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Research Paper

The g - Semi Inverses of Interval Valued Fuzzy S0ft Matrices

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Abstract:*In this paper, we discuss various K-g-Semi inverses associated with a K-regular interval- valued fuzzy soft matrix, the existence and contraction of Semi-Inverse, {1,2} inverses of Interval Valued fuzzy matrix are determined in terms of the row and column spaces.*

Keywords:*Fuzzy soft matrix, Interval Valued fuzzy soft Matrix.(IVFSM), Generalised inverse (g-inverse) of Interval Valued fuzzy soft Matrix .*

I. Introduction

We deal with interval – valued fuzzy matrices (IVFM) that is, matrices whose entries are intervals and all the intervals are subintervals of the interval [0,1]. Recently the concept of IVFM a generalization of fuzzy matrix was introduced and developed by shymal and pal[5], by extending the max. min operations on fuzzy algebra F=[0,1], for elements, a, b $\in$F, a+b= max{a,b} and a-b=min{a,b}. In [2], Meenakshi and Kaliraja have represented and IVFM as an interval matrix of its lower and upper limit fuzzy matrices. In [3], Meenakshi and Jenita have introduced the concept of K-regular fuzzy matrix and discussed about inverses associated with a K-regular fuzzy matrix as a generalization of results on regular fuzzy matrix developed in [1]. A matrix A $\in F_n$, the set of all nxn fuzzy matrices is said to be right(left) K-regular if there exists x(y)$\in F_n$, such that A^k x A = A^k(AYA^k=A^k) , x(y) is called a right(left) K-g inverses of A, Where K is a positive Integer. Recently we have expended characterization of various inverses of K-Regular interval – valued fuzzy matrices in [6].In [13] P.Rajesajeswari and P.Dhanalakshmi have introduced Interval Valued fuzzy soft matrix , its types with examples and some new operation. In [14] Dr.N.Sarala and M.Prabhavathi have introduced the concept of generalized inverse (g-inverse) for $IVFSM_S$.

In this paper, we discuss various g-Semi inverses of K- regular interval-valued fuzzy soft matrices. In section 2, some basic definitions and results needed are given. In section 3, characterization of various Semi K-inverses of K-regular IVFM are determined.

II. PRELIMINARIES

In this section, some basic definitions and results needed are given Let $(IVFM)_n$ denotes the set of nxn Interval-valued fuzzy sot matrices.

Definition 2.1

Let U ={c_1,c_2,c_3…c_m} be the Universe set and E be the set of parameters given by E ={e_1,e_2,e_3…e_n}. Let A ⊆ E and (F,A) be a fuzzy soft set in the fuzzy soft class (U,E). Then fuzzy soft set (F,A) in a matrix form as A_{mxn} =$[a_{ij}]_{mxn}$ or A=$[a_{ij}]$ i=1,2,…m, j=1,2,3,…n

$$\text{Where } a_{ij} = \begin{cases} \mu_j(c_i) & \text{if } e_j \in A \\ 0 & \text{if } e_j \notin A \end{cases}$$

$\mu_j(c_i)$ represents the membership of c_i in the fuzzy set $F(e_j)$.

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Interval valued fuzzy soft set 2.2

Let U be an initial Universe set and E be the set of parameters, let A ⊆ E. A pair (F,A) is called Interval valued fuzzy soft set over U where F is a mapping given by F: $A \rightarrow I^U$, where I^U denotes the collection of all Interval valued fuzzy subsets of U.

Interval valued fuzzy soft matrix 2.4

Let U ={$c_1,c_2,c_3\ldots c_m$} be the Universe set and E be the set of parameters given by E ={$e_1,e_2,e_3\ldots e_n$}. Let A ⊆ E and (F,A) be a interval valued fuzzy soft set over U, where F is a mapping given by F: $A \rightarrow I^U$, where I^U denotes the collection of all Interval valued fuzzy subsets of U. Then the Interval valued fuzzy soft set can expressed in matrix form as

$\tilde{A}_{mxn}=[a_{ij}]_{mxn}$ or $\tilde{A} = [a_{ij}]$ i= 1,2,…m, j=1,2,…n

$$\text{Where } a_{ij} = \begin{cases} [\mu_{jL}(c_i), \mu_{jU}(c_i)] & \text{if } e_j \in A \\ [0,0] & \text{if } e_j \notin A \end{cases}$$

Lemma 2.1

For $\tilde{A} = [\tilde{A}_L , \tilde{A}_U]$ Є $(IVFSM)_{mn}$ and $\tilde{B} = [\tilde{B}_L ,\tilde{B}_U]$Є $(IVFSM)_{np}$, The following hold.

