



Bitopological separation axioms via $S^{**}G$ -open set

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ABSTRACT

In this paper, we define a ii -open set in a bitopological space as follows: Let (X, T_1, T_2) be a bitopological space, a subset A of X is called a $(TT_2$ - ii - open set) if there exists $U, V \subseteq X$ and $U \in T_1, V \in T_2$ such that: $A = \text{int}^2(V) \cap \text{int}^1(U)$. We study some characteristics and properties of this class. We also explain the relation between ii -open sets and open sets, i -open sets and a -open sets in bitopological space. Next, we define ii -continuous mappings on bitopological spaces with some properties.

Keywords: a - open set, i - open set, ii - open set, bitopological space. , Pairwise $S^{**}G$ - Separation axioms

I. Introduction

Let A be a subset of the topological space (X, τ) , then the union of all $S^{**}g$ open sets contained in the subset A of X is called the $S^{**}g$ interior of A and denoted by $S^{**}g \text{ int}(A)$. The intersection of all $S^{**}g$ closed sets X containing a subset A of X is called the $S^{**}g$ closure of A and is denoted by $S^{**}g \text{ cl}(A)$. In this chapter we will consider pairwise $S^{**}g$ - R_i spaces [$i = 0, 1$], pairwise $S^{**}g$ - T_i spaces [$i = 0, 1, 2, 3, 4, 5$], pairwise $S^{**}g$ - regular spaces, even $S^{**}g$ - Urysohn spaces, even $S^{**}g$ - normal spaces, even $S^{**}g$ - completely normal spaces in bitopological spaces.

Pairwise $S^{**}G$ - Separation axioms

In this section, the concept of pairwise $S^{**}g$ -separation axioms is introduced and its basic properties in bitopological spaces are discussed.

Definition Let (X, τ_1, τ_2) be a bitopological space and let $A \subseteq X$ then the intersection of all τ_i - $S^{**}g$ - closed sets of X containing a subset A of X is called τ_i - $S^{**}g$ closure of A and is denoted by τ_i - $S^{**}g \text{ cl}(A)$.

Definition Let (X, τ_1, τ_2) be a bitopological space and let $A \subseteq X$ then the union of all τ_i - $S^{**}g$ open sets contained in a subset A of X is called τ_i - $S^{**}g$ interior of A and is denoted by τ_i - $S^{**}g \text{ int}(A)$.

Definition A bitopological space (X, τ_1, τ_2) is pairwise $S^{**}g$ - R_0 if for each

i - $S^{**}g$ - open set $G, x \in G$ implies τ_j - $S^{**}g$ - $\text{cl}(\{x\}) \subseteq G$, where $i, j = 1, 2$ and $i \neq j$.

Example Let $X = \{a, b, c\}, \tau_1 = \{ \emptyset, X, \{a, c\} \}$ and $\tau_2 = \{ \emptyset, X, \{b, c\} \}$.

Clearly the space (X, τ_1, τ_2) is pairwise $S^{**}g$ - R_0 .

Theorem In a bitopological space (X, τ_1, τ_2) the following statements are

equivalent :-

$(X, 1, 2)$ is pairwise $s^{**}g$ - R_0 .

For any i - $s^{**}g$ - closed set F and a point $x \in F$, there exists a $U \in s^{**}gO(X, j)$ such that $x \in U$ and $F \cap U = \emptyset$ for $i, j = 1, 2$ and $i \neq j$.

For any i - $s^{**}g$ - closed set F and a point $x \in F$, j - $s^{**}g$ - $cl(\{x\}) \cap F = \emptyset$, for $i, j = 1, 2$ and $i \neq j$.

Proof. i) ii) : Let F be a i - $s^{**}g$ - closed set F and a point $x \in F$. Then by

i), j - $s^{**}g$ - $cl(\{x\}) \cap F = \emptyset$, where $i, j = 1, 2$ and $i \neq j$. Let $U \in s^{**}gO(X, j)$ such that $x \in U$ and $F \cap U = \emptyset$.

iii) : Let F be a i - $s^{**}g$ - closed set F and a point $x \in F$. Suppose the given conditions hold. Since $U \in s^{**}gO(X, j)$, $U \cap j$ - $s^{**}g$ - $cl(\{x\}) = \emptyset$. Then $F \cap j$ - $s^{**}g$ - $cl(\{x\}) = \emptyset$, where $i, j = 1, 2$ and $i \neq j$.

i) : Let $G \in s^{**}gO(X, i)$ and $x \in G$. Now $X - G$ is i - $s^{**}g$ closed and $x \in X - G$. By iii), j - $s^{**}g$ - $cl(\{x\}) \cap (X - G) = \emptyset$ and hence j - $s^{**}g$ - $cl(\{x\}) \cap G = \emptyset$ for $i, j = 1, 2$ and $i \neq j$. Therefore, the space $(X, 1, 2)$ is pairwise $s^{**}g$ - R_0 .

