above we get

$$M(u, v, t) = 1 \Rightarrow u = v.$$

**Corollary 2.5.** Let  $(X, M, *)$  be a g. f. m. s. where  $M$  is triangular. Assume that  $S, T$  are two self-maps on  $X$  such that  $\frac{1}{M(Tx, Ty, t)} - 1 \leq \psi \left( a \left(\frac{1}{M(x, y, t)} - 1\right) + b \left(\frac{M(x, y, t) * M(y, Ty, t)}{M(x, Tx, t) * M(x, Ty, 2t)} - 1\right) + d \left(\frac{1}{M(x, Tx, t)} - 1 + \frac{1}{M(y, Ty, t)} - 1\right) \right)$ ,

For all  $x, y \in X$  and  $t > 0$ ,  $a, b, d \geq 0$  with  $a + b + 2d < 1$ . Then  $T$  has a unique fixed point.

**Corollary 2.6.** Let  $(X, M, *)$  be a g. f. m. s. where  $M$  is triangular. Assume that  $S, T$  are two self-maps on  $X$  such that

$$\begin{aligned} \frac{1}{M(Tx, Ty, t)} - 1 &\leq \psi \left( a \left(\frac{1}{M(x, y, t)} - 1\right) + c \left(\frac{M(x, Tx, t)}{M(x, Ty, 2t)} - 1 + \frac{M(y, Ty, t)}{M(x, Ty, 2t)} - 1\right) + d \left(\frac{1}{M(x, Tx, t)} - 1 \right. \right. \\ &\quad \left. \left. + \frac{1}{M(y, Ty, t)} - 1\right) \right) \end{aligned}$$

For all  $x, y \in X$  and  $t > 0$ ,  $a, c, d \geq 0$  with  $a + 2c + 2d < 1$ . Then  $T$  has a unique fixed point.

**Corollary 2.7.** Let  $(X, M, *)$  be a g. f. m. s. where  $M$  is triangular. Assume that  $S, T$  are two self-maps on  $X$  such that  $\frac{1}{M(Tx, Ty, t)} - 1 \leq a \left(\frac{1}{M(x, y, t)} - 1\right) + b \left(\frac{M(x, y, t) * M(y, Ty, t)}{M(x, Tx, t) * M(x, Ty, 2t)} - 1\right) + d \left(\frac{1}{M(x, Tx, t)} - 1 + \frac{1}{M(y, Ty, t)} - 1\right)$ ,

For all  $x, y \in X$  and  $t > 0$ ,  $a, b, d \geq 0$  with  $a + b + 2d < 1$ . Then  $T$  has a unique fixed point.

**Corollary 2.8.** Let  $(X, M, *)$  be a g. f. m. s. where  $M$  is triangular. Assume that  $S, T$  are two self-maps on  $X$  such that

$$\begin{aligned} \frac{1}{M(Tx, Ty, t)} - 1 &\leq a \left(\frac{1}{M(x, y, t)} - 1\right) + c \left(\frac{M(x, Tx, t)}{M(x, Ty, 2t)} - 1 + \frac{M(y, Ty, t)}{M(x, Ty, 2t)} - 1\right) + d \left(\frac{1}{M(x, Tx, t)} - 1 \right. \\ &\quad \left. + \frac{1}{M(y, Ty, t)} - 1\right) \end{aligned}$$

For all  $x, y \in X$  and  $t > 0$ ,  $a, c, d \geq 0$  with  $a + 2c + 2d < 1$ .

Then  $T$  has a unique fixed point.

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