(i) $\tilde{A}^T = [\tilde{A}_L^T , \tilde{A}_U^T]$

(ii) $\tilde{A}\tilde{B} = [\tilde{A}_L \tilde{B}_L , \tilde{A}_U \tilde{B}_U]$

(iii) $\tilde{A} + \tilde{B} = [\tilde{A}_L + \tilde{B}_L, \tilde{A}_U + \tilde{B}_U]$

(iv) $[\alpha+\beta](\tilde{A} + \tilde{B}) = [\alpha\tilde{A}_L +\beta \tilde{B}_L ,\alpha \tilde{A}_U+\beta \tilde{B}_U]$, for $\alpha \leq \beta$ Є FSM

(v) $[\alpha ,\beta]\tilde{A} = [\alpha\tilde{A}_L , \beta\tilde{A}_U]$, for $\alpha \leq \beta$ Є FSM

(vi) $\alpha\tilde{A} = [\alpha\tilde{A}_L ,\alpha \tilde{A}_U]$, for α Є FSM

Theorem :2.1

Let $\tilde{A} = [\tilde{A}_L , \tilde{A}_U]$ Є $(IVFSM)_{mn}$ .Then Ã is regular IVFSM ⟺ $\tilde{A}_L$ and $\tilde{A}_U$Є FSM_{mn} are regular.

Theorem : 2.2

Let $\tilde{A} = [\tilde{A}_L , \tilde{A}_U]$ be an $(IVFSM)_{mn}$

Then

(i) $\mathcal{R}(A) = [\mathcal{R}(\tilde{A}_L),\mathcal{R}(\tilde{A}_U)]$ Є $(IVFSM)_{In}$

(ii) $\mathcal{C}(A) = [\mathcal{C}(\tilde{A}_L), \mathcal{C}(\tilde{A}_U)]$ Є $(IVFSM)_{Im}$

Theorem : 2.3

For Ã , B̃ Є $(IVFSM)_{mn}$.

(i) $\mathcal{R}(\tilde{B}) \leq (\tilde{A}) \Leftrightarrow \tilde{B} = \tilde{X}\tilde{A}$ for some x Є $(IVFSM)_m$.

(ii) $\mathcal{C}(\tilde{B}) \leq (\tilde{A}) \Leftrightarrow \tilde{B} = \tilde{A}\tilde{Y}$ for some Y Є $(IVFSM)_n$.

Theorem : 2.4

For Ã Є $(IVFSM)_{mn}$, B̃ Є $(IVFSM)_{mn}$. the following hold .

$\mathcal{R}(\tilde{A}\tilde{B}) \subseteq \mathcal{R}(\tilde{A})$ are $\mathcal{C}(\tilde{A}\tilde{B}) \subseteq (\tilde{B})$.

Definition : 2.4

For Ã Є $(IVFSM)_{mn}$if there exists X̃Є $(IVFSM)_{mn}$ such that.

(i) $\tilde{A}\tilde{X}\tilde{A} = \tilde{A}$

(ii) $\tilde{X}\tilde{A}\tilde{X} = \tilde{X}$

(iii) $(\tilde{A}\tilde{X})^T = (\tilde{A}\tilde{X})$

(iv) $(\tilde{X}\tilde{A})^T = (\tilde{X}\tilde{A})$,Then X̃ is called a g- inverse of A .

Theorem : 2.5

Let A Є $(IVFSM)_{mn}$and X̃Є A {1} Then X̃ЄÃ {2} if and only if $\mathcal{R}(\tilde{A}\tilde{X}) = \mathcal{R}(\tilde{X})$.

Theorem : 2.6

For Ã Є $(IVFSM)_{mn}$Ã has a {1,3} inverse of and only if $\tilde{A}^T A$ is a regular IVFSM and $\mathcal{R}(\tilde{A}^T A) = \mathcal{R}(\tilde{A})$.