Definition A space $(X, 1, 2)$ is said to be pairwise $s^{**}g$ - R_1 if for each $x, y \in X$,

i - $s^{**}g$ - $cl(\{x\}) \neq j$ - $s^{**}g$ - $cl(\{y\})$, there exist disjoint sets $U \in s^{**}gO(X, j)$ and $V \in s^{**}gO(X, i)$ such that i - $s^{**}g$ - $cl(\{x\}) \cap U = \emptyset$ and j - $s^{**}g$ - $cl(\{y\}) \cap V = \emptyset$ where $i, j = 1, 2$ and $i \neq j$.

Example Let $X = \{a, b, c\}$, $\tau_1 = \{ \emptyset, X, \{b, c\} \}$ and $\tau_2 = \{ \emptyset, X, \{a\} \}$. Clearly the space $(X, 1, 2)$ is pairwise $s^{**}g$ - R_1 .

Theorem If $(X, 1, 2)$ is pairwise $s^{**}g$ - R_1 , then it is pairwise $s^{**}g$ - R_0 .

Proof. Suppose that $(X, 1, 2)$ is pairwise $s^{**}g$ - R_1 . Let U be a j - $s^{**}g$ - open set and $x \in U$. If $y \in X - U$, then $y \in X - U$ and $x \in j$ - $s^{**}g$ - $cl(\{y\})$. Therefore, for each point $y \in X - U$, i - $s^{**}g$ - $cl(\{x\}) \neq j$ - $s^{**}g$ - $cl(\{y\})$. Since $(X, 1, 2)$ is pairwise $s^{**}g$ - R_1 , there exist a j - $s^{**}g$ - open set U_y and a i - $s^{**}g$ - open set V_y such that i - $s^{**}g$ - $cl(\{x\}) \cap U_y = \emptyset$, j - $s^{**}g$ - $cl(\{y\}) \cap V_y = \emptyset$ and $U_y \cap V_y = \emptyset$, where $i, j = 1, 2$ and $i \neq j$. Let $A = \{V_y / y \in X - U\}$, then $X - U = \bigcup A$, $x \in A$ and A is j - $s^{**}g$ - open set. Therefore, j - $s^{**}g$ - $cl(\{x\}) \cap X - A = \emptyset$. Hence $(X, 1, 2)$ is pairwise $s^{**}g$ - R_0 .

Theorem A space $(X, 1, 2)$ is pairwise $s^{**}g$ - R_1 if and only if for every pair of points x and y of X such that i - $s^{**}g$ - $cl(\{x\}) \neq j$ - $s^{**}g$ - $cl(\{y\})$, there exists a j - $s^{**}g$ - open set U and i - $s^{**}g$ - open set V such that $x \in V$, $y \in U$ and $U \cap V = \emptyset$, where $i, j = 1, 2$ and $i \neq j$.

Proof. Suppose that (X, τ_1, τ_2) is pairwise $S^{**}g$ -R1. Let x, y be points of X such

that $S^{**}g\text{-cl}(\{x\}) \cap S^{**}g\text{-cl}(\{y\}) = \emptyset$, where $i, j = 1, 2$ and $i \neq j$. Then there exist a

i - $S^{**}g$ -open set U and j - $S^{**}g$ -open set V such that $x \in U$ and $y \in V$ and $U \cap V = \emptyset$.

On the other hand, suppose there exist a j - $S^{**}g$ -open set U and i - $S^{**}g$ -open set V such that $x \in V$, $y \in U$ and $U \cap V = \emptyset$, where $i, j = 1, 2$ and $i \neq j$. Since every pairwise $S^{**}g$ -R1 space is every pairwise $S^{**}g$ -R0, i - $S^{**}g\text{-cl}(\{x\}) \cap j$ - $S^{**}g\text{-cl}(\{y\}) = \emptyset$ from which we infer that i - $S^{**}g\text{-cl}(\{x\}) \cap i$ - $S^{**}g\text{-cl}(\{y\}) = \emptyset$ for $i, j = 1, 2$ and $i \neq j$.

