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Review Paper

Bianchi Type-I Inflationary Cosmological Model for Stiff Perfect Fluid Distribution in General Relativity

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ABSTRACT: *We have investigated Bianchi type-I inflationary cosmological model for stiff perfect fluid distribution in general relativity. To obtain the deterministic solution of the model, we assume that the expansion (θ) is proportional to the shear (σ) which leads to $A=(BC)^n$ and potential $V(\phi)$ is constant. The behavior of the model from physical and geometrical aspects is also discussed.*

KEYWORDS: *Stiff fluid, Bianchi Type-I, Inflationary Universe, Perfect fluid, Cosmology.*

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I. INTRODUCTION

The inflationary scenario explains several mysteries of modern cosmology like homogeneity, isotropy, horizon problem and the flatness of the observed universe. Inflation means extremely rapid expansion of early universe by a factor of 10^{78} in volume driven by negative pressure vacuum energy density. Guth's [12] introduced the concept of inflation and suggested that rapid expansion is due to false vacuum energy and after inflation, the universe is filled with bubbles. Various authors viz. Abbott and Wise [1], Albrecht and Steinhardt [2], Burd and Barow [11], La and Steinhardt [13], Linde [14], Mijic et al.[15] have investigated inflationary cosmological models using homogeneous and isotropic space-time in different versions.

Stein-Schabes [24] has pointed out that inflationary scenario is possible when potential has flat region and Higgs field evolves slowly but the universe expands in an exponential way due to vacuum field energy. Sharma and Poonia [19] investigated Bianchi type-I inflationary cosmological model with bulk viscosity in general relativity. Bali and Singh [9] investigated LRS Bianchi type-I stiff fluid inflationary universe with variable bulk viscosity. Inflationary universe in Bianchi type-II space-time with Higgs field and flat potential in general relativity is studied by Singh [23]. Rothman and Ellis [18] explained that we could have a solution of the isotropy problem if we work with anisotropic metric and these can be inflated in very general circumstances. Poonia and Sharma [17] investigated inflationary scenario in Bianchi type-II space with bulk viscosity in general relativity. Bali and Poonia [6], Bali [3,4], Bali and Jain [5] have discussed inflationary cosmological models in general relativity using Bianchi type-I space time in which the potential is considered as constant. Inflationary scenario in Bianchi type-V space time with bulk viscosity and dark energy in radiation dominated phase is studied by Bali and Goyal [7]. Sharma and Poonia [20] investigated cosmic inflation in Bianchi type-IX space with bulk viscosity. Some more cosmological models are also investigated by Bali and Saraf [8], Naidu et al. [16], Tripathi et al. [25], Tyagi and Singh [27], Tyagi et al. [26], Bali and Tyagi [10], Singh and Tiwari [21], Singh et al. [22] and Verma and Shri Ram [28] to name a few. Bianchi type-I space-time play a significant role in description of early stage of evolution of universe.

Impelled by the above mentioned studies, we have investigated the Bianchi Type-I inflationary cosmological model for stiff perfect fluid distribution in the presence of massless scalar field with a flat region in which potential is constant. For the complete solution of the field equation, we assume that expansion (θ) is proportional to the shear (σ) and $V(\phi)$ is constant. The physical and geometrical aspects of the model are also discussed.

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II. THE METRIC AND FIELD EQUATION

We consider Bianchi Type-I space-time in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2 \qquad (1)$$

in which $A(t)$, $B(t)$ and $C(t)$ are cosmic scale functions.

We assume the co-ordinate to be co-moving so that

$$v^1 = 0 = v^2 = v^3,\ v^4 = 1$$

In case of gravity minimally coupled to a scalar field V(ϕ), as given by Stein-Schabes [24], we have

$$S = \int \sqrt{-g}\left[R - \frac{1}{2} g^{ij}\partial_i \phi \partial_j \phi - V(\phi)\right] \mathrm{d}^4 x \qquad (2)$$

The Einstein's field equation (in gravitational units $8\pi G = c = 1$), in the case of massless scalar field ϕ with potential $V(\phi)$ are given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \qquad (3)$$

with

$$T_{ij} = (\rho + p) v_i v_j + p g_{ij} + \partial_i \phi \partial_j \phi - \left[\frac{1}{2}\partial_r \phi \partial^r \phi + V(\phi)\right] g_{ij} \qquad (4)$$

Here ρ is the energy density, p the pressure, ϕ is Higgs field, V the potential.