III. g – SEMI INVERSES OF K - REGULAR INTERVAL VALUED FUZZY SOFT MATRICES

In this section, we shall introduce the concept of semi – g inverses associated with a k regular IVFM as an extension of k – g inverses of a k – regular fuzzy matrix [3] and as a generalization of generalized inverses of a regular IVFM [2].

Definition: 3.1.

Ã matrix $\tilde{A} \in IVFSM_n$ is said to be right k – regular if there exists $\tilde{X} \in (IVFSM)_n$ such that $\tilde{A}^k \tilde{X} \tilde{A} = \tilde{A}^k$ and $\tilde{X}^k \tilde{A} \tilde{X} = \tilde{X}^k$. For some positive integer k . then $\tilde{X}^k$ is called a k – semi inverses (or) $\{1_r^k, 2_r^k\}$ inverses of Ã. set of $\{1_r^k, 2_r^k\}$ inverses of Ã is denoted as $\tilde{A}\{1_r^k, 2_r^k\} = \{ \tilde{X} / \tilde{A}^k \tilde{X} \tilde{A} = \tilde{A}^k$ and $\tilde{X}^k \tilde{A} \tilde{X} = \tilde{X}^k\}$.

Definition: 3.2.

A matrix $\tilde{A} \in$ IVFSM, is said to be left –k – regular if there exists $\tilde{X} \in (IVFM)_n$ such that $\tilde{A} \tilde{X} \tilde{A}^k = \tilde{A}^k$ and $\tilde{X} \tilde{A} \tilde{X}^k = \tilde{X}^k$, for some the integer k. then X̃ is called a k – semi – inverses (or) $\{1_\ell^k, 2_\ell^k\}$ inverses of Ã. set of $\{1_\ell^k, 2_\ell^k\}$ inverse of Ã is denoted as $\tilde{A}\{1_\ell^k, 2_\ell^k\} = \{ x / \tilde{A} \tilde{X} \tilde{A}^K = \tilde{A}^k$ and $\tilde{X} \tilde{A} \tilde{X}^K = \tilde{X}^k\}$.

Theorem: 3.1.

Let $\tilde{A} = [\tilde{A}_L, \tilde{A}_U] \in (IVFSM)_n$. Then Ã has a $\{1_r^k\}$ and $\{2_r^k\}$ inverses <=> $\tilde{A}_L$ and $\tilde{A}_U \in F_n$ have $\{1_r^k\}$and $\{2_r^k\}$ inverses.

Proof :

Let $\tilde{A}_U = [\tilde{A}_L, \tilde{A}_U] \in (IVFSM)_n$ and $\tilde{X} = [\tilde{X}_L, \tilde{X}_U] \in (IVFSM)_n$.

Since, Ã has a$\{1_r^k\}$ and $\{2_r^k\}$ inverse, then

Three exists $\tilde{X} \in IVFSM_n$ such that,

$\tilde{A}^k \tilde{X} \tilde{A} = \tilde{A}^k$ and $\tilde{X}^k \tilde{A} \tilde{X} = \tilde{X}^k$.

Then by lemma $\Leftrightarrow [\tilde{A}_L^k \tilde{X}_L \tilde{A}_L, \tilde{A}_U^k \tilde{X}_L \tilde{A}_U] = [\tilde{A}_L^k, \tilde{A}_U^k]$ and $[\tilde{X}_L^k \tilde{A}_L \tilde{X}_L, \tilde{X}_U^k \tilde{A}_U \tilde{X}_U] = [\tilde{X}_L^k, \tilde{X}_U^k]$

$\Leftrightarrow \tilde{A}_L^k \tilde{X}_L \tilde{A}_L = \tilde{A}_L^k$ and $\tilde{A}_L^k \tilde{X}_U \tilde{A}_U = \tilde{A}_U^k$;$\tilde{X}_L^k \tilde{A}_L \tilde{X}_L = \tilde{X}_L^k$ and $\tilde{X}_U^K \tilde{A}_U \tilde{X}_U = \tilde{X}_U^k$;

Hence, $\tilde{A} \in (IVFSM)_n$ has a $\{1_r^k\}$ and $\{2_r^k\}$ are inverses

$\Leftrightarrow$ $\tilde{A}_L$ and $\tilde{A}_U$ have $\{1_r^k\}$ and $\{2_r^k\}$ inverses.