Remark The converse of theorem need not be true in general. The space

(X, τ_1, τ_2) [in Example 2.2.1.] is pairwise $S^{**}g$ -R0 but not pairwise $S^{**}g$ -R1.

Theorem In a bitopological space (X, τ_1, τ_2) the following statements are

equivalent :

(X, τ_1, τ_2) is pairwise $S^{**}g$ -R1

For any two distinct points $x, y \in X$, i - $S^{**}g\text{-cl}(\{x\}) \cap j$ - $S^{**}g\text{-cl}(\{y\}) = \emptyset$ implies that there exists a i - $S^{**}g$ -closed set F_1 and a j - $S^{**}g$ -closed

set F_2 such that $x \in F_1, y \in F_2, x \notin F_2, y \notin F_1$ and $X = F_1 \cup F_2, i, j = 1, 2$ and $i \neq j$.

Proof. (i) \Rightarrow (ii) : Suppose that (X, τ_1, τ_2) is pairwise $S^{**}g$ -R1. Let $x, y \in X$

such that i - $S^{**}g\text{-cl}(\{x\}) \cap j$ - $S^{**}g\text{-cl}(\{y\}) \neq \emptyset$. By Theorem 2.2.1, then there exist

disjoint sets $V = S^{**}gO(X, \tau_i)$, $U = S^{**}gO(X, \tau_j)$ such that $x \in U, y \in V$ and

$U \cap V = \emptyset$, where $i, j = 1, 2$ and $i \neq j$. Then $F_1 = X - V$ is a i - $S^{**}g$ -closed set

and $F_2 = X - U$ is a j - $S^{**}g$ -closed set such that $x \in F_1, x \notin F_2, y \in F_2, y \notin F_1$,

F_2 and $X = F_1 \cup F_2$ where $i, j = 1, 2$ and $i \neq j$.

(ii) \Rightarrow (i) : Let $x, y \in X$ such that i - $S^{**}g\text{-cl}(\{x\}) \cap j$ - $S^{**}g\text{-cl}(\{y\}) \neq \emptyset$ where $i, j = 1, 2$ and $i \neq j$. By (ii), there exists a i - $S^{**}g$ -closed set F_1 and a j - $S^{**}g$ -closed set F_2 such that $X = F_1 \cup F_2, x \in F_1, y \in F_2, x \notin F_2, y \notin F_1$. Therefore, $x \in X - F_2 = U = S^{**}gO(X, \tau_i)$ and $y \in X - F_1 = V = S^{**}gO(X, \tau_j)$

which implies that i - $S^{**}g\text{-cl}(\{x\}) \cap j$ - $S^{**}g\text{-cl}(\{y\}) = \emptyset$ and $U \cap V = \emptyset$ where $i, j = 1, 2$ and $i \neq j$.

Definition A bitopological space X is called pairwise $S^{**}g$ -T0 if for any

pair of distinct points x, y of X , there exists a set which is either i - $S^{**}g$ -open

or τ_j - $S^{**}g$ -open containing one of the points but not the other, where $i, j = 1, 2$ and $i \neq j$.

Theorem A bitopological space X is called pairwise $S^{**}g$ - T_0 if either (X, τ_1) or (X, τ_2) is $S^{**}g$ - T_0 .

Proof. The proof is obvious.

Theorem The product of an arbitrary family of pairwise $S^{**}g$ - T_0 space is pairwise $S^{**}g$ - T_0 .

Proof Let $(X, \tau_1, \tau_2) = \prod_{\alpha \in \Delta} (X_\alpha, \tau_{1\alpha}, \tau_{2\alpha})$, where $(X_\alpha, \tau_{1\alpha}, \tau_{2\alpha})$ are the product topologies on X generated by $\tau_{1\alpha}$ and $\tau_{2\alpha}$ respectively and $X = \prod_{\alpha \in \Delta} X_\alpha$. Let $x = (x_\alpha)$ and $y = (y_\alpha)$ be two distinct points of X . Hence $x_\alpha \neq y_\alpha$ for some α . But $(X_\alpha, \tau_{1\alpha}, \tau_{2\alpha})$ is pairwise $S^{**}g$ - T_0 , there exist either a $\tau_{1\alpha}$ - $S^{**}g$ -open set

U_α containing x_α but not y_α or a $\tau_{2\alpha}$ - $S^{**}g$ -open set V_α containing y_α but not x_α . Define $U = \prod_{\lambda \neq \alpha} (X_\lambda \times U_\alpha)$ and $V = \prod_{\lambda \neq \alpha} (X_\lambda \times V_\alpha)$. Then U is τ_1 - $S^{**}g$ -open and V is τ_2 - $S^{**}g$ -open. Also, U contains x but not y .