The conservation relation leads to

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,\partial^\mu \phi\right) = -\frac{dV}{d\phi} \qquad (5)$$

The Einstein's field equation (3) for the line-element (1) leads to non-linear differential equations as follows

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = -p - \frac{1}{2}\phi_4^2 + V(\phi) \qquad (6)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = -p - \frac{1}{2}\phi_4^2 + V(\phi) \qquad (7)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = -p - \frac{1}{2}\phi_4^2 + V(\phi) \qquad (8)$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} = \rho + \frac{1}{2}\phi_4^2 + V(\phi) \qquad (9)$$

The equation (5) for scalar field (ϕ) leads to

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$$\phi_{44} + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) \phi_4 = -\frac{dV}{d\phi} \qquad (10)$$

where suffix '4' indicates derivative with respect to time t.

III. SOLUTION OF FIELD EQUATIONS

The field equations (6) to (9) represent a system of four independent equations with unknown parameters A, B, C, ρ, p, ϕ. To obtain the deterministic solution, we assume the following conditions:

(i) V(ϕ) is constant

i.e. V(ϕ) = K (11)

(ii) Shear (σ) is proportional to expansion (θ) which leads to

$$A = (BC)^n \qquad (12)$$

From equations (10), (11) and (12) we have

$$\phi_4 = \frac{m}{A^{\frac{n+1}{n}}} \qquad (13)$$

where m is constant of integration.

The scale factor R^3 for line-element (1) is given by

$$R^3 = A^{\frac{n+1}{n}} \qquad (14)$$

Form equations (6) and (9), we get

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C} \right) = \rho - p + 2K \qquad (15)$$

We assume that the model is filled with stiff fluid distribution which leads to

$$\rho = p \qquad (16)$$

Equations (12), (15) and (16) together lead to

$$\frac{(BC)_{44}}{BC} + n\left[\frac{(BC)_4}{BC} \right]^2 = 2K \qquad (17)$$

Now let us consider $BC = \mu$ and $\frac{B}{C} = \nu$, in equation (17) which leads to

$$\mu_{44} + n\frac{\mu_4^2}{\mu} = 2K\mu \qquad (18)$$

From equation (18), we get

$$\frac{\mu^n d\mu}{\sqrt{M}\sqrt{\left(\mu^{n+1}\right)^2+\frac{E}{M}}}=dt \tag{19}$$

where $M=\frac{2K}{n+1}$ and E is constant of integration.

On integrating (19), we get

$$\frac{1}{\sqrt{M}}\int\frac{\mu^n d\mu}{\sqrt{\left(\mu^{n+1}\right)^2+\frac{E}{M}}}=\int dt+F=t+F \tag{20}$$

Equation (20) leads to

$$\mu^{n+1}=P\sinh lT \tag{21}$$

Where F is constant of integration and $l=(n+1)\sqrt{M}$, $T=t+F$, $P=\sqrt{\frac{E}{M}}$.

From equations (7) and (8), we get

$$\frac{B_{44}}{B}-\frac{C_{44}}{C}+\frac{A_4}{A}\left(\frac{B_4}{B}-\frac{C_4}{C}\right)=0 \tag{22}$$

Using equation (12) in (22), we have

$$C^2\left(\frac{B}{C}\right)_4=\frac{J}{\mu^n} \tag{23}$$

where J is constant of integration.

From equations (21) and (23), we get

$$\frac{v_4}{v}=\frac{J}{P}\left(\cos echlT\right) \tag{24}$$

After integration equation (24) leads to

$$v=N\tanh^{\frac{J}{lP}}\left(\frac{lT}{2}\right) \tag{25}$$

where N is constant of integration

Therefore,

$$A^2=P^{\frac{2n}{n+1}}\left(\sinh^{\frac{2n}{n+1}}lT\right) \tag{26}$$

$$B^2 = NP^{\frac{1}{n+1}}\left(\sinh^{\frac{1}{n+1}} lT\right)\left(\tanh^{\frac{J}{lP}}\frac{lT}{2}\right) \tag{27}$$

and

$$C^2 = \frac{P^{\frac{1}{n+1}}}{N}\left(\sinh^{\frac{1}{n+1}} lT\right)\left(\coth^{\frac{J}{lP}}\frac{lT}{2}\right) \tag{28}$$