Theorem: 3.2

Let $\tilde{A} \in (IVFSM)_m$ and k be a positive integer $\tilde{X} \in \tilde{A}\{1_r^k\} \Leftrightarrow \tilde{X}^T \in \tilde{A}^T\{1_r^k\}$ and $\tilde{X} \in \tilde{A}\{2_r^k\} \Leftrightarrow \tilde{X}^T \in \tilde{A}^T\{2_\ell^k\}$

Proof:

$\tilde{X} \in \tilde{A}\{1_r^k\} \Leftrightarrow \tilde{A}^k \tilde{X} \tilde{A} = \tilde{A}^k$ and

$\Leftrightarrow (\tilde{A}^k X \tilde{A})^T = (\tilde{A}^k)^T$

$\Leftrightarrow \tilde{A}^T X^T (\tilde{A}^T)^k = (\tilde{A}^k)^T$

$\Leftrightarrow \tilde{X}^T \in \tilde{A}^T\{1_\ell^k\}$

and

$\tilde{X} \in \tilde{A}\{2_r^k\} \Leftrightarrow \tilde{X}^k \tilde{A} \tilde{X} = \tilde{A}^k$

$\Leftrightarrow (\tilde{X}^k \tilde{A} \tilde{X})^T = (\tilde{A}^k)^T$

$\Leftrightarrow \tilde{X}^T \tilde{A}^T (\tilde{X}^T)^K = (\tilde{X}^T)^K$

$\Leftrightarrow \tilde{X}^T \in \tilde{A}^T\{2_\ell^k\}$

Theorem: 3.3.

(i) Let $\tilde{A} \in (IVFSM)_{mxn}$ and $\tilde{X} \in A\{1_r^k\}$ then

$X \in \tilde{A}\{2_r^k\}$ if and only if $(A^k \tilde{X}) = (\tilde{X})$

(ii) Let $\tilde{A} \in (IVFSM)_{mxn}$ and $X \in \tilde{A}\{1_r^k\}$ then

$X \in \tilde{A}\{2_\ell^k\}$ if and only if $(\tilde{X}) = \mathcal{C}(\tilde{X}\tilde{A}^k)$

Proof:

Since $\tilde{A} = [\tilde{A}_L, \tilde{A}_U]$ and $\tilde{X} = [\tilde{X}_L \tilde{X}_U] \in$ (IVFSM)

$\tilde{X} \in \tilde{A}\{2_r^k\} \Leftrightarrow X^K \tilde{A} \tilde{X} = \tilde{X}^K$ then by eqn(3)

$\Leftrightarrow \tilde{X}_L^K \tilde{A}_L X_L = \tilde{X}_L^K$ and $\tilde{X}_U^K \tilde{A}_U \tilde{X}_U = \tilde{X}_U^K$,

$\tilde{X}_L \in \tilde{A}_L\{2_r^k\}$ and $\tilde{X}_U \in \tilde{A}_U\{2_r^k\}$

$\Leftrightarrow \tilde{A}_L \in \tilde{X}_L\{1_r^k\}$ and $\tilde{A}_U \in \tilde{X}_U\{1_r^k\}$.

$\Leftrightarrow (\tilde{X}_L) = (\tilde{A}_L^K \tilde{X}_L)$ and $(\tilde{X}_U) = \mathcal{R}(\tilde{A}_U^K \tilde{X}_U)$

$\Leftrightarrow (\tilde{A}^K \tilde{X}) = (\tilde{X})$ (by lemma:2.4)

$\Leftrightarrow (\tilde{A}^K \tilde{X}) = (\tilde{X})$.