Definition A bitopological space X is called pairwise $S^{**}g$ - T_1 if for every distinct points x, y of X , there is a τ_i - $S^{**}g$ -open set U and a τ_j - $S^{**}g$ -open set V such that $x \in U, y \notin U$ and $y \in V, x \notin V$, where $i, j = 1, 2$ and $i \neq j$.

Example Let $X = \{a, b, c\}$, $\tau_1 = \{ \emptyset, X, \{a, c\} \}$ and $\tau_2 = \{ \emptyset, X, \{b, c\} \}$.

Clearly τ_1 - $S^{**}gO(X) = \{ \emptyset, X, \{a\}, \{c\}, \{a, c\} \}$ and τ_2 - $S^{**}gO(X) = \{ \emptyset, X, \{b\}, \{c\}, \{b, c\} \}$. Then the bitopological space (X, τ_1, τ_2) is called pairwise $S^{**}g$ - T_1 .

Remark Since a bitopological space (X, τ_1, τ_2) is pairwise $S^{**}g$ - T_1 if and only if the singletons are τ_j - $S^{**}g$ -closed, it is clear that every pairwise $S^{**}g$ - T_1 is pairwise $S^{**}g$ - R_0 . But the converse is not true in general as it can be seen from the following example:

Example Let $X = \{a, b, c\}$, $\tau_1 = \tau_2 = \{ \emptyset, X, \{a\}, \{b, c\} \}$. It is clear that τ_1 - $S^{**}gO(X) = \tau_2$ - $S^{**}gO(X) = \{ \emptyset, X, \{a\}, \{b, c\} \}$. Then the bitopological space (X, τ_1, τ_2) is pairwise $S^{**}g$ - R_0 but not pairwise $S^{**}g$ - T_1 .

Remark The following example shows that the notions pairwise $S^{**}g$ - T_0 -ness and pairwise $S^{**}g$ - R_0 -ness are independent.

Example Let $X = \{a, b, c, d\}$, $\tau_1 = \tau_2 = \{ \emptyset, X, \{a\}, \{a, b\} \}$. It is clear that

τ_1 - $S^{**}gO(X) = \tau_2$ - $S^{**}gO(X) = \{ \emptyset, X, \{a\}, \{a, b, c\}, \{a, b\}, X \}$. Then the

bitopological space (X, τ_1, τ_2) is pairwise $S^{**}g$ - T_0 but not (X, τ_1, τ_2) is pairwise

$S^{**}g$ - R_0 . Also the bitopological space (X, τ_1, τ_2) as in example is pairwise

$S^{**}g$ - R_0 but not pairwise $S^{**}g$ - T_0 .

Corollary A bitopological space X is pairwise $S^{**}g$ - T_1 iff if it is pairwise $S^{**}g$ - T_0 and pairwise $S^{**}g$ - R_0 .

Lemma If every finite subset of a bitopological space (X, τ_1, τ_2) is τ_j -

$S^{**}g$ closed then it is pairwise $S^{**}g$ - T_1 .

Proof Let $x, y \in X$ such that $x \neq y$. Then by hypothesis, $\{x\}$ and $\{y\}$ are τ_j - $S^{**}g$ - closed sets in X . Hence $X \setminus \{x\}$ and $X \setminus \{y\}$ are τ_i - $S^{**}g$ - open subsets of X such that $x \in X \setminus \{x\}$ and $y \in X \setminus \{y\}$. Therefore, (X, τ_1, τ_2) pairwise $S^{**}g$ - T_1 .

Theorem The product of an arbitrary family of pairwise $S^{**}g$ - T_1 space is pairwise $S^{**}g$ - T_1 .