Hence, by suitable transformation of coordinates, metric (1) reduces to

$$ds^2 = -dT^2 + P^{\frac{2n}{n+1}}\left(\sinh^{\frac{2n}{n+1}} lT\right)dX^2 + P^{\frac{1}{n+1}}\left(\sinh^{\frac{1}{n+1}} lT\right)\left[\tanh^{\frac{J}{lP}}\frac{lT}{2}dY^2 + \coth^{\frac{J}{lP}}\frac{lT}{2}dZ^2\right] \tag{29}$$

where $x = X$, $\sqrt{N}y = Y$, $\frac{z}{\sqrt{N}} = Z$.

IV. PHYSICAL AND GEOMETRICAL ASPECTS

For the model (29), the rate of Higgs field

$$\phi = \log\tanh^{\frac{m}{(n+1)\sqrt{E}}}\left(\frac{\sqrt{2K(n+1)}}{2}T\right) + W \tag{30}$$

where W is constant of integration.

For the model (29), pressure (p), density (ρ), the spatial volume (R^3), the expansion (θ), shear (σ), decelerating parameter (q) and Hubble parameter (H) are given by

$$\begin{aligned}\rho = p = &\left\{\frac{K(4nE + E - 2)}{E(n+1)}\right\}\coth^2\sqrt{2K(n+1)}T \\ &- \frac{J^2K}{8E(n+1)}\left\{\coth^2\sqrt{\frac{K(n+1)}{2}}T + \tanh^2\sqrt{\frac{K(n+1)}{2}}T\right\} + \frac{K(J^2+4)}{4E(n+1)} - K\end{aligned} \tag{31}$$

$$R^3 = \sqrt{\frac{E(n+1)}{2K}}\sinh\sqrt{2K(n+1)}T \tag{32}$$

$$\theta = (n+1)\sqrt{E}\coth\sqrt{2K(n+1)}T \tag{33}$$

$$\sigma = \frac{1}{\sqrt{3}}\sqrt{\left[\left\{E\left(n^2 - n + \frac{1}{4}\right) + \frac{3J^2E(n+1)}{8K}\right\}\coth^2\sqrt{2K(n+1)}T - \frac{3J^2E(n+1)}{8K}\right]} \tag{34}$$

$$q = 2 - 3\tanh^2\sqrt{2K(n+1)}T \tag{35}$$

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$$H=\frac{(n+1)\sqrt{E}}{3}\coth\sqrt{2K(n+1)}T \tag{36}$$

From equations (33) and (34), we get

$$\frac{\sigma}{\theta}=\frac{1}{\sqrt{3}}\sqrt{\left[\frac{\left(n^2-n+\frac{1}{4}\right)}{(n+1)^2}+\frac{3J^2K}{2E^2(n+1)^3}\left(\sec h^2\sqrt{2K(n+1)}T\right)\right]} \tag{37}$$

V. CONCLUSION

The model (29) starts expanding with Big-bang at $T=0$. The expansion θ decreases as time increases for $n>0$ and it approaches to constant as $T\to\infty$.

The Spatial Volume (R^3) increases as time increases for $n>-1$. It represents inflationary scenario of universe containing massless scalar field with flat potential.

Since $\underset{T\to\infty}{Lim}\frac{\sigma}{\theta}\neq 0$, hence the model represents anisotropic space-time in general.

The Hubble parameter (H) decreases as time increases for $n>-1$. The Hubble parameter is initially large but leads to finite quantity for large values of time. The energy density and pressure are initially large and decreases later on.

The deceleration parameter (q) is constant initially i.e. at $T=0$ and decreases as time T increases. When $T\to\infty$, $q\to-1$ which represent accelerating model.

The rate of Higgs field (ϕ) is initially too large but decreases with time and vanishes for large value of time for $n>-1$. The rate of Higgs field evolves slowly but the universe expands. The model has cigar type singularity at $T=0$.

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