Conversely,

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Let $(\tilde{A}^K\tilde{X}) = (\tilde{X})$ then by lemma(2.4)
$(\tilde{X}) \leq (\tilde{A}^K\tilde{X})$
$\Leftrightarrow \tilde{X} = \tilde{Y}\tilde{A}^K\tilde{X}$ for some $\tilde{Y} \in IVFSM_{mxn}$
$\tilde{X}\tilde{A}\ \tilde{X}^K = \tilde{Y}\tilde{A}^K\ \tilde{X}\tilde{A}\ \tilde{X}^K$
$= \tilde{Y}\tilde{A}^K\tilde{X}^K$
$= \tilde{Y}\tilde{A}^K\tilde{X}^K$
$= \tilde{Y}\tilde{A}^K\tilde{X}\ \tilde{X}^{K-1}$
$= \tilde{X}\ \tilde{X}^{K-1}$
$= \tilde{X}^K$
Then $\tilde{X} \in \tilde{A}\ \{2_r^k\}$
(ii). Proof in similar to (i) and here omitted.

Corollary 3.2.1:
For $\tilde{A} \in IVFSM_{mxn}$, if $\tilde{X} \in \tilde{A}\ \{1_r^k, 2_r^k\}$ then
$\rho(\tilde{A}) = \rho(\tilde{X})$
Proof:
Since $\tilde{X} \in \tilde{A}\ \{1_r^X\}$ $\tilde{A}$ is a right k – regular fuzzy soft matrix. $\tilde{A}^k\tilde{X}$ is regular, being idempotent.
.: By theorem 2.2 $\rho_r(\tilde{A}) = \rho_r(\tilde{A}^k\tilde{X})$ since $\tilde{X} \in \tilde{A}\ \{1_r^k, 2_r^k\}$
By theorem 3.2 $(\tilde{A}^k\tilde{X}) = \mathcal{R}(\tilde{X})$
.: $\rho_r(\tilde{A}) = \rho_r(\tilde{A}^k\tilde{X}) = \rho_r(\tilde{X})$.
Hence the corollary.

REFERENCES

[1]. Kim, K.H. and Roush, F.N., Generalized fuzzy matrices, Fuzzy sets and systems, 4(1980), 293 – 315.
[2]. Meenakshi, AR., and Kaliraja,M., Regular Interval – valued Fuzzy Matrices, Advances in Fuzzy Mathematics, 5 (1)(2010), 7 – 15.
[3]. Meenakshi, AR., and Jenita, p., Generalized Regular Fuzzy Matrices, Iranian Journal of fuzzy Systems, 8 (2), (2011) 133 – 141.
[4]. Meenakshi, AR., and Poongodi, P., Generalized Regular Interval Valued Fuzzy Matrices, International Journal of Fuzzy Mathematics and Systems, 2 (1) (2012), 29 – 36.
[5]. Shyamal. A.K. and Pal.M., Interval – Valued Fuzzy Matrices, Journal of Fuzzy Mathematics, 14 (3) (2006) 582 – 592.
[6]. Meenakshi, AR., and Kaliraja,M., g-inverses of Interval – valued Fuzzy Matrices, j in Fuzzy Mathematics,vol.45 ,No.1,2013,83-92
[7]. Meenakshi, AR., and Kaliraja,M.,inverses of k- Regular Interval – valued Fuzz Matrices, Advances in Fuzzy Mathematics,
[8]. D.r.N.Sarala and M.Prabhavathi an application of Interval valued fuzzy soft matrixin medical Diagnosis. IOSR Journal of Mathematics Vol.11,Issue 1 ver.VI (Jan-Feb.2015), pp: 1-6
[9]. D.r.N.Sarala and M.Prabhavathi An a Application of Interval Valued Fuzzy soft matrix in decision making problemInternational Journal of Mathematics trends and technology(IJMTT) v 21 (1):1-10May2015.
[10]. D.r.N.Sarala and M.PrabhavathiRegular Interval Valued Fuzzy soft matrix International Journal of Science and Research (IJSR) ISSN (Online): 2319-7064 Index Copernicus Value (2013): 6.14 | Impact Factor (2014): 5.611
[11]. P. RajarajeswariandP.Dhanalakshmi Interval - valued fuzzy soft matrix theory, Annals of Pure and Applied Mathematics, Vol. 7 (2014) 61 – 72.
[12]. D.r.N.Sarala and M.PrabhavathiRegular Interval Valued Fuzzy soft matrix International Journal of jamal academic research Feb (201657-62.