Proof Let $(X, \tau_1, \tau_2) = \prod_{\alpha \in \Delta} (X_\alpha, \tau_{1\alpha}, \tau_{2\alpha})$, where τ_1 and τ_2 are the product topologies on X generated by $\tau_{1\alpha}$ and $\tau_{2\alpha}$ respectively and $X = \prod_{\alpha \in \Delta} X_\alpha$. Let $x = (x_\alpha)$ and $y = (y_\alpha)$ be two distinct points of X . Hence $x_\alpha \neq y_\alpha$ for some

But $(X_\lambda, \tau_{1\lambda}, \tau_{2\lambda})$ is pairwise $S^{**}g$ - T_1 , there exist a $\tau_{1\lambda}$ - $S^{**}g$ - open set U containing x_λ but not y_λ and there exist a $\tau_{2\lambda}$ - $S^{**}g$ - open set V_α containing y_λ but not x_λ . Define $U = \prod_{\lambda \neq \alpha} (X_\lambda \times U_\alpha)$ and $V = \prod_{\lambda \neq \alpha} (Y_\lambda \times V_\alpha)$. Then U is τ_1 - $S^{**}g$ - open set and V is τ_2 - $S^{**}g$ - open set. Also, U contains x but not y and V contains y but not x .

Theorem A bitopological space X is called pairwise $S^{**}g$ - T_1 if either

(X, τ_1) or (X, τ_2) is $S^{**}g$ - T_1 .

Proof. Let (X, τ_1, τ_2) be pairwise $S^{**}g$ - T_1 space. Let x, y be two distinct points

of X , then there exists a τ_1 - $S^{**}g$ - open set U such that $x \in U, y \notin U$. Thus,

(X, τ_1) is $S^{**}g$ - T_1 . Similarly, (X, τ_2) is $S^{**}g$ - T_1 . Converse is obvious.

Definition A bitopological space X is called pairwise $S^{**}g$ - T_2 or

pairwise $S^{**}g$ - Hausdorff if given distinct points x, y of X , there is a τ_i - $S^{**}g$

- open set U and a τ_j - $S^{**}g$ - open set V such that $x \in U, y \in V, U \cap V = \emptyset$

where $i, j = 1, 2$ and $i \neq j$.

Corollary A bitopological space X is pairwise $S^{**}g$ - T_2 iff if it is pairwise $S^{**}g$ - T_1 and pairwise $S^{**}g$ - R_1 .

Theorem Every pairwise $S^{**}g$ - T_2 space is pairwise $S^{**}g$ - T_1 space. **Proof.** Let X is pairwise $S^{**}g$ - T_2 space. Since X is pairwise $S^{**}g$ - T_2 space,

there exists a τ_i - $S^{**}g$ - open set U and a τ_j - $S^{**}g$ - open set V such that x

$x \in U, y \in V, U \cap V = \emptyset$, where $i, j = 1, 2$ and $i \neq j$. $x \in U$, but $y \notin U$ and $y \in V$

but $x \notin V$. X is pairwise $S^{**}g$ - T_1 space, there is a τ_i - $S^{**}g$ - open set U and

a τ_j - $S^{**}g$ - open set V such that $x \in U, y \in V, U \cap V = \emptyset$, where $i, j =$

$1, 2$ and $i \neq j$.

In general the converse of the above theorem need not be true and it can be seen from the following example.

Example Let $X = \{a, b, c\}$, $\tau_1 = \{ \emptyset, X, \{a, c\} \}$, $\tau_2 = \{ \emptyset, X, \{b, c\} \}$. Clearly the bitopological space (X, τ_1, τ_2) is pairwise $S^{**}g$ - T_1 but not pairwise $S^{**}g$ - T_2 .

Remark Every pairwise $S^{**}g$ - T_1 space is pairwise $S^{**}g$ - T_0 .

Theorem If a space (X, τ_1, τ_2) is pairwise $S^{**}g$ - T_2 , then it is pairwise $S^{**}g$ - R_1 .

Proof. Let (X, τ_1, τ_2) be pairwise $S^{**}G$ - T2. Then for any two distinct points $x,$

y of X , their exist a τ_i - $S^{**}G$ - open set U and a τ_j - $S^{**}G$ - open set V such that x

$U, y \in V$ and $U \cap V = \emptyset$ where $i, j = 1, 2$ and $i \neq j$. If (X, τ_1, τ_2) is pairwise $S^{**}G$ - T1, then τ_i - $S^{**}G$ - $\text{cl}(\{x\})$ and τ_j - $S^{**}G$ - $\text{cl}(\{y\})$ and thus τ_i

$S^{**}G$ - $\text{cl}(\{x\}) \neq \tau_j$ - $S^{**}G$ - $\text{cl}(\{y\})$, where $i, j = 1, 2$ and $i \neq j$. Thus for any

distinct pair of points x, y of X such that τ_i - $S^{**}G$ - $\text{cl}(\{x\}) \neq \tau_j$ - $S^{**}G$ - $\text{cl}(\{y\})$ where $i, j = 1, 2$ and $i \neq j$, there exists a τ_j - $S^{**}G$ - open set U and τ_i - $S^{**}G$ - open

set V such that $x \in V, y \in U$ and $U \cap V = \emptyset$ where $i, j = 1, 2$ and $i \neq j$. Hence (X, τ_1, τ_2) is pairwise $S^{**}G$ - R1.

Remark The converse of the above theorem is not true in general that is pairwise $S^{**}G$ - R1 space is not pairwise $S^{**}G$ - T2 space.

Remark If a bitopological space X pairwise $S^{**}G$ - T_i , then it is pairwise $S^{**}G$ - $T_{i-1}, i = 1, 2$.

Definition Let X be a bitopological space. Then τ_i is $S^{**}G$ - regular w.r.to

τ_j if for each point x in X and each τ_i - $S^{**}G$ - closed set P such that $x \in P$ there

is a τ_i - $S^{**}G$ - open set U and a τ_j - $S^{**}G$ - open set V disjoint from U such that

$x \in U$ and $P \cap V = \emptyset$. X is pairwise $S^{**}G$ - regular if τ_i is $S^{**}G$ - regular w.r.to τ_j and

τ_j is $S^{**}G$ - regular w.r.to τ_i .

Remark A pairwise $S^{**}G$ - regular space need not be a pairwise $S^{**}G$ - T1 space as seen by next example.

Example Let $X = \{a, b, c\}, \tau_1 = \{ \emptyset, X, \{a\} \}, \tau_2 = \{ \emptyset, X, \{b, c\} \}$. Clearly the bitopological space (X, τ_1, τ_2) is pairwise $S^{**}G$ - regular but not a pairwise $S^{**}G$ - T1 space. Since $\{b\}$ is not τ_2 - $S^{**}G$ - closed.

Definition X is pairwise $S^{**}G$ - T3 if it is pairwise $S^{**}G$ - regular and pairwise $S^{**}G$ - T1.

Remark Pairwise $S^{**}G$ - T3 \implies Pairwise $S^{**}G$ - T2.

Theorem Every pairwise $S^{**}G$ - T0, pairwise $S^{**}G$ - regular space is pairwise $S^{**}G$ - T1 and hence pairwise $S^{**}G$ - T3.

Example Let X be a pairwise $S^{**}G$ - T3 space. Then X is also a pairwise

$S^{**}G$ - T2 space. Let $a, b \in X$. Since X is a pairwise $S^{**}G$ - T1 space, $\{a\}$ is a τ_i -

$S^{**}G$ - closed set. Since a and b are distinct. By pairwise $S^{**}G$ - regularity,

τ_i - $S^{**}G$ - open set U and a τ_j - $S^{**}G$ - open set V such that $\{a\} \cap U = \emptyset$ and $b \in V$.

Hence X is pairwise $S^{**}G$ - T3.

Definition A bitopological space X is called pairwise $S^{**}G$ - Urysohn, if for any two points x and y of X such that $x \neq y$, there exists a τ_i - $S^{**}G$ - open set

U and a τ_j - $S^{**}G$ - open set V such that $x \in U, y \in V, \tau_i$ - $S^{**}G$ - $\text{cl}(U) \cap \tau_j$ - $S^{**}G$ - $\text{cl}(V) = \emptyset$ where $i, j = 1, 2$ and $i \neq j$.

Example Let $X = \{a, b, c\}, \tau_1 = \{ \emptyset, X, \{a\} \}$ and $\tau_2 = \{ \emptyset, X, \{a\}, \{b, c\} \}$.

It is clear that τ_i - $S^{**}G$ -open set $\{a, c\}$, $\{c\}$ and τ_j - $S^{**}G$ -open sets are

$\{b, c\}$, $\{a\}$. Then the bitopological space X is called pairwise $S^{**}G$ -

Urysohn.

Remark Obviously, pairwise $S^{**}G$ - T_3 pairwise $S^{**}G$ -Urysohn pairwise $S^{**}G$ - T_2 .

Definition X is said to be pairwise $S^{**}G$ -normal if for each τ_i - $S^{**}G$ -

closed set A and τ_j - $S^{**}G$ -closed set B with $A \cap B = \emptyset$, there exists a τ_i - $S^{**}G$

open set $V \supset B$ and there exists a τ_j - $S^{**}G$ -open set $U \supset A$ such that $U \cap V = \emptyset$, where $i, j = 1, 2$ and $i \neq j$.

Example Let $X = \{a, b, c\}$, $\tau_1 = \tau_2 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Clearly the bitopological space (X, τ_1, τ_2) is pairwise normal but not pairwise $S^{**}G$ -normal as well as pairwise $S^{**}G$ -regular.

Definition A pairwise $S^{**}G$ -normal, pairwise $S^{**}G$ - T_1 space is called pairwise $S^{**}G$ - T_4 space.

Example Let X be a pairwise $S^{**}G$ - T_4 space. Then X is also a pairwise

$S^{**}G$ - T_3 space. Suppose that F is a τ_j - $S^{**}G$ -closed subset of X and $p \in X$

does not belong to F . Since X is a pairwise $S^{**}G$ - T_1 space, $\{p\}$ is a τ_j - $S^{**}G$ -

closed set. Since F and $\{p\}$ are disjoint. By pairwise $S^{**}G$ -normality, τ_i -

$S^{**}G$ -open set G and a τ_j - $S^{**}G$ -open set H such that $F \cap G = \emptyset$ and $p \in H$.

Hence X is pairwise $S^{**}G$ - T_4 .

Definition A bitopological space X is said to be a pairwise $S^{**}G$ -completely normal provided that whenever A and B are subsets of X such that

τ_i - $S^{**}G$ - $\text{cl}(A) \cap B = \emptyset$ and $A \cap \tau_j$ - $S^{**}G$ - $\text{cl}(B) = \emptyset$ there exists a τ_j - $S^{**}G$ -

open set U and a τ_i - $S^{**}G$ -open set V such that $A \subset U$, $B \subset V$, $U \cap V = \emptyset$,

where $i, j = 1, 2$ and $i \neq j$.

Definition A pairwise $S^{**}G$ - T_1 space, pairwise $S^{**}G$ -completely normal bitopological space is called pairwise $S^{**}G$ - T_5 space.

Theorem Every pairwise $S^{**}G$ -completely normal space is pairwise $S^{**}G$ -normal.

Proof. Let X be a pairwise $S^{**}G$ -completely normal bitopological space. Let

A be a τ_i - $S^{**}G$ -closed set and B be a τ_j - $S^{**}G$ -closed set such that $A \cap B = \emptyset$.

. Then τ_i - $S^{**}G$ - $\text{cl}(A) \cap B = A \cap B = \emptyset$ and $A \cap \tau_j$ - $S^{**}G$ - $\text{cl}(B) = A \cap B = \emptyset$.

. By complete $S^{**}G$ -normality, there exists a τ_j - $S^{**}G$ -open set u and a τ_i -

$S^{**}G$ -open set V such that $A \subset U$, $B \subset V$, $U \cap V = \emptyset$. Hence X is pairwise $S^{**}G$ -normal.

Theorem Every pairwise completely normal space is pairwise $S^{**}G$ -completely normal.

Proof. The proof is obvious.

Theorem If a bitopological space (X, τ_1, τ_2) is pairwise $S^{**}g$ - completely

normal then every subspace is pairwise $S^{**}g$ - normal.

Proof. Let (X, τ_1, τ_2) be pairwise $S^{**}g$ - completely normal and (Y, τ_1, τ_2) be

a subspace. Let F_1 and F_2 be disjoint $S^{**}g$ - closed in τ_1 and τ_2 respectively.

F_1 is τ_1 - $S^{**}g$ - closed $F = \tau_1 - S^{**}gcl(F_1)$. Then $F_1 \cap \tau_2 - S^{**}gcl(F_2) = \tau_1$

$S^{**}gcl(F_1) \cap \tau_2 - S^{**}gcl(F_2) = (Y \cap \tau_1 - S^{**}gcl(F_1)) \cap \tau_2 - S^{**}gcl(F_2) = \tau_2$

$S^{**}gcl(F_2) \cap \tau_1 - S^{**}gcl(F_1) = F_1 \cap F_2 = \emptyset$. Similarly we can show that, $\tau_1 - S^{**}gcl(F_1) \cap F_2 = \emptyset$. Thus F_1, F_2 is a pairwise $S^{**}g$ - separated pair of X . By pairwise $S^{**}g$ - complete normality, there exists disjoint sets $G_1 \cap \tau_1$ and $G_2 \cap \tau_2$ such that, $F_2 \cap \tau_1 \subseteq G_1, F_1 \cap \tau_2 \subseteq G_2$. Then $F_2 \cap Y \subseteq G_1, F_1 \cap Y \subseteq G_2, (Y \cap G_1) \cap (Y \cap G_2) = \emptyset$ and $Y \cap G_1 \cap \tau_1, Y \cap G_2 \cap \tau_2$. Hence (Y, τ_1, τ_2) is pairwise $S^{**}g$ - normal.

Definition A subset A of a space (X, τ_1, τ_2) is said to be bi - $S^{**}g$ - open if it is both i - $S^{**}g$ open and j - $S^{**}g$ open, where $i, j = 1, 2$ and $i \neq j$.

Theorem Every pairwise $S^{**}g$ - closed, pairwise $S^{**}g$ - continuous image of a pairwise $S^{**}g$ - normal space is pairwise $S^{**}g$ - normal. on to

Proof. Let (X, τ_1, τ_2) be a pairwise $S^{**}g$ - normal space. Let $f : (X, \tau_1, \tau_2) \rightarrow$

(Y, τ_1^*, τ_2^*) be a pairwise $S^{**}g$ - closed, pairwise $S^{**}g$ - continuous mapping. Let A and B be two disjoint subsets of Y , where A is τ_1^* - $S^{**}g$ - closed and B is

τ_2^* - $S^{**}g$ - closed. Then $f^{-1}(A)$ is τ_1 - $S^{**}g$ - closed and $f^{-1}(B)$ is τ_2 - $S^{**}g$ - closed. Also $A \cap B = f^{-1}(A \cap B) = f^{-1}(\emptyset) = \emptyset$. Since X is pairwise $S^{**}g$

- normal, there exists disjoint sets G_A and G_B such that $f^{-1}(A) \subseteq G_A, f^{-1}(B) \subseteq G_B$,

where G_A is τ_2 - $S^{**}g$ - open and G_B is τ_1 - $S^{**}g$ - open. Let $G_A^* = \{y : f^{-1}(y) \subseteq G_A\}$

and $G_B^* = \{y : f^{-1}(y) \subseteq G_B\}$. Then $G_A^* \cap G_B^* = \emptyset, A \subseteq G_A^*, B \subseteq G_B^*$ and since

$G_A^* = Y - f(X - G_A), G_B^* = Y - f(X - G_B)$. Here G_A^* is τ_2^* - $S^{**}g$ - open and G_B^*

is τ_1^* - $S^{**}g$ - open. Hence (Y, τ_1^*, τ_2^*) is pairwise $S^{**}g$ - normal.

Theorem Every bi - $S^{**}g$ - closed subspace of a pairwise $S^{**}g$ - normal space is pairwise $S^{**}g$ - normal.

Proof. Let (Y, τ_1, τ_2) be a bi - $S^{**}g$ closed subspace of a pairwise $S^{**}g$ - normal

space (X, τ_1, τ_2) . Let A be a i - $S^{**}g$ - closed set and B be a j - $S^{**}g$ - closed

set disjoint from A . Since the space Y is bi - $S^{**}g$ - closed, A is i - $S^{**}g$ closed

and j - $S^{**}g$ - closed, where $i, j = 1, 2$ and $i \neq j$. By pairwise $S^{**}g$ - normality

of (X, τ_1, τ_2) , there exists a j - $S^{**}g$ - open set U and a i - $S^{**}g$ - open set V

such that $A \subseteq U, B \subseteq V, U \cap V = \emptyset$. Thus, $A \cap Y \subseteq U \cap Y$ and $B \cap Y \subseteq V \cap Y$

$V \cap Y \cap (U \cap Y) \cap (V \cap Y) = \emptyset$. Also, $U \cap Y$ is j - $S^{**}g$ - open and $V \cap Y$ is i - $S^{**}g$ - open. Thus, there exists i - $S^{**}g$ - open set $V \cap Y$ and a j - $S^{**}g$ - open set $U \cap Y$ such that $A \cap Y \subseteq (U \cap Y), B \cap Y \subseteq (V \cap Y), (U \cap Y) \cap (V \cap Y) = \emptyset$.

Hence (Y, τ_1, τ_2) is pairwise $S^{**}g$ - normal